

# Bounded rationality, default and borrowing constraints

W. Daher\*    V.F. Martins-da-Rocha†    Mário Páscoa‡  
Y. Vailakis§

August 16, 2006

An important feature of the classical Arrow-Debreu model is that it imposes very weak informational requirements on the participants of the economy. Decision makers are assumed to know only their own characteristics. No assumptions are imposed on what they know or believe about other agents' characteristics and beliefs. Market prices convey all the relevant information about the economic environment and guide the economic agents to take their decisions.

While keeping the informational structure at a minimum level, the Arrow-Debreu model has often been questioned for its unrealism compared to the actual operation of markets. Among others, many of the criticisms have challenged the formal extension of the model to uncertain dynamic environments. The approach to model time and uncertainty appears to be too demanding since it requires the presence of an inordinate number of markets. At the beginning of time agents need to observe the prices at all conceivable states and dates. Related to this is the observation that the theory requires in principle a complete system of financial markets which appears to be too detailed to have any practical significance. Such a financial structure puts away the rich institutional structure of actual financial markets for allocating resources and providing opportunities for risk-sharing.

Allowing for a sequence of spot commodities and financial markets addresses some of these issues. Equilibrium allocations can now be imple-

---

\*Cermsem, Université Paris-1 and Mathematics Department at the American University of Beirut

†Ceremade, Université Paris-Dauphine

‡Faculdade de Economia, Universidade Nova de Lisboa

§CNRS-Cermsem, Université Paris-1 and School of Business and Economics, University of Exeter

mented by a simpler and more realistic structure of markets. However, the transition to a sequential markets model is not without cost, since it requires more stringent informational requirements on the participants of the economy. Traders need to form expectations not only about their own characteristics but also about future prices.

The *Perfect Foresight approach* to model sequential markets, attributed to Radner (1967) (see also Radner (1972) and Radner (1982)),<sup>1</sup> proposes an equilibrium concept based on the hypothesis that agents are able to forecast correctly the equilibrium prices. Although traders need not agree on the joint probability distribution of future events, they must believe that a unique future price will appear at each event. That is, agents are required to have degenerate and common price expectation functions (see Radner (1982)). In addition, the expectations are self-fulfilling in the sense that they correspond to the equilibrium prices at each event.

The hypothesis itself imposes a strong rationality on the participants of the economy. Agents need to observe not only the current prices but in addition the actual map from states to future prices. But this in turn implies that agents fully understand their economic environment and have a capacity for computation and communication far beyond what is realistic.

These considerations led a sizable literature in economics to propose a *bounded rationality* approach to model competition in uncertain dynamic environments. A common characteristic of bounded-rationality models is that agents have no structural information about their economic environment. Therefore, the models are consistent with the informational economy underlying the static Arrow-Debreu model. Agents have beliefs which become now part of the primitives of the economy. These beliefs determine their actions and feed back into the actual evolution of the economic variables.

*Temporary equilibrium models*, originated in the works of Grandmont (see Grandmont (1970), Grandmont (1977) and Green (1973)), constitute an important part of the bounded rationality paradigm. The conceptual framework of temporary equilibrium takes into account the possibility that agents may have neither correct nor common and degenerate expectations about the evolution of future prices. The equilibrium achieved at a given moment is only temporary since only the current actions are coordinated by spot prices. No coordination is required for future plans which may be incompatible if agents have incorrect expectations.

---

<sup>1</sup>If agents are using equilibrium prices to make inferences about the environment, then perfect foresight equilibrium takes the special form of a so called rational expectations equilibrium (Radner (1979)).

The temporary equilibrium approach has provided a coherent framework as an alternative to the perfect foresight paradigm. Moreover, it has permitted the investigation of a variety of new themes that were difficult to be explained using rational models: the role of money and the existence of liquidity traps, quantity rationing and price stickiness, the possibility of a positive role for government intervention.

Two are the most serious drawbacks of the temporary equilibrium approach. The first point to note is related to the possibility of *arbitrage* in spot markets. In the absence of *borrowing constraints* the size of traded contracts cannot be limited. For some patterns of expectations an agent may find it profitable to take an extreme short position. It is then clear that the existence of equilibrium cannot be guaranteed for all expectation patterns.

The problem then turns out to find necessary conditions on expectation patterns that are consistent with equilibrium in spot markets. Several assumptions have been proposed in the literature depending on the particular environment under study (see Green (1973), Hart (1974), Grandmont (1977), Hammond (1983)). Roughly speaking, all imply that traders need to partially agree on a sufficient large set of future prices. These conditions, known as *overlapping expectation conditions*, appear to be rather stringent since they imply an unrealistic uniform perception for future price uncertainty. Subsequent research has challenged the role and relevance of overlapping expectations. Milne (1980) argues for borrowing constraints that reflect lenders' perception for default risk. Alternatively, Stahl (1985a) (see also Stahl (1985b)) emphasizes the need for institutional borrowing constraints.

The second point to note is related to the issue of *bankruptcy*. Even when traders' expectations overlap, the achieved equilibrium outcome in spot markets may lead to bankruptcy in subsequent periods due to inaccurate price expectations. Grandmont in his excellent survey (see Grandmont (1982)) points out this problem and argues for the design of *default rules* in temporary equilibrium models. These rules may restore equilibrium in future periods and lead to equilibrium allocations that reflect the extent of bankruptcy as a decision of the agents themselves.

The paper follows the bounded rationality approach as an alternative to model sequential competitive markets. Our main motivation is to provide a framework that keeps the desirable characteristics of bounded-rationality models (informational economy) and simultaneously addresses the shortcomings encountered in temporary equilibrium models (overlapping expectations, bankruptcy).

We believe that a fruitful market mechanism to prevent bankruptcy in

future periods and to ensure existence of equilibrium at the current period has to allow for the possibility of default while simultaneously protecting short-sales of assets through collateral requirements. In traditional temporary equilibrium models the only penalty a bankrupt agent may suffer, is a zero consumption bundle in the period of default. There are several reasons that justify the need for default penalties. First, without default, the creditors' willingness to lend depends crucially on their expectations to be repaid. Second, with low or zero default penalties, borrowers may not choose to repay their debts even though they are able. In either case the possibility of bankruptcy increases. It is therefore intuitive that allowing for default penalties which are proportional to the extent of bankruptcy leaves room for restoring equilibrium in future periods. An analogous argument applies if we allow for the possibility of collateralized promises. The intuition is that even if agents made bad expectations about future prices, the market will seize their collateral at the second period and the economy will not collapse. In addition, the endogenous nature of collateral requirements has an additional side impact. It allows for the dispense of overlapping expectation conditions.

The implications of default and collateral date back many years to the work of Dubey, Geanakoplos, and Zame (1990), Kehoe and Levine (1993), Zame (1993), and more recently to Araujo, Páscoa, and Torres-Martinez (2002), Dubey, Geanakoplos, and Shubik (2005). This literature highlights important features of default that may be desirable with incomplete opportunities for risk sharing. However, with no exception, all studies rely on a perfect foresight formulation. Therefore, the models can still be questioned on the basis of the informational requirements that justify rationality. In addition, the informational requirements are even more demanding in these models since agents have to be able to predict not only the future prices but also the delivery rates for all assets.

This study provides a reappraisal of the implications of default and collateral in a different setting that abstracts from the rational paradigm by allowing agents to be less sophisticated. We formulate and analyze a two-period model that is in close relation with the traditional temporary equilibrium models, but it deviates from them by allowing for durable goods, collateral and the possibility of default.

We propose a new equilibrium notion (*beliefs equilibrium*) that is simultaneously free of stringent informational requirements and consistent with market clearing in both periods. The equilibrium existence result is builded gradually in order to highlight the role played by default and collateral in achieving the equilibrium outcome. It is shown that under overlapping ex-

expectations default always implies the existence of beliefs equilibria. In addition, the reliance on collateral to secure loans, allows us to dispense with the overlapping expectation conditions. A final remark concerns with the development of the existence proof. Although technically is more demanding, the argument draws on a variety of standard methods.

Section 2 presents the theoretical framework and outlines the basic model. In section 3 we formally define the model's equilibrium concept (beliefs equilibrium) and show how this concept differentiates, in terms of informational requirements, from the corresponding perfect foresight solution. The existence proof is builded gradually in sections 4 and 5. In this way we formally disentangle the role of default and collateral and show that each has a bite. Section 4 investigates under which condition a family of beliefs is a temporary equilibrium. Given that a necessary condition is the absence of arbitrage opportunities in the first period, we present three alternative sets of assumptions that exclude arbitrage, namely, exogenous bounds on short sales, overlapping expectations and collateral. It is shown that collateral provides a more realistic way to exclude arbitrage opportunities and insure the existence of temporary equilibrium. Exogenous bounds are difficult to justify economically. On the other hand, overlapping expectations put serious restrictions on expectations patterns. Most important, when default is allowed, overlapping expectations is no more a sufficient condition for the existence of temporary equilibrium. This is because it does not preclude the possibility that for given default penalties agents may go simultaneously long and short in the same security. We provide an example that highlights this point. In section 5 we turn our attention to the existence of equilibrium at the second period. We present a simple example of an economy with overlapping expectations and no default that admits a temporary equilibrium but a beliefs equilibrium always fails to exist. The example shows that bankruptcy is possible when some consumers with price expectations far away from the equilibrium prices take an extreme sort position in the spot markets. We subsequently show that allowing for default is sufficient to restore equilibrium at the second period, since it excludes the situation described in our example. Section 6 concludes.

## References

- ARAUJO, A., M. R. PÁSCOA, AND J.-P. TORRES-MARTINEZ (2002): "Collateral avoids Ponzi schemes in incomplete markets," *Econometrica*, 70(4), 1613–1638.

- DUBEY, P., J. GEANAKOPOLOS, AND M. SHUBIK (2005): “Default and punishment in general equilibrium,” *Econometrica*, 73(1), 1–37.
- DUBEY, P., J. GEANAKOPOLOS, AND W. R. ZAME (1990): “Default and efficiency in a General Equilibrium Model with incomplete markets,” Discussion paper, Cowles Foundation.
- GRANDMONT, J.-M. (1970): “On the temporary competitive equilibrium,” Ph.D. Dissertation, University of California, Berkeley.
- (1977): “Temporary general equilibrium theory,” *Econometrica*, 45(3), 535–572.
- (1982): “Temporary General Equilibrium Theory,” in *Handbook of mathematical economics, Vol. II*, vol. 19 of *Handbooks in Econom.*, pp. 879–922. North-Holland, Amsterdam.
- GREEN, J. R. (1973): “Temporary general equilibrium in a sequential trading model with spot and futures transactions,” *Econometrica*, 41, 1103–1123.
- HAMMOND, P. J. (1983): “Overlapping expectations and Hart’s condition for equilibrium in a securities model,” *Journal of Economic Theory*, 31(1), 170–175.
- HART, O. D. (1974): “On the existence of equilibrium in a securities model,” *Journal of Economic Theory*, 9(3), 293–311.
- KEHOE, T., AND D. K. LEVINE (1993): “Debt-constrained asset markets,” *Review of Economic Studies*, 60, 865–888.
- MILNE, F. (1980): “Short-selling, default risk and the existence of equilibrium in a securities model,” *International Economic Review*, 21(2), 255–267.
- RADNER, R. (1967): “Equilibre des marchés à terme et au comptant en cas d’incertitude,” *Cahiers d’Econométrie*, 4, 35–52, (C.N.R.S., Paris).
- (1972): “Existence of equilibrium of plans, prices, and price expectations in a sequence of markets,” *Econometrica*, 40, 289–303.
- (1979): “Rational expectations equilibrium: generic existence and the information revealed by prices,” *Econometrica*, 47(3), 655–678.

- (1982): “Equilibrium under uncertainty,” in *Handbook of mathematical economics, Vol. II*, vol. 20 of *Handbooks in Econom.*, pp. 923–1006. North-Holland, Amsterdam.
- STAHL, D. (1985a): “Bankruptcies in temporary equilibrium forward markets with and without institutional restrictions,” *Review of Economic Studies*, 52(3), 459–471.
- (1985b): “Relaxing the sure-solvency conditions in temporary equilibrium models,” *Journal of Economic Theory*, 37(1), 1–18.
- ZAME, W. R. (1993): “Efficiency and the role of default when security markets are incomplete,” *American Economic Review*, 83, 1142–1164.