Existence of Bifurcation in Macroeconomic Dynamics: Grandmont Was Right

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Extended Abstract

Motivation: Grandmont (1985) found that the parameter space of even the simplest, most classical models are stratified into bifurcation regions. But by demonstrating that fact with a classical model in which all policies are Ricardian equivalent and all solutions are Pareto optimal, he was not able to reach conclusions about the policy relevance of his dramatic discovery. The result was years of controversy, as evidenced by papers appearing in Barnett, Deissenberg, and Feichtinger (2004) and Barnett, Geweke, and Shell (2005).

Barnett and He (1999,2002) subsequently found transcritical, codimension two, and Hopf bifurcation boundaries within the parameter space of the policy relevant Bergstrom and Wymer continuous-time dynamic macroeconometric model of the UK economy. That model was produced from a system of second order differential equations.

Because of the Lucas critique, there is increasing interest in Euler equation models. Since the Bergstrom and Wymer model is a structural model, rather than an Euler equations model, He and Barnett (2002) have continued their investigation of policy relevant bifurcation by searching the parameter space of the Leeper and Sims (1994) Euler equations macroeconometric model. Unexpectedly, they have found the existence of a singularity bifurcation boundary within the model's parameter space. Although known in engineering, singularity has not previously been encountered in economics. We believe that other economists, who in the future explore other Euler equation models, are likely to find singularity bifurcation boundaries. We make clear the mathematical nature of singularity bifurcation and why it is likely to be common in the class of modern Euler equation models rendered important by the Lucas critique.

In addition to Euler equation models, which have become fundamental in policy-relevant macroeconomic models, there has been growing interest in recent years in new Keyensian models. As a result, in this paper we also explore bifurcation within the class of new Keynesian models. We find the possibility of Hopf bifurcation, with the setting of the policy parameters influencing the existence and location of the bifurcation boundary. We further study forward looking, backward looking, and hybrid models having both forward and backwards looking features. We investigate different monetary policy rules relative to bifurcation. In each case, we solve numerically for the location and properties of the bifurcation boundary and its dependency upon policy rule parameter settings.

Summary: Grandmont found that the parameter space of the most classical dynamic models is stratified into an infinite number of subsets supporting an infinite number of different kinds of dynamics, from monotonic stability at one extreme to chaos at the other extreme, and with all forms of multiperiodic dynamics between. This result changed prior views that different kinds of economic dynamics can only be produced by different kinds of structures.

Since this dramatic transformation of views was criticized for lack of policy relevance, Barnett and He (1999, 2002) investigated a continuous time traditional Keynesian structural model (Bergstrom’s model), and found results supporting Grandmont’s conclusions, but with a policy relevant model.

Subsequent criticism of Keynesian structural models by the Lucas critique have motivated development of Euler equation models having policy-invariant deep parameters. Barnett and He are investigating bifurcation of the Leeper and Sims model, which is the best known of the policy relevant Euler equations macroeconomic models. We continue that investigation in this paper. These results further confirm Grandmont’s views, but with an unexpected form of bifurcation: singularity bifurcation.

Even more recently, interest in policy in some circles has moved away from Euler equations models to New Keynesian models. Barnett and Duzhak are investigating bifurcation of New Keynesian models and are finding Hopf bifurcation. A central proposition used in this part of our research is proof of a theorem on the existence of Hopf bifurcation.

Conclusion: Beginning with Grandmont’s findings with a classical model, we continue to follow the path through policy relevant traditional Keynesian models, more recent Euler equation macroeconomic models, and New Keynesian models. All bifurcate. At this stage of our research, we now can conclude that Grandmont’s conclusions hold for all categories of dynamic macroeconomic models, from the oldest to the newest.

Grandmont was right.

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