Overlapping generations and idiosyncratic risk: can prices reveal the best monetary policy?

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Abstract

Incomplete markets in overlapping generations leaves room for Pareto improvement. I analyze a model with two goods and idiosyncratic shocks. Intervention can make agents better off in two different situations: one in which agents face shocks on endowments and the other in which shocks affect preferences. I compare market equilibrium with the constrained efficient equilibrium allocation and show that, when shocks are symmetric either only on endowments or only on preferences, prescription of optimal monetary policy depends exclusively on information about relative prices. Monetary policy may be expansionist or contractionist in order to bring prices to their constrained optimal point.

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1 Introduction

I analyze a general equilibrium model of overlapping generations with two goods in which identical agents live for two periods and face idiosyncratic risk either on endowments or on preferences. The only available asset is money and there is a policy maker whose intervention is constrained to existent markets. Consumption of agents in their second period of life includes two goods while all people from the young generation consume only one of these two commodities. The model varies by considering either shocks on old's endowments or on their preferences. Shocks are either positive or negative, with the same absolute value. I show that the pattern of the young generation's consumption, determined by which good they demand, affects the constrained optimal monetary policy. As in the standard definition of constrained optimality,[1] only reallocations of existent assets are allowed. In this model, only spot and money markets are open.

To obtain a Pareto improvement the policy maker must induce changes of relative prices, unattainable without intervention. Prices' reaction to monetary policy is sufficient information for determining if the intervention must be expansionist or contractionist.

I also consider degenerated cases. The degenerated case of shocks on endowments is the one in which old agents may have zero units of one commodity. The analog for shocks on preferences is when in some possible event only one good is desired by the old agent. Results obtained remain valid in the degenerated case of shocks on endowments. However, this is not true for shocks on preferences. Besides information about relative prices, the policy maker must be aware about agents' preferences.

Hart (1975) shows that when markets are incomplete, competitive equilibria are typically Pareto inefficient. It is natural to expect that economies in which agents cannot insure for every possible contingency do not converge to optimal equilibria. Geanakoplos and Polemarchakis (1986) formalize the notion of Pareto constrained optimality. An allocation is constrained efficient if, with the same financial structure, no other allocation can make agents better off, in the strict sense for at least one of them. They show that when assets pay in a numeraire commodity, competitive equilibria are generically constrained suboptimal. Polemarchakis and Carvajal (2005) extend this result for the case of idiosyncratic risks. Since constrained sub-optimality is proven, I investigate what type of policy should be implemented. The advantage of the model I present is that the information necessary for the implementation of the constrained optimal policy is publicly available, since it consists on the reaction relative prices with respect to the

[1]See Geanakoplos and Polemarchakis (1986).

policy.

The second section presents the general model. In the third section I analyze the case of risk on endowment and in the fourth section I consider risk on preferences. In both sections there is an example followed by the presentation of the model. The constrained optimal policy is analyzed in section 5 and the last section presents the degenerated cases. Proofs are left to appendix.

2 The General Model

Generations overlap and in each generation there is a continuum of individuals indexed by i in the interval $[0, 1]$ with identical preferences. Each person lives for two periods. Old people derive utility from two goods: a and b. To simplify notation I also use a and b to denote old's initial endowments. Young generation's endowment is denoted by e, which can be either good a or good b. I denote numeraire as the good consumed by young generation.

For shocks on endowments and shocks on preferences, I consider two cases: one in which the numeraire is the same good as the one subject to a shock, so that $e = a_1$, and another in which the numeraire is the same commodity as the one not subject to shocks, implying $e = b_1$. Preferences of young and old people are different: young generation wants to consume only the good they are endowed with. Utility functions are $u^i_g = u^i_g : \Re^g_{++} \rightarrow \Re$, where g indicates if people are from the young generation ($g = 1$) or from the old generation ($g = 2$).

Individuals face some type of uncertainty, so that either a positive or a negative shock can be realized at their last period of life. These shocks are represented by a parameter ϵ whose value is in the interval $(0, a)$. The absolute value of this parameter is the same for all individuals. The probability of realization of a positive shock is π, $\pi \in [0, 1]$, while a negative shock occurs with probability $1 - \pi$. The amount of numeraire consumed by a young agent i is x^i_1. In case of a positive shock old consumers demand x^i_{a+} of good a and $x^i_{b_+}$ of good b. Similarly, demands in case of realization of a negative shock are x^i_{a-} and $x^i_{b_-}$. When shocks affect endowments, individuals calculate their expected utility during their lives as:

$$U^i = u^i_1(x^i_1) + \pi u^i_2(x^i_{a_+}, x^i_{b_+}) + (1 - \pi)u^i_2(x^i_{a_-}, x^i_{b_-}) \quad (2.1)$$

If shocks affect preferences, expected utility of agent i is:

$$U^i = u_1^i(x_1^i) + \pi u_2^i(x_{a_+}^i + \epsilon, x_{b_+}^i) + (1-\pi)u_2^i(x_{a_-}^i - \epsilon, x_{b_-}^i)$$

In the model of endowments, shocks affect good a by determining different distributions of it across agents. Although good b's endowment is state independent, its demand varies according to the shock realized. When shocks occur on preferences, utility functions are directly affected by the shock parameter ϵ. With a positive shock, individual i prefers good b better than when a negative shock is realized.

Assumption 1. *The function u_g^i, $g = 1, 2$, satisfies:*

i. continuity and smoothness: u_g^i is C^∞ on $\Re_{++}^g$.

ii. monotonicity[2]: for two commodity bundles $x, y \in \Re_+^g$, $x \gg y \Rightarrow u_g^i(x^i) > u_g^i(y^i)$.

iii. strict concavity: for any $x, y \in \Re_+^g$, and any $\alpha \in (0,1), u_g^i(\alpha x^i + (1-\alpha)y^i) > \alpha u_g^i(x^i) + (1-\alpha)u_g^i(y^i)$.

From now on I omit superscripts i when not necessary. Individuals' probabilities of receiving a shock are such that no aggregate uncertainty exists, that is, I assume that π and $1-\pi$ are the proportion of the population who receives a positive and a negative shock, respectively. Idiosyncratic shocks' distributions are not required to be independent among consumers.

Besides exchanging commodities in a spot market, young agents can trade a nominal asset, called money. The purchase of one unit of this asset gives the consumer the right to receive one unit back when old. No other market exists other than spot and money markets. By using exclusively the markets already available in this economy, it is possible to make transfers whose outcome is a preferred allocation in the strict sense for all consumers.[3]

Let m, $m \in \Re$, be the amount of money agents carry from young to old age and τ be the transfer made by the policy maker. The monetary transfer can be expressed as a function of m: $\tau = \delta m$, $\delta \in \Re$. This transfer is made exclusively to old agents and there is no discrimination, in the sense that everybody receive the same amount of money τ in their last period of life. Note that τ can be either positive or negative. When it is positive, old agents have more money than what they saved when young.

[2]I use standard notation for inequalities of vectors: $x \gg y$ means that every component of x is strictly greater than every coordinate of y; $x > y$ indicates that all coordinates of x are greater than its respective coordinates of y and at least one coordinate of x is strictly greater than its respective coordinate in y; $x \geq y$ denotes that all elements of x are at least as great as its respective coordinates in y.

[3]Except in a set of economies with measure zero.

On the contrary, they end up with less money than what was saved in their first period of life. Prices grow at the same rate as money, since there is no aggregate risk and all generations are identical. For $l_g = a, b$, with $g = 1, 2$, let p_{l_1} be the price of good l that an individual faces when young and p_{l_2} be the price of good l that the same individual faces when old. Then, for every generation $p_{l_2} = (1+\delta)p_{l_1}$, as stated in the following assumption.

Assumption 2. *In an overlapping generations model without population growth and with idiosyncratic risk, if money is the only available financial asset, then equilibrium prices grow at the same rate as money.*

From assumption 2, in the absence of a social planner, prices are constant along the time. Since there is no population growth and agents have identical preferences, it is reasonable to expect that their maximization problems do not vary. This would not be true if prices were allowed to change. Then, I assume that without intervention every generation takes as given the same level of prices. In the case of an intervention that satisfies $\tau = \delta m$, observe that at each period of time consumers' endowments of assets differ by δm from the previous period. If prices grow at this rate, each generation demands $(1+\delta)$ times their previous generation's demand for money.

Young agents decide about x_1, x_{a_+}, x_{a_-}, x_{b_+}, x_{b_-} and m. They have no influence on the decision of τ. Let $\bar{\mathbf{x}}^{\mathbf{i}} = (\bar{x}_1,\ \bar{x}_{a_+},\ \bar{x}_{a_-},\ \bar{x}_{b_+},\ \bar{x}_{b_-})$ be the equilibrium allocation of goods and $V^i(e, a, b, \delta)$ be individual's indirect utility function.[4] Because agents are identical, in the sense that they face the same type of risk and have the same expected utilities, the planner's problem is to find money's growth rate δ that satisfies:

$$\arg\max_{\delta} V^i$$

This problem is static, since the proportion of people who receive a positive or negative shock does not vary across time.

Call expansionist the monetary policy that sets a positive value for δ, neutral the one that establishes $\delta = 0$, and contractionist the one that defines δ to be negative. For a given policy δ, Pareto improvement occurs if and only if $\frac{\partial V}{\partial \delta} d\delta > 0$. Hence, the optimal policy is expansionist when $\frac{\partial V}{\partial \delta} > 0$, neutral when $\frac{\partial V}{\partial \delta} = 0$ and contractionist when $\frac{\partial V}{\partial \delta} < 0$.

[4] To make notation simpler I omit the arguments of the function V^i, which are endowments e, a and b, and money's growth rate, δ.

3 Shocks on Endowments

3.1 An Example of Shocks on Endowments

Endowments of old agents are uncertain. With half probability, individual's endowment of good a increases by ϵ, $\epsilon \in (0, a)$ and, with the same probability, it decreases by ϵ. Endowment of good b is not subject to risk. Individuals' preferences are represented by:

$$U = x_1 + \frac{\gamma}{2}\ln x_{a_+} + \frac{(1-\gamma)}{2}\ln x_{b_+} + \frac{\gamma}{2}\ln x_{a_-} + \frac{(1-\gamma)}{2}\ln x_{b_-} \tag{3.1}$$

γ is a preference parameter that lies in $(0, 1)$;

x_1 is young age's consumption that can be either of good a or of good b;

x_{a_+} (x_{a_-}) is the amount of good a consumed at the old age in the state in which a good (bad) shock is realized;

x_{b_+} (x_{b_-}) is the amount of good b consumed at the old age in the state in which a good (bad) shock is realized.

Under the possibility of a transfer τ, $\tau \in \Re$, that satisfies $\tau = \delta m$, budget constraints are:

$$\begin{aligned} &px_1 + m = pe \\ &(1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_a \epsilon + p_b b) \\ &(1+\delta)(p_a x_{a_-} + p_b x_{b_-}) = m + \tau + (1+\delta)(p_a a - p_a \epsilon + p_b b) \end{aligned} \tag{3.2}$$

m represents the amount of money the young agent carries for his second period of life; it can assume positive or negative values.

e is the young's initial endowment, which is necessarily the same good as the one he wants to consume, x_1; when x_1 is consumption of good a, $e = a$; otherwise, $e = b$;

p is the price of the good consumed by young agents; $p = p_a$ when $e = a_1$ and $p = p_b$ when $e = b_1$;

$(1 + \delta)p_l$ is the price of good l consumed by agents when old, $l = a, b$;

δ is money's growth rate;

ϵ is the value of the shock on endowment, $0 < \epsilon < a$;

a is the old's endowment of good a that increases or decreases by ϵ, according to the realization of a positive or negative shock;

b is the old's endowment of good b.

The first equation in (3.2) represents young's budget constraint. Besides consuming x_1, he decides how much money to save, which is m. The second equation is the old's budget constraint when a positive shock is realized. The value of his consumption of goods a and b, represented by the left-hand side of (3.2), must be equal to the value of his initial endowments, plus the value of his savings and received transfers. Since there was a positive shock, his endowment of commodity a is increased by the amount ϵ. Similarly, the third equation is the old's budget constraint in case of realization of a negative shock. The left-hand side represents the value of his expenditure, while the right-hand side is the value of his savings, received transfers and endowments after the realization of a negative shock of magnitude ϵ.

Since there is no aggregate risk, individuals take as given the same prices when they face a positive or a negative shock. Half of the old generation demands x_{a_+} and x_{b_+}, while half of them demands x_{a_-} and x_{b_-}.

Consider first the case in which the numeraire is a, the good subject to idiosyncratic shocks. In this case, system (3.2) becomes:

$$\begin{aligned} & p_a x_1 + m = p_a a_1 \\ & (1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a(a+\epsilon) + p_b b) \\ & (1+\delta)(p_a x_{a_-} + p_b x_{b_-}) = m + \tau + (1+\delta)(p_a(a-\epsilon) + p_b b) \end{aligned} \tag{3.3}$$

The partial derivative of the indirect utility function with respect to δ evaluated at 0 is:

$$\left.\frac{\partial V}{\partial \delta}\right|_{\delta=0} = \frac{(1-\gamma)\left(1-\sqrt{1+4\epsilon^2}\right)}{2\sqrt{1+4\epsilon^2}} < 0$$

Because $\left.\frac{\partial V}{\partial \delta}\right|_{\delta=0} < 0$, the optimal policy must decrease the amount of money the old generation carries comparing to what they have saved.

Consider now the case in which the numeraire, that is, young agent's consumption good, is b, the commodity whose endowment is certain. The system of budget constraints is similar to (3.2), with the appropriate substitutions in the first equation: $p = p_b$ and $e = b_1$:

$$\begin{aligned} & p_b x_1 + m = p_b b_1 \\ & (1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a(a+\epsilon) + p_b b) \\ & (1+\delta)(p_a x_{a_-} + p_b x_{b_-}) = m + \tau + (1+\delta)(p_a(a-\epsilon) + p_b b) \end{aligned}$$

Again, we are interested on the sign of the partial derivative of the indirect utility function V with respect to δ, which, at $\delta = 0$ is:

$$\left.\frac{\partial V}{\partial \delta}\right|_{\delta=0} = \frac{\gamma^2\epsilon^2(1-\gamma)}{a^2-\gamma^2\epsilon^2} > 0$$

Assumptions about endowment a, shock ϵ and preference parameter γ assure the above inequality: remember that $\gamma \in (0,1)$ and $\epsilon \in (0,a)$. The constrained optimal policy is expansionist. Equilibrium relative price is displayed in appendix and so are equilibrium demands calculated as a function of δ.

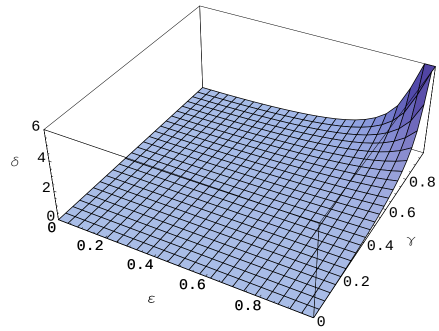
In both cases the prescription of policy tells about consumers' savings. When a is the numeraire, individuals save less than they should. Without intervention, young's consumption x_1 exceeds by $\frac{\gamma}{2(1+\sqrt{1+4\epsilon^2})}\left(\frac{\delta}{1+\delta}\right)$ what he would demand if there was an intervention with negative money's growth rate $\delta < 0$.

By defining a policy of money's growth rate $\delta^* < 0$, the monetary authority makes people of the old generation poorer, inducing higher level of savings. Observe that a negative δ implies that money's nominal rate of return is negative. However, this is not perceived by consumers, because they see δ as a function of the aggregate demand for money, not of their own portfolio decision, m^i.

When b is the numeraire and money grows at a rate $\delta > 0$, consumption of x_1 differs from its consumption without intervention by $\frac{\delta(1-\gamma)a^2}{(1+\delta)(a^2-\epsilon^2\gamma^2)}$. In this case, agents save too much when $\delta = 0$. A positive transfer to old agents turns them richer and induces a lower level of savings.

The following graph shows different values for the constrained optimal policy according to the parameters ϵ and γ when good b is the numeraire.

Fig. 1: The optimal policy when b is the numeraire ($a = 1$)



The figure shows that the optimal policy increases monotonically with ϵ and γ, the shock and preference parameters. It is quite intuitive that the higher the shock, the higher the distortion between market equilibrium and the contrained Pareto optimal point. Concerning the preference parameter γ, we see that when individuals have strong preferences for a, that is, for higher values of γ, constrained optimal transfers τ must assume a higher proportion of consumers' demand for money.

3.2 The Model of Shocks on Endowments

Endowments are uncertain: when young, individuals expect that, with some probability π, their endowment of good a will be augmented by ϵ; otherwise, it will be diminished by the same amount, ϵ. There is no shock on the endowment of the other good, b. Following notation of sections 2 and 3.1, preferences can be represented by:

$$U = u_1(x_1) + \pi u_2(x_{a_+}, x_{b_+}) + (1-\pi)u_2(x_{a_-}, x_{b_-}) \tag{3.4}$$

Consumers face the following budget constraints:

$$\begin{aligned} &px_1 + m = pe \\ &p_a x_{a_+} + p_b x_{b_+} = m + p_a(a+\epsilon) + p_b b \\ &p_a x_{a_-} + p_b x_{b_-} = m + p_a(a-\epsilon) + p_b b \end{aligned} \tag{3.5}$$

The variable p is equal to either p_a or p_b, according to which good is the numeraire. Similarly, e can be either a_1 or b_1. For each agent i, consider $\mathcal{B_E}^i$ as the set of allocations $\mathbf{x^i} = (x_1^i, x_{a_+}^i, x_{a_-}^i, x_{b_+}^i, x_{b_-}^i)$ and m^i that satisfy (3.5) for given prices $\mathbf{p} = (p_a, p_b)$. Formally,

$$\mathcal{B_E}^i = \left\{ (\mathbf{x^i}, m^i) : \begin{array}{c} px_1 + m = pe \\ p_a x_{a_+} + p_b x_{b_+} = m + p_a(a+\epsilon) + p_b b \\ p_a x_{a_-} + p_b x_{b_-} = m + p_a(a-\epsilon) + p_b b \end{array} \right\}$$

Let $(\mathbf{x^i}(p_a,p_b), m^i(p_a,p_b))$ be a plan for individual i. Agents choose plans in $\mathcal{B_E}^i$ in order to maximize their utilities. At every period of time there is one young generation's plan, yielding a sequence of plans along the time, indexed by t. Let this sequence be $\{(\overline{\mathbf{x}}^{\mathbf{i}}, \overline{m}^i)_t\}_{t=0}^{\infty}$.

Definition 1. *A competitive market equilibrium with idiosyncratic risk on endowments is a sequence at time t of a collection of individuals i's plans and two prices* $\{(\overline{\mathbf{x}}^{\mathbf{i}}, \overline{m}^i)_t, (p_a, p_b)_t\}_{t=0}^{\infty}$ *that satisfies:*
i. $\overline{\mathbf{x}}^{\mathbf{i}} \in \arg\max \{U^i(\overline{\mathbf{x}}^{\mathbf{i}}) | \overline{\mathbf{x}}^{\mathbf{i}} \in \mathcal{B_E}^i\}$ *for all individuals and, at each period of time,*
ii.a. $\overline{x}_1^i + \pi\overline{x}_{a_+}^i + (1-\pi)\overline{x}_{a_-}^i = a_1^i + a^i$ *when good a is the numeraire, or*
ii.b. $\pi\overline{x}_{a_+}^i + (1-\pi)\overline{x}_{a_-}^i = a^i$ *when good b is the numeraire.*[5]

When there is a social planner who uses τ as an instrument policy, consumers' budget constraints become:

$$\begin{aligned} &px_1 + m = pe \\ &(1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_a\epsilon + p_b b) \\ &(1+\delta)(p_a x_{a_-} + p_b x_{b_-}) = m + \tau + (1+\delta)(p_a a - p_a\epsilon + p_b b) \end{aligned} \tag{3.6}$$

In the presence of a social planner, the definition of equilibrium is the same as before,

[5] I do not state commodity b's market clearing condition, since it is satisfied once market a clears.

with the budget constraints in $\mathcal{B}_{\mathcal{E}}{}^i$ substituted by equations (3.6). Variables τ and δ are given to consumers. Equilibrium is characterized by a system of market clearing and agents' first order conditions, denoted by E_a when commodity a is the numeraire or by E_b, when b is the numeraire. These equilibrium conditions are presented in Result 2 in appendix.

The planner must choose δ that maximizes individuals' utilities restricted to system E_a when a is the numeraire or E_b when b is the numeraire. Relative prices depend on δ, the policy maker's choice variable, and the indirect utility is also a function of δ.

The first order condition for $\max_{\delta} V^i$ is:

$$\begin{aligned} &\left[\pi\tfrac{\partial u_2}{\partial x_{b_+}} + (1-\pi)\tfrac{\partial u_2}{\partial x_{b_-}}\right]\left[\tfrac{\partial(m/p_b)}{\partial\delta} - \tfrac{1}{1+\delta}\tfrac{p}{p_b}\tfrac{\partial(m/p)}{\partial\delta}\right] + \\ &+\left[\pi\tfrac{\partial u_2}{\partial x_{b_+}}(a+\epsilon-x_{a_+}) + (1-\pi)\tfrac{\partial u_2}{\partial x_{b_-}}(a-\epsilon-x_{a_-})\right]\tfrac{\partial(p_a/p_b)}{\partial\delta} = 0 \end{aligned} \tag{3.7}$$

Equation (3.7) may be rewritten according to which good young generation consumes. Let the first case be the one in which commodity a is the numeraire, so that $p = p_a$.

From budget constraints (3.6), at $\delta = 0$, the left hand side of equation (3.7) becomes:

$$\left.\frac{\partial V}{\partial\delta}\right|_{\delta=0} = -\left[\pi(b-x_{b+})\frac{\partial u_2}{\partial x_{b_+}} + (1-\pi)(b-x_{b_-})\frac{\partial u_2}{\partial x_{b_-}}\right]\left(\frac{p_b}{p_a}\right)\frac{\partial(p_a/p_b)}{\partial\delta} \tag{3.8}$$

If, instead, young generation consumes commodity b, the planners' first order condition evaluated at 0 is:

$$\left.\frac{\partial V}{\partial\delta}\right|_{\delta=0} = -\left[\pi\left(b-x_{b_+}+\frac{m}{p_b}\right)\frac{\partial u_2}{\partial x_{b_+}} + (1-\pi)\left(b-x_{b_-}+\frac{m}{p_b}\right)\frac{\partial u_2}{\partial x_{b_-}}\right]\left(\frac{p_b}{p_a}\right)\frac{\partial(p_a/p_b)}{\partial\delta} \tag{3.9}$$

Proposition 1. *In an overlapping generations model in which agents live for two periods and face symmetric idiosyncratic risk on endowments, the sign of the derivative of the relative price p_a/p_b with respect to the money's growth rate δ, $\frac{\partial(p_a/p_b)}{\partial\delta}$, evaluated at $\delta = 0$, determines the constrained optimal policy. When this derivative is positive (negative), the optimal policy is contractionist (expansionist). There must be no intervention if relative prices are not affected by monetary transfers.*

Proof: see appendix.

4 Shocks on Preferences

4.1 An Example of Shocks on Preferences

When young, people own and consume only one good. They are not sure about their preferences in the future. The information they have is that, with half probability, their utility derived from good a is augmented by a shock parameter ϵ and, with half probability it is diminished by the same parameter ϵ. Their preferences can be represented by:

$$U = x_1 + \frac{\gamma}{2}\ln(x_{a_+} + \epsilon) + \frac{(1-\gamma)}{2}\ln x_{b_+} + \frac{\gamma}{2}\ln(x_{a_-} - \epsilon) + \frac{(1-\gamma)}{2}\ln x_{b_-}, \text{ where:}$$

γ is a preference parameter that lies in $(0,1)$;

x_1 is young age's consumption that can be either of good a or of good b;

x_{a_+} (x_{a_-}) is the amount of good a consumed at the old age, in the state in which good a's utility is augmented (diminished);

x_{b_+} (x_{b_-}) is the amount of good b consumed at the old age, in the state in which good a's utility is augmented (diminished);

ϵ is a shock parameter satisfying $0 < \epsilon < a$.

With the possibility of transfers τ of money, budget constraints are:

$$\begin{aligned}
&px_1 + m = pe \\
&(1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_b b) \\
&(1+\delta)(p_a x_{a_-} + p_b x_{b_-}) = m + \tau + (1+\delta)(p_a a + p_b b)
\end{aligned}$$

Consider first the case in which the numeraire is good a, the one subject to idiosyncratic shocks. When $\delta = 0$, the optimal policy is to make transfers such that $\tau < 0$, that is, to set $\delta < 0$. This result comes from the calculation of $\frac{\partial V}{\partial \delta}$ evaluated at $\delta = 0$:

$$\left.\frac{\partial V}{\partial \delta}\right|_{\delta=0} = \frac{(1-\gamma)}{2}\frac{(1-\sqrt{1+4\epsilon^2})}{\sqrt{1+4\epsilon^2}} < 0$$

When b is the numeraire, the effect of a monetary transfer at $\delta = 0$ is:

$$\left.\frac{\partial V}{\partial \delta}\right|_{\delta=0} = \frac{\gamma^2\epsilon^2}{a^2 - \gamma^2\epsilon^2} > 0$$

Equilibrium relative price and demand functions that determine V in cases of numeraire a and b are in appendix. As in Example 3.1 of shocks on endowments, when good a is the numeraire there are too little savings, since without intervention x_1 is lower than it should be. When b is the numeraire, opposite result prevails.

4.2 The Model of Shocks on Preferences

When old, people can have augmented or diminished utility derived from consumption of good a. In the first case, their utilities are a function of what they consume of this good, plus a parameter ϵ. In the second case, the function's argument is what they consume of a, subtracted by the same parameter ϵ. Let x_1 be young's consumption, x_{l_+} (x_{l_-}) be consumption of good $l = a,\ b$ when the agent has augmented (diminished) utility derived from good a. Shocks on preferences satisfy $0 < \epsilon < a$ and utility functions representing these preferences are:

$$U = u_1(x_1) + \pi u_2(x_{a_+} + \epsilon, x_{b_+}) + (1 - \pi)u_2(x_{a_-} - \epsilon, x_{b_-})$$

Consumers face the following budget constraints:

$$\begin{aligned} px_1 + m &= pe \\ p_a x_{a_+} + p_b x_{b_+} &= m + p_a a + p_b b \\ p_a x_{a_-} + p_b x_{b_-} &= m + p_a a + p_b b \end{aligned} \tag{4.1}$$

Endowments do not vary between states: young individuals own e, which can be either commodity a or commodity b, while old people always have a and b. Then, either $p = p_a$ or $p = p_b$.

For each agent i, consider $\mathcal{B_P}^i$ as the set of allocations $\mathbf{x^i} = (x^i_1, x^i_{a+}, x^i_{a-}, x^i_{b+}, x^i_{b-})$ and m^i that satisfy (4.1) for given prices $\mathbf{p} = (p_a, p_b)$. Formally,

$$\mathcal{B_P}^i = \left\{ (\mathbf{x^i}, m^i) : \begin{aligned} px_1 + m &= pe \\ p_a x_{a_+} + p_b x_{b_+} &= m + p_a a + p_b b \\ p_a x_{a_-} + p_b x_{b_-} &= m + p_a a + p_b b \end{aligned} \right\}$$

Let $(\mathbf{x^i}(p_a, p_b), m^i(p_a, p_b))$ be a plan for individual i. Agents choose plans in $\mathcal{B_P}^i$ in order to maximize their utilities. At every time t there is one young generation of people deciding about their plans. This yields a sequence of plans along the time.

Definition 2. *A competitive market equilibrium with idiosyncratic risk on preferences is a sequence at time t of a collection of individuals i's plans and two prices* $\{(\overline{\mathbf{x}}^{\mathbf{i}}, \overline{m}^i)_t, (p_a, p_b)_t\}_{t=0}^{\infty}$ *that satisfies:*
i. $\overline{\mathbf{x}}^{\mathbf{i}} \in \arg\max \{U^i(\overline{\mathbf{x}}^{\mathbf{i}}) | \overline{\mathbf{x}}^{\mathbf{i}} \in \mathcal{B_P}^i\}$ *for all individuals and, at each period of time,*

ii.a. $\overline{x}_1^i + \pi\overline{x}_{a_+}^i + (1-\pi)\overline{x}_{a_-}^i = a_1^i + a^i$ *when good a is the numeraire, or*
ii.b. $\pi\overline{x}_{a_+}^i + (1-\pi)\overline{x}_{a_-}^i = a^i$ *when good b is the numeraire.*

When intervention is possible, the social planner chooses τ to be the amount of money received by the old generation. Budget constraints become:

$$\begin{aligned} &px_1 + m = pe \\ &(1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_b b) \\ &(1+\delta)(p_a x_{a_-} + p_b x_{b_-}) = m + \tau + (1+\delta)(p_a a + p_b b) \end{aligned} \tag{4.2}$$

With intervention, competitive equilibrium is as in Definition 2, with the budget constraints in $\mathcal{B}_{\mathcal{P}}{}^i$ replaced by equations (4.2). Result 4 in appendix presents these equilibrium conditions when either a or b are the numeraire.

The planner's problem of choosing a policy instrument τ that maximizes individuals' utilities is to solve $\max\limits_{\delta} V^i$. The first order condition of this problem is:

$$\begin{aligned} &-\tfrac{p}{p_b}\Big[\pi\tfrac{\partial u_2}{\partial x_{b_+}} + (1-\pi)\tfrac{\partial u_2}{\partial x_{b_-}}\Big]\tfrac{\partial(m/p)}{\partial\delta} + \pi\tfrac{\partial u_2}{\partial x_{b_+}}\Big[\tfrac{\partial(m/p_b)}{\partial\delta} + (a - x_{a_+})\tfrac{\partial(p_a/p_b)}{\partial\delta}\Big] + \\ &+(1-\pi)\tfrac{\partial u_2}{\partial x_{b_-}}\Big[\tfrac{\partial(m/p_b)}{\partial\delta} + (a - x_{a_-})\tfrac{\partial(p_a/p_b)}{\partial\delta} + \tfrac{p_a}{p_b}\Big] = 0 \end{aligned} \tag{4.3}$$

When a is the numeraire, the left hand side of this equation, evaluated at $\delta = 0$, is equivalent to:

$$\left.\frac{\partial V}{\partial\delta}\right|_{\delta=0} = -\left[\pi(b - x_{b_+})\frac{\partial u_2}{\partial x_{b_+}} + (1-\pi)(b - x_{b_-})\frac{\partial u_2}{\partial x_{b_-}}\right]\left(\frac{p_b}{p_a}\right)\frac{\partial(p_a/p_b)}{\partial\delta}$$

When b is the numeraire, at $\delta = 0$ (4.3) becomes:

$$\left.\frac{\partial V}{\partial\delta}\right|_{\delta=0} = -\left[\pi\left(b - x_{b_+} + \frac{m}{p_b}\right)\frac{\partial u_2}{\partial x_{b_+}} + (1-\pi)\left(b - x_{b_-} + \frac{m}{p_b}\right)\frac{\partial u_2}{\partial x_{b_-}}\right]\left(\frac{p_b}{p_a}\right)\frac{\partial(p_a/p_b)}{\partial\delta}$$

The expressions of $\frac{\partial V}{\partial\delta}$ evaluated at $\delta = 0$ are the same as those for the model of shocks on endowments, namely, equations (3.8) and (3.9).

Proposition 2. *In an overlapping generations model in which agents live for two periods and face symmetric idiosyncratic risk on preferences, the sign of the derivative of the relative price p_a/p_b with respect to the money's growth rate δ, $\frac{\partial(p_a/p_b)}{\partial\delta}$, evaluated at $\delta = 0$, determines the constrained optimal policy. When this derivative is positive (negative), the optimal policy is contractionist (expansionist). There must be no intervention if relative prices are not affected by monetary transfers.*

Proof: exactly the same proof of proposition 1.

5 The Constrained Optimal Policy

As shown by the Examples in sections 3.1 and 4.1 of shocks on endowments and on preferences, there are situations in which printing money is efficient, others in which it is better to decrease the amount of money transferred from one generation to another.

In fact, in both models the sign of $\frac{\partial V}{\partial \delta}$ depends on the sign of the derivative of relative prices with respect to the policy instrument, $\frac{\partial p_a/p_b}{\partial \delta}$. The expressions for $\frac{\partial V}{\partial \delta}$ at $\delta = 0$ vary depending on what young people own and consume, that can be either good a or good b. However, they are the same for the models of shocks on endowments and on preferences:

Numeraire	$\left.\frac{\partial V}{\partial \delta}\right\vert_{\delta=0}$
a	$-\left[\pi(b - x_{b_+})\frac{\partial u_2}{\partial x_{b_+}} + (1-\pi)(b - x_{b_-})\frac{\partial u_2}{\partial x_{b_-}}\right]\left(\frac{p_b}{p_a}\right)\frac{\partial(p_a/p_b)}{\partial \delta}$
b	$-\left[\pi\left(b - x_{b_+} + \frac{m}{p_b}\right)\frac{\partial u_2}{\partial x_{b_+}} + (1-\pi)\left(b - x_{b_-} + \frac{m}{p_b}\right)\frac{\partial u_2}{\partial x_{b_-}}\right]\left(\frac{p_b}{p_a}\right)\frac{\partial(p_a/p_b)}{\partial \delta}$

Propositions 1 and 2 show that equilibrium relative price p_a/p_b is too high without intervention. For $\frac{\partial(p_a/p_b)}{\partial \delta} > 0$, the constrained optimal policy is contractionist. By setting $\delta < 0$ the policy maker induces a decrease of the relative price $\frac{p_a}{p_b}$. By the same argument, $\frac{\partial(p_a/p_b)}{\partial \delta} < 0$ determines a choice of $\delta > 0$ and the effect of the constrained optimal policy is to reduce $\frac{p_a}{p_b}$.

Regardless of which good is the numeraire, a decrease in the relative price $\frac{p_a}{p_b}$ increases old generation's consumption of good a relatively to b. This is easily verified by the first order conditions:

$$\frac{\partial u_2}{\partial x_{a_+}} = \frac{p_a}{p_b}\frac{\partial u_2}{\partial x_{b_+}} \quad \text{and} \quad \frac{\partial u_2}{\partial x_{a_-}} = \frac{p_a}{p_b}\frac{\partial u_2}{\partial x_{b_-}}.$$

These results are important because they tell us that relative price contains sufficient information for prescription of the optimal monetary policy. The policy maker does not need to know about agents' preferences, only about the response of prices to the policy. If, by setting $\delta > 0$, an increase in relative price is observed, then the policy must be reversed, that is, the optimal intervention would be to set $\delta < 0$.

Conditions for a Pareto improvement are basically two:

- Agents must see the transfer τ as independent of their choice of m, although it is in fact defined as $\tau = \delta m$. Since there is a continuum of agents, one agent's choice does not affect the equilibrium allocation. Then, agents see τ as a function of the aggregate amount of money in the economy, not of their own choice of m.

- The policy maker must know how relative prices react to the policy: in any case the policy maker must induce a decrease of the relative price of a, the good subject to a shock.

The optimal policy δ still depends on preference parameters. However, the social planner can induce a convergence to the best value of δ by trying small changes of it and observing the reaction of relative prices. The optimal point would be achieved when no reaction is observed anymore.[6]

6 Degenerated Cases

In the previous sections, shocks were assumed to lie on the interval $(0, a)$. What if the shocks are stronger than that and achieve limit cases? In this section I analyze extreme situations, such as the one in which old's endowment of good a is zero in some state of the world. Concerning preferences, there can be a contingency in which old agents derive utility only from one good. I call this situations as degenerated. Everything follows the general model of section 2, except for the assumption $\epsilon \in (0, a)$.

6.1 Degenerated Shocks on Endowments

6.1.1 An Example of Degenerated Shocks on Endowments

Agents are uncertain about their future endowment of good a. With half probability they do not own any unit of it. With half probability, they have a units of this good. Utility functions are the same as equation (3.1), the example of shocks on endowments. Budget constraints are:

$$\begin{aligned} &px_1 + m = pe \\ &(1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_b b) \\ &(1+\delta)(p_a x_{a_-} + p_b x_{b_-}) = m + \tau + (1+\delta)p_b b \end{aligned} \tag{6.1}$$

[6] This conclusion depends crucially on the assumption of risk aversion. If agents were risk neutral, then $\frac{\partial u_2}{\partial x_{b_+}} = \frac{\partial u_2}{\partial x_{b_-}}$ and intervention would not be necessary.

When good a is the numeraire, the social planner's first order condition, evaluated at $\delta = 0$, is:

$$\left.\frac{\partial V}{\partial \delta}\right|_{\delta=0} = \frac{-(1-\gamma)a^2}{2\left(1+\sqrt{1+a^2}\right)\sqrt{1+a^2}} < 0$$

The constrained optimal policy is contractionist. In case of numeraire b, when there is no intervention, indirect utility varies with δ according to the following expression:

$$\left.\frac{\partial V}{\partial \delta}\right|_{\delta=0} = \frac{\gamma^2}{1+\gamma} > 0$$

The policy maker shoul implement an expansionist intervention. Equilibrium prices and demands are displayed in Result 5 in appendix. The results are comparable to those in section 3.1, where idiosyncratic shocks are symmetric on wealth. They are exactly the same when in section 3.1 ϵ is substituted by $a/2$ and a is divided by 2. Observe that budget constraints are exactly the same after these substitutions.

6.1.2 The Model of Degenerated Shocks on Endowments

Individuals have the same preferences and expect that, with a probability π they do not receive endowment of good a when old. Otherwise, they have a and b as initial endowment. Let x_{a_-} and x_{b_-} be their demands for the former case, and x_{a_+} and x_{b_+} be their demands for the latter. Utility functions are expressed exactly as in equation (3.4) in the model of shocks on endowments. Budget constraints are:

$$\begin{aligned} px_1 + m &= pe \\ p_a x_{a_+} + p_b x_{b_+} &= m + p_a a + p_b b \\ p_a x_{a_-} + p_b x_{b_-} &= m + p_b b \end{aligned} \tag{6.2}$$

For each agent i, consider ${\mathcal{B}_{\mathcal{DE}}}^i$ as the set of allocations and m^i that satisfy (6.2) for given prices p_a and p_b:

$${\mathcal{B}_{\mathcal{DE}}}^i = \left\{ (\mathbf{x^i}, m^i) : \begin{aligned} px_1 + m &= pe \\ p_a x_{a_+} + p_b x_{b_+} &= m + p_b b \\ p_a x_{a_-} + p_b x_{b_-} &= m + p_a a + p_b b \end{aligned} \right\}$$

Defining $(\mathbf{x^i}(p_a, p_b), m^i(p_a, p_b))$ as a plan for agent i, there is a sequence of plans along the time and competitive equilibrium is defined as:

Definition 3. *A competitive market equilibrium with degenerated idiosyncratic risk on endowments is a sequence at time t of a collection of individuals i's plans and two prices*

$\{(\overline{\mathbf{x}}^{\mathbf{i}}, \overline{m}^i)_t, (p_a, p_b)_t\}_{t=0}^{\infty}$ *that satisfies:*
i. $\overline{\mathbf{x}}^{\mathbf{i}} \in \arg\max \{U^i(\overline{\mathbf{x}}^{\mathbf{i}}) | \overline{\mathbf{x}}^{\mathbf{i}} \in \mathcal{B}_{\mathcal{DE}}{}^i\}$ $\forall$ *individuals and, at each period of time,*
ii.a. $\overline{x}_1^i + \pi \overline{x}_{a+}^i + (1-\pi)\overline{x}_{a-}^i = a_1^i + \pi a^i$ *when good a is the numeraire, or*
ii.b. $\pi \overline{x}_{a+}^i + (1-\pi)\overline{x}_{a-}^i = \pi a^i$ *when good b is the numeraire.*

When intervention is possible, the concept of equilibrium is the same, except that equations (6.2) are replaced by the system (6.1).

Result 6 in appendix shows the equations that characterize competitive equilibrium with intervention. The expressions of $\frac{\partial V}{\partial \delta}$ evaluated at $\delta = 0$ are the same as equation (3.8) for numeraire a, and as (3.9) for numeraire b. In words, the result obtained for shocks on endowments stated in Proposition 1 is also valid in its degenerated version. In fact it is only a particular case of the model of symmetric shocks when a is divided by 2 and ϵ is substituted by $a/2$.

6.2 Degenerated Shocks on Preferences

6.2.1 An Example of Degenerated Shocks on Preferences

Shocks on preferences are such that, when young, agents don't know if they will derive utility from two goods, a and b, or only from good a. Preferences are represented by:

$$U = x_1 + \frac{1}{2}\ln x_{a_-} + \frac{1}{2}[\gamma \ln x_{a_+} + (1-\gamma)\ln x_{b_+}]$$

x_{a_-} is the demand for a when the old agent wants to consume only good a;
x_{a_+} (x_{b_+}) is the demand for good a (b) when the old agent likes both goods.
Young agents must observe:

$$px_1 + m = pe$$

Budget constraints of people from the old generation vary according to the shock received. For those who like both goods, it is:

$$p_a x_{a_+} + p_b x_{b_+} = m + \tau + p_a a + p_b b$$

If the agent likes only good a, budget constraint is:

$$p_a x_{a_-} = m + \tau + p_a a + p_b b$$

The numeraire can be either good a or good b. In the former case, budget constraints are:

$$\begin{aligned} &p_a x_1 + m = p_a a_1 \\ &(1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_b b) \\ &(1+\delta) p_a x_{a_-} = m + \tau + (1+\delta)(p_a a + p_b b) \end{aligned} \tag{6.3}$$

Result 7 in appendix lists equilibrium relative price and equilibrium demands. When the social planner takes the derivative of the indirect utility function with respect to δ, first order condition is:

$$\frac{\partial V}{\partial \delta} = \frac{1+\gamma}{2(1+\delta)^2} - \frac{1+\gamma}{2(1+\delta)} = 0$$

Clearly, the value of δ that maximizes V is zero.

When the numeraire is good b, budget constraints are like (6.3), except for the first equation, substituted by $p_b x_1 + m = p_b b_1$.

Result 7 in appendix presents relative price and equilibrium demands as functions of δ. Social planner's first order condition is:

$$\frac{\partial V}{\partial \delta} = \frac{1-\gamma}{2(1+\delta)^2} - \frac{1-\gamma}{2(1+\delta)} = 0$$

Again, it determines that the best choice is to set $\delta = 0$. In both cases there is no room for intervention.

6.2.2 The Model of Degenerated Shocks on Preferences

There is uncertainty on preferences: with some probability π an old agent derives utility just from good a. Otherwise, both goods, a and b, are desirable. Utilities satisfy Assumption 1 and are represented by:

$$U = u_1(x_1) + \pi u_2(x_{a_-}, 0) + (1-\pi) u_2(x_{a_+}, x_{b_+})$$

Consumers face the following budget constraints:

$$\begin{aligned} &p x_1 + m = p e \\ &p_a x_{a_-} = m + p_a a + p_b b \\ &p_a x_{a_+} + p_b x_{b_+} = m + p_a a + p_b b \end{aligned} \tag{6.4}$$

Denote as ${\mathcal{B}_{\mathcal{DP}}}^i$ the set of allocations $\mathbf{x^i} = (x_1^i, x_{a_+}^i, x_{a_-}^i, x_{b_+}^i, x_{b_-}^i)$ and m^i that satisfy

(6.4) for given prices $\mathbf{p} = (p_a, p_b)$. Formally,

$$\mathcal{B}_{\mathcal{DP}}{}^i = \left\{ (\mathbf{x^i}, m^i) : \begin{array}{l} px_1 + m = pe \\ p_a x_{a_-} = m + p_a a + p_b b \\ p_a x_{a_+} + p_b x_{b_+} = m + p_a a + p_b b \end{array} \right\}$$

A plan for individual i is represented by $(\mathbf{x^i}(p_a, p_b), m^i(p_a, p_b))$, such that along the time t there is a sequence of individuals plans.

Definition 4. *A competitive market equilibrium with degenerated idiosyncratic risk on preferences is a sequence at time t of a collection of individuals i's plans and two prices $\{(\overline{\mathbf{x}}^{\mathbf{i}}, \overline{m}^i)_t, (p_a, p_b)_t\}_{t=0}^{\infty}$ that satisfies:*
i. $\overline{\mathbf{x}}^{\mathbf{i}} \in \arg\max \{U^i(\overline{\mathbf{x}}^{\mathbf{i}}) | \overline{\mathbf{x}}^{\mathbf{i}} \in \mathcal{B}_{\mathcal{DP}}{}^i\}$ $\forall$ *individuals and, at each period of time,*
ii.a. $\overline{x}^i + \pi \overline{x}^i_{a_+} + (1-\pi)\overline{x}^i_{a_-} = a^i_1 + a^i$ *when good a is the numeraire, or*
ii.b. $\pi \overline{x}^i_{a_+} + (1-\pi)\overline{x}^i_{a_-} = a^i$ *when good b is the numeraire.*

In the presence of monetary transfers τ, agents' budget constraints become:

$$\begin{array}{l} px_1 + m = pe \\ (1+\delta)(p_a x_{a_-}) = m + \tau + (1+\delta)(p_a a + p_b b) \\ (1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_b b) \end{array} \tag{6.5}$$

The equilibrium is characterized by market clearing and consumers' first-order conditions, summarized by the system of equations shown in Result 8 in appendix. With intervention, instead of facing restrictions (6.4), equations (6.5) characterize consumers' budget constraints. The social planner optimizes the welfare function over δ.

In case of numeraire a, the planner's first-order condition is:

$$\begin{array}{l} \frac{\delta}{1+\delta}\left((1-\pi)\frac{\partial u_2}{\partial x_{a_+}} + \pi \frac{\partial u_2}{\partial x_{a_-}}\right)\frac{\partial (m/p_a)}{\partial \delta} + \\ \left((1-\pi)(b - x_{b_+})\frac{\partial u_2}{\partial x_{a_+}} + \pi b \frac{\partial u_2}{\partial x_{a_-}}\right)\frac{\partial (p_b/p_a)}{\partial \delta} = 0 \end{array} \tag{6.6}$$

If, instead, b is the numeraire, the relevant rule is:

$$\begin{array}{l} \frac{\delta}{1+\delta}\left((1-\pi)\frac{\partial u_2}{\partial x_{a_+}} + \pi \frac{\partial u_2}{\partial x_{a_-}}\right)\frac{\partial (m/p_a)}{\partial \delta} + \\ (1-\pi)\left(b - x_{b_+} + \frac{1}{1+\delta}\frac{m}{p_b}\right)\frac{\partial u_2}{\partial x_{a_+}}\frac{\partial (p_b/p_a)}{\partial \delta} + \pi\left(b + \frac{1}{1+\delta}\frac{m}{p_b}\right)\frac{\partial u_2}{\partial x_{a_-}}\frac{\partial (p_b/p_a)}{\partial \delta} = 0 \end{array} \tag{6.7}$$

Given the results obtained in Example 6.2.1 and the general constrained suboptimality proved by Polemarchakis and Carvajal (2005), it becomes interesting to confirm that the neutrality verified in section 6.2.1 does not hold for other types of utility functions.

When a is the numeraire, the partial derivative $\frac{\partial V}{\partial \delta}$ evaluated at $\delta = 0$ is:

$$\left.\frac{\partial V}{\partial \delta}\right|_{\delta=0} = -\left[\pi b \frac{\partial u_2}{\partial x_{a-}} + (1-\pi)(b - x_{b+})\frac{\partial u_2}{\partial x_{a+}}\right]\left(\frac{p_b}{p_a}\right)^2 \frac{\partial (p_a/p_b)}{\partial \delta} \tag{6.8}$$

and when numeraire is good b:

$$\left.\frac{\partial V}{\partial \delta}\right|_{\delta=0} = -\left[\pi\left(b + \frac{m}{p_b}\right)\frac{\partial u_2}{\partial x_{a-}} + (1-\pi)\left(b - x_{b+} + \frac{m}{p_b}\right)\frac{\partial u_2}{\partial x_{a+}}\right]\left(\frac{p_b}{p_a}\right)^2 \frac{\partial (p_a/p_b)}{\partial \delta} \tag{6.9}$$

Proposition 3. *In an overlapping generations model in which agents live for two periods and face degenerated idiosyncratic risk on preferences, the optimal policy limited to transfers of money depends on the sign of the derivative of relative price with respect to the money's growth rate,* $\frac{\partial (p_a/p_b)}{\partial \delta}$*, evaluated at* $\delta = 0$*. It also depends on the difference between the marginal utility of the commodity that suffers a shock, a, for the two types of old individuals:* $\frac{\partial u_2}{\partial x_{a_-}}$*, which is of the old who consume only good a, and* $\frac{\partial u_2}{\partial x_{a_+}}$*, the marginal utility of people who consume both goods. This difference must be evaluated at* $\delta = 0$*.*

Proof: see appendix.

Even if interventions affect relative prices, there can be situations in which it does not increase welfare. The case of degenerated shocks on preferences in section 6.2.1 is an example: in equilibrium, the marginal utilities of both types of old consumers with respect to good a is equal, even when evaluated at $\delta \neq 0$.

When relative price p_a/p_b increases with the monetary policy, it must be that the marginal utility of those who consume only good a is lower than of the remaining old population. The population who consumes only a always prefers a low relative price p_a/p_b. Since on the margin they are better off than the other old agents, the increase in relative price creates a Pareto improvement.

When shocks on preferences achieve an extreme case in which preferences can possibly become degenerated, conditions for a success of the policy are stronger. Besides the information about relative price, the policy maker must know about old generation's preferences.

Situations in which there are idiosyncratic shocks on endowments or in which the shocks on preferences are not of the degenerated form seem to be much more common, suggesting that Pareto improvement interventions may be often implementable.

A Appendix - Proofs and Results

Proof. **Proposition 1**

Consider equation (3.8):

$$\left.\frac{\partial V}{\partial \delta}\right|_{\delta=0} = -\left[\pi(b - x_{b_+})\frac{\partial u_2}{\partial x_{b_+}} + (1-\pi)(b - x_{b_-})\frac{\partial u_2}{\partial x_{b_-}}\right]\left(\frac{p_b}{p_a}\right)\frac{\partial(p_a/p_b)}{\partial \delta}$$

Feasibility implies that $b = \pi x_{b_+} + (1-\pi)x_{b_-}$. Then, the expression in brackets becomes:

$$\pi(1-\pi)\left[(x_{b_-} - x_{b_+})\frac{\partial u_2}{\partial x_{b_+}} + (x_{b_+} - x_{b_-})\frac{\partial u_2}{\partial x_{b_-}}\right] \tag{A.1}$$

By concavity and monotonicity, if $x_{b_-} > x_{b_+}$, then $\frac{\partial u_2}{\partial x_{b_+}} > \frac{\partial u_2}{\partial x_{b_-}}$. Rewriting (A.1) as:

$$\pi(1-\pi)(x_{b_-} - x_{b_+})\left(\frac{\partial u_2}{\partial x_{b_+}} - \frac{\partial u_2}{\partial x_{b_-}}\right)$$

the expression is positive for $x_{b_-} \neq x_{b_+}$. I obtain similar result when $x_{b_+} > x_{b_-}$. For $\frac{\partial(p_a/p_b)}{\partial \delta} > 0$, then $\frac{\partial V}{\partial \delta}$ evaluated at $\delta = 0$ is negative.

In case of the numeraire being good b, substitution of $\pi x_{b_+} + (1-\pi)x_{b_-} - m/p_b$ in the place of b yields (A.1). As already argued, this expression is always positive. The optimal policy is defined by the derivative of prices with respect to δ. □

Proof. **Proposition 3**

When good a is the numeraire, $b = (1-\pi)x_{b_+}$, implying that equation (6.8) can be rearranged as:

$$-\pi b\left[\frac{\partial u_2}{\partial x_{a_-}} - \frac{\partial u_2}{\partial x_{a_+}}\right]\left(\frac{p_b}{p_a}\right)^2\frac{\partial(p_a/p_b)}{\partial \delta} \tag{A.2}$$

Since $\frac{\partial u_2}{\partial x_{a_-}}$ and $\frac{\partial u_2}{\partial x_{a_+}}$ are positive, the signal of $\frac{\partial V}{\partial \delta}$ depends on the difference between these marginal utilities and also on the signal of the partial derivative of relative price with respect to δ.

The same argument applies to the case of numeraire b, since $b = (1-\pi)x_{b_+} - m/p_b$ implies that equation (6.9) is equivalent to (A.2). □

Result 1. ***Equilibrium Price and Demands of Example 3.1 - Shocks on Endowments***

	Numeraire a	***Numeraire b***
$\frac{p_a}{p_b}$	$\frac{b}{1-\gamma}\left(\frac{2(1+\delta)}{1+\sqrt{1+4(1+\delta)^2\epsilon^2}}\right)$	$\frac{a\gamma}{(1+\delta)(a^2-\gamma^2\epsilon^2)}$
$\frac{m}{p}$	$\frac{\gamma(1+\sqrt{1+4(1+\delta)^2\epsilon^2})}{2(1+\delta)} - a$	$\frac{a^2(1-\gamma)}{(1+\delta)(a^2-\epsilon^2\gamma^2)}$
x_1	$a_1 + a - \frac{\gamma(1+\sqrt{1+4(1+\delta)^2\epsilon^2})}{2(1+\delta)}$	$b_1 + b - \frac{(1-\gamma)a^2}{(a^2-\epsilon^2\gamma^2)(1+\delta)}$
x_{a_+}	$\frac{\gamma(1+\sqrt{1+4(1+\delta)^2\epsilon^2})}{2(1+\delta)} + \gamma\epsilon$	$a + \gamma\epsilon$
x_{a_-}	$\frac{\gamma(1+\sqrt{1+4(1+\delta)^2\epsilon^2})}{2(1+\delta)} - \gamma\epsilon$	$a - \gamma\epsilon$
x_{b_+}	$b + \frac{2(1+\delta)b\epsilon}{1+\sqrt{1+4(1+\delta)^2\epsilon^2}}$	$\frac{(1-\gamma)a}{(1+\delta)(a-\epsilon\gamma)}$
x_{b_-}	$b - \frac{2(1+\delta)b\epsilon}{1+\sqrt{1+4(1+\delta)^2\epsilon^2}}$	$\frac{(1-\gamma)a}{(1+\delta)(a+\epsilon\gamma)}$

Result 2. ***Equilibrium System - Section 3.2 - Shocks on Endowments***

Depending on a or b being the numeraire, equilibrium is, respectively, E_a or E_b:

$$E_a = \begin{cases} \frac{(1+\delta)u_1'}{p_a} = \frac{\pi}{p_a}\frac{\partial u_2}{\partial x_{a_+}} + \frac{1-\pi}{p_a}\frac{\partial u_2}{\partial x_{a_-}} \\ \frac{\partial u_2}{\partial x_{a_+}} = \frac{p_a}{p_b}\frac{\partial u_2}{\partial x_{b_+}} \\ \frac{\partial u_2}{\partial x_{a_-}} = \frac{p_a}{p_b}\frac{\partial u_2}{\partial x_{b_-}} \\ p_a x_1 + m = p_a a_1 \\ (1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_a\epsilon + p_b b) \\ (1+\delta)(p_a x_{a_-} + p_b x_{b_-}) = m + \tau + (1+\delta)(p_a a - p_a\epsilon + p_b b) \\ \overline{x}_1 + \pi\overline{x}_{a_+} + (1-\pi)\overline{x}_{a_-} - a_1 - a = 0 \end{cases}$$

$$E_b = \begin{cases} \frac{(1+\delta)u_1'}{p_b} = \frac{\pi}{p_a}\frac{\partial u_2}{\partial x_{a_+}} + \frac{1-\pi}{p_a}\frac{\partial u_2}{\partial x_{a_-}} \\ \frac{\partial u_2}{\partial x_{a_+}} = \frac{p_a}{p_b}\frac{\partial u_2}{\partial x_{b_+}} \\ \frac{\partial u_2}{\partial x_{a_-}} = \frac{p_a}{p_b}\frac{\partial u_2}{\partial x_{b_-}} \\ p_b x_1 + m = p_b b_1 \\ (1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_a\epsilon + p_b b) \\ (1+\delta)(p_a x_{a_-} + p_b x_{b_-}) = m + \tau + (1+\delta)(p_a a - p_a\epsilon + p_b b) \\ \pi\overline{x}_{a_+} + (1-\pi)\overline{x}_{a_-} - a = 0 \end{cases}$$

Result 3. ***Equilibrium Demands of Example 4.1 - Shocks on Preferences***

	Numeraire a	***Numeraire b***
$\frac{p_a}{p_b}$	$\frac{2b(1+\delta)}{(1-\gamma)(1+\sqrt{1+4(1+\delta)^2\epsilon^2})}$	$\frac{a\gamma}{(1+\delta)(a^2-\gamma^2\epsilon^2)}$
$\frac{m}{p}$	$\frac{\gamma\left(1+\sqrt{1+4\epsilon^2(1+\delta)^2}\right)}{2(1+\delta)} - a$	$\frac{(1-\gamma)a^2}{(1+\delta)(a^2-\gamma^2\epsilon^2)} - b$
x_1	$a_1 + a - \frac{\gamma\left(1+\sqrt{1+4\epsilon^2(1+\delta)^2}\right)}{2(1+\delta)}$	$b_1 + b - \frac{(1-\gamma)a^2}{(1+\delta)(a^2-\gamma^2\epsilon^2)}$
x_{a_+}	$\frac{\gamma\left(1+\sqrt{1+4\epsilon^2(1+\delta)^2}\right)}{2(1+\delta)} - (1-\gamma)\epsilon$	$a - (1-\gamma\epsilon)$
x_{a_-}	$\frac{\gamma\left(1+\sqrt{1+4\epsilon^2(1+\delta)^2}\right)}{2(1+\delta)} + (1-\gamma)\epsilon$	$a + (1-\gamma\epsilon)$
x_{b_+}	$b\left(1 + \frac{2(1+\delta)\epsilon}{1+\sqrt{1+4\epsilon^2(1+\delta)^2}}\right)$	$\frac{(1-\gamma)(a^2+a\epsilon\gamma)}{(1+\delta)(a^2-\gamma^2\epsilon^2)}$
x_{b_-}	$b\left(1 - \frac{2(1+\delta)\epsilon}{1+\sqrt{1+4\epsilon^2(1+\delta)^2}}\right)$	$\frac{(1-\gamma)(a^2-a\epsilon\gamma)}{(1+\delta)(a^2-\gamma^2\epsilon^2)}$

Result 4. ***Equilibrium System - Section 4.2 - Shocks on Preferences***

Depending on a or b being the numeraire, equilibrium is, respectively, F_a or F_b:

$$F_a = \begin{cases} \frac{(1+\delta)u_1'}{p_a} = \frac{\pi}{p_a}\frac{\partial u_2}{\partial x_{a_+}} + \frac{1-\pi}{p_a}\frac{\partial u_2}{\partial x_{a_-}} \\ \frac{\partial u_2}{\partial x_{a_+}} = \frac{p_a}{p_b}\frac{\partial u_2}{\partial x_{b_+}} \\ \frac{\partial u_2}{\partial x_{a_-}} = \frac{p_a}{p_b}\frac{\partial u_2}{\partial x_{b_-}} \\ p_a x_1 + m = p_a a_1 \\ (1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_b b) \\ (1+\delta)(p_a x_{a_-} + p_b x_{b_-}) = m + \tau + (1+\delta)(p_a a + p_b b) \\ \overline{x}_1 + \pi\overline{x}_{a_+} + (1-\pi)\overline{x}_{a_-} - a_1 - a = 0 \end{cases}$$

$$F_b = \begin{cases} \frac{(1+\delta)u_1'}{p_b} = \frac{\pi}{p_a}\frac{\partial u_2}{\partial x_{a_+}} + \frac{1-\pi}{p_a}\frac{\partial u_2}{\partial x_{a_-}} \\ \frac{\partial u_2}{\partial x_{a_+}} = \frac{p_a}{p_b}\frac{\partial u_2}{\partial x_{b_+}} \\ \frac{\partial u_2}{\partial x_{a_-}} = \frac{p_a}{p_b}\frac{\partial u_2}{\partial x_{b_-}} \\ p_b x_1 + m = p_b b_1 \\ (1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_b b) \\ (1+\delta)(p_a x_{a_-} + p_b x_{b_-}) = m + \tau + (1+\delta)(p_a a + p_b b) \\ \pi \overline{x}_{a_+} + (1-\pi)\overline{x}_{a_-} - a = 0 \end{cases}$$

Result 5. ***Equilibrium Demands of Example 6.1.1 - Degenerated Shocks on Endowments***

	Numeraire a	***Numeraire b***
$\frac{p_a}{p_b}$	$\frac{2b(1+\delta)}{(1-\gamma)\left(1+\sqrt{1+(1+\delta)^2a^2}\right)}$	$\frac{2\gamma}{a(1+\delta)(1-\gamma^2)}$
$\frac{m}{p}$	$\frac{\gamma\left(1+\sqrt{1+a^2(1+\delta)^2}\right)}{2(1+\delta)} - \frac{a}{2}$	$\frac{1}{(1+\delta)(1+\gamma)} - b$
x_1	$a_1 + \frac{a}{2} - \frac{\gamma\left(1+\sqrt{1+a^2(1+\delta)^2}\right)}{2(1+\delta)}$	$b_1 + b - \frac{1}{(1+\delta)(1+\gamma)}$
x_{a_+}	$\gamma\left(\frac{1+\sqrt{1+a^2(1+\delta)^2}}{2(1+\delta)} + \frac{a}{2}\right)$	$\frac{(1+\gamma)a}{2}$
x_{a_-}	$\gamma\left(\frac{1+\sqrt{1+a^2(1+\delta)^2}}{2(1+\delta)} - \frac{a}{2}\right)$	$\frac{(1-\gamma)a}{2}$
x_{b_+}	$b + \frac{ab(1+\delta)}{1+\sqrt{1+(1+\delta)^2a^2}}$	$\frac{1}{1+\delta}$
x_{b_-}	$b - \frac{ab(1+\delta)}{1+\sqrt{1+(1+\delta)^2a^2}}$	$\frac{(1-\gamma)}{(1+\delta)(1+\gamma)}$

Result 6. ***Equilibrium System - Section 6.1.2 - Degenerated Shocks on Endowments***

Depending on a or b being the numeraire, equilibrium is, respectively, G_a or G_b:

$$G_a = \begin{cases} \frac{(1+\delta)u_1'}{p_a} = \frac{\pi}{p_a}\frac{\partial u_2}{\partial x_{a_+}} + \frac{1-\pi}{p_a}\frac{\partial u_2}{\partial x_{a_-}} \\ \frac{\partial u_2}{\partial x_{a_+}} = \frac{p_a}{p_b}\frac{\partial u_2}{\partial x_{b_+}} \\ \frac{\partial u_2}{\partial x_{a_-}} = \frac{p_a}{p_b}\frac{\partial u_2}{\partial x_{b_-}} \\ p_a x_1 + m = p_a a_1 \\ (1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_b b) \\ (1+\delta)(p_a x_{a_-} + p_b x_{b_-}) = m + \tau + (1+\delta) + p_b b \\ \overline{x}_1 + \pi \overline{x}_{a_+} + (1-\pi)\overline{x}_{a_-} - a_1 - \pi a = 0 \end{cases}$$

$$G_b = \begin{cases} \frac{(1+\delta)u_1'}{p_b} = \frac{\pi}{p_a}\frac{\partial u_2}{\partial x_{a_+}} + \frac{1-\pi}{p_a}\frac{\partial u_2}{\partial x_{a_-}} \\ \frac{\partial u_2}{\partial x_{a_+}} = \frac{p_a}{p_b}\frac{\partial u_2}{\partial x_{b_+}} \\ \frac{\partial u_2}{\partial x_{a_-}} = \frac{p_a}{p_b}\frac{\partial u_2}{\partial x_{b_-}} \\ p_b x_1 + m = p_b b_1 \\ (1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_b b) \\ (1+\delta)(p_a x_{a_-} + p_b x_{b_-}) = m + \tau + (1+\delta) + p_b b \\ \pi \overline{x}_{a_+} + (1-\pi)\overline{x}_{a_-} - \pi a = 0 \end{cases}$$

Result 7. ***Equilibrium Demands of Example 6.2.1 - Degenerated Shocks on Preferences***

	Numeraire a	***Numeraire b***
$\frac{p_a}{p_b}$	$\frac{2b(1+\delta)}{1-\gamma}$	$\frac{1+\gamma}{2a(1+\delta)}$
$\frac{m}{p}$	$\frac{(1+\gamma)}{2(1+\delta)} - a$	$\frac{1-\gamma}{2(1+\delta)} - b$
x_1	$a_1 + \frac{a}{2} - \frac{\gamma\left(1+\sqrt{1+a^2(1+\delta)^2}\right)}{2(1+\delta)}$	$b_1 + b - \frac{1}{(1+\delta)(1+\gamma)}$
x_{a_+}	$\frac{\gamma}{1+\delta}$	$\frac{2a\gamma}{1+\gamma}$
x_{a_-}	$\frac{1}{1+\delta}$	$\frac{2a}{1+\gamma}$
x_{b_+}	$2b$	$\frac{1-\gamma}{1+\delta}$

Result 8. ***Equilibrium System - Section 6.2.2 - Degenerated Shocks on Preferences***

Let this system be called H_a or H_b, where its subscript indicates which commodity is the numeraire:

$$H_a = \begin{cases} \frac{(1+\delta)u_1'}{p_a} = \frac{(1-\pi)}{p_a}\frac{\partial u_2}{\partial x_{a_+}} + \frac{\pi}{p_a}\frac{\partial u_2}{\partial x_{a_-}} \\ \frac{1}{p_a}\frac{\partial u_2}{\partial x_{a_+}} = \frac{1}{p_b}\frac{\partial u_2}{\partial x_{b_+}} \\ p_a x_1 + m = p_a a_1 \\ (1+\delta)(p_a x_{a_-}) = m + \tau + (1+\delta)(p_a a + p_b b) \\ (1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_b b) \\ \overline{x}_1 + \pi \overline{x}_{a_-} + (1-\pi)\overline{x}_{a_+} - e - a = 0 \end{cases}$$

$$H_b = \begin{cases} \frac{(1+\delta)u_1'}{p_b} = \frac{(1-\pi)}{p_a}\frac{\partial u_2}{\partial x_{a_+}} + \frac{\pi}{p_a}\frac{\partial u_2}{\partial x_{a_-}} \\ \frac{1}{p_a}\frac{\partial u_2}{\partial x_{a_+}} = \frac{1}{p_b}\frac{\partial u_2}{\partial x_{b_+}} \\ p_b x_1 + m = p_b b_1 \\ (1+\delta)(p_a x_{a_-}) = m + \tau + (1+\delta)(p_a a + p_b b) \\ (1+\delta)(p_a x_{a_+} + p_b x_{b_+}) = m + \tau + (1+\delta)(p_a a + p_b b) \\ \pi \overline{x}_{a_-} + (1-\pi)\overline{x}_{a_+} - a = 0 \end{cases}$$

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