Imperfect Enforcement and Foreign Investment: A Rationale for Multilateral Organizations

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March 1999

Abstract
This paper examines the role of multilateral organizations in directing private capital flows when enforcement is inadequate. We analyze the investment decision of a country that faces a repayment incentive constraint. It is well known that “under investment” occurs when this constraint binds. As a consequence of the market failure (i.e., inadequate contractual enforcement), there is scope for policy intervention designed to promote private investment. We consider two policies that are frequently used by multilateral organizations: technical assistance and loan subsidies. We show that these policies improve private investment flows and welfare, and hence provide a rationale for the operation of organizations such as the World Bank in conjunction with private markets.

Key words: Enforcement, Foreign Investment and Multilateral Organizations.

JEL Classification: F34.

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1. Introduction

Are multilateral organizations such as the World Bank necessary? This question has been asked frequently in recent years. Private foreign investment in developing countries has grown significantly, increasing from $43 billion in 1990 to about $256 billion in 1997. As a consequence, some have argued that multilateral organizations may no longer be necessary since “the market” will direct capital flows appropriately. In this paper we analyze the role of such organizations when enforcement of contracts is inadequate and under investment occurs as a result of this market failure. When under investment is caused by enforcement problems, we show that it can be mitigated by two policies often used by multilateral organizations: technical assistance and loan subsidies. We show that these policies promote the flow of private capital and enhance welfare.

One of the crucial problems pertaining to international lending is the lack of a supra-national legal authority that can enforce private contracts across borders. As a result, foreign capital is subject to endogenous expropriation risk: settings where borrowers may simply choose not to repay their debt. This indicates that debt repayment in an international setting is contingent on the borrowers’ willingness to pay rather than ability to pay (cf., Eaton and Gersovitz (1981)). In their seminal paper, Eaton and Gersovitz showed that under investment occurs when foreign capital is subject to endogenous expropriation risk. Rational investors anticipate this, which imposes a repayment incentive constraint on the optimal contracting problem. This constraint causes investment to fall short of the unconstrained optimal level. Because an effective supra-national judicial system with the authority to enforce contracts is unlikely, we consider policy interventions that mitigate the under investment distortion caused by the incentive constraint. We study the effects of technical assistance and loan subsidies, two policies often used by multilateral organizations and show that these policies reduce expropriation risk and can, in principle, restore investment to its unconstrained optimal level. Thus, by providing these services, multilateral organizations indirectly serve as enforcers of private contracts in an international setting.

The paper considers a model of imperfect enforcement and shows that a country’s investment problem can be reduced to a comparison of the country’s discount factor $\beta$ and two critical thresholds, $\beta^*$ and $\beta^\prime$, that are derived in Asiedu and Villamil (1999). These thresholds segment the unit interval on

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1 Although there was a decline in portfolio equity flows in the last quarter of 1997 as a result of the Asian crisis, an increase in other forms of investment resulted in a net increase in foreign investment.
3 Expropriation is the violation of any condition of an investment agreement. For example, government default on foreign investment contracts or guarantees, preventing domestic residents from honoring obligations to foreign creditors, etc.
which $\beta$ is defined into three disjoint sub-intervals that correspond to the three possible equilibria for investment flows: autarky, under investment and unconstrained optimal investment. Since $\beta$ is a “deep structural parameter” that is difficult to alter at a point in time, we focus on policy interventions that can alter the thresholds. We show that technical assistance and loan subsidies affect these thresholds by decreasing $\beta$ and $\beta^*$ and thereby increasing the “good” sub-interval where unconstrained investment occurs and shrinking the “bad” sub-interval where autarky occurs. Thus, these policies increase the likelihood that a country will be able to access private capital markets and/or sustain the unconstrained optimal level of investment. Moreover, these policies increase welfare.

Our work is clearly related to recent work on self-enforcing contracts by Atkeson (1991), Eaton (1993), Thomas and Worrall (1994), and Krasa and Villamil (1999). In addition, it is related to the well-known Folk Theorem that for $\beta$ sufficiently close to one, expropriation will never occur (cf., Chari and Kehoe (1990)). While this Folk Theorem is true in our model, our perspective is different. Since $\beta$ is often difficult to alter at a point in time (as noted above), we examine how specific policy interventions by multilateral organizations can be used to alter the thresholds $\beta$ and $\beta^*$. Finally, our analysis is related to recent work by Parente and Prescott (1999a and 1999b) on barriers to riches. They argue (1999a, p. 1) that poor countries are poor because some individuals or groups are able to block the adoption of superior technologies, and that without this ability the country “will accumulate any human and physical capital needed to operate the better technologies.” Their thesis assumes a market economy with well-defined property rights.4 Our analysis is complementary in that it considers the under investment problem that occurs when property rights are not well defined or enforced.5

Section 2 specifies the model. Section 3 shows that technical assistance and loan subsidies, can play a crucial role in facilitating private capital flows to developing countries when enforcement is problematic. Finally, Section 4 concludes. All proofs are in the Appendix.

2. The Model and Equilibria

Consider an economy with an infinite time horizon and agents of two types: domestic and foreign. Domestic agents are identical and have a common risk neutral utility function.6 The domestic

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4 Contractual enforcement is a problem in most developing countries. On a scale of 1-10, the degree to which contracts are enforceable, averaged over the period 1990-95, was as low as 2.3 for Haiti, 5.8 for Nigeria, and 7.3 for China compared to 10 for Switzerland and Japan (c.f., International Country Risk Guide).

5 Parente and Prescott (1999a, p. 11) discuss promises written into investment codes in many Sub-Saharan countries where under investment is prevalent. As they note, it is not clear whether the promises in these codes are credible (because they are unenforceable).

6 We focus on the risk neutral case to study the effect of “pure enforcement problems” on investment. Under investment would be even more severe if the host used international capital markets for consumption insurance (i.e.,
country has access to a constant returns to scale production function $F(.)$ which requires two inputs, labor and capital. Let $f(.)$ denote output per capita, which satisfies the Inada conditions. Labor is provided inelastically by domestic residents. Risk neutral foreign agents provide capital to supplement domestic capital and earn a competitive return given by the gross world interest rate $r$, which we assume is constant over time. Let $k$, $k'$, and $k^d$ be the aggregate, foreign and domestic capital stock per capita, respectively. Assume that capital depreciates completely, that the domestic capital stock is constant, and that the foreign capital stock is elastic (i.e., we consider a small open economy). Assume further that the country’s domestic capital stock is less than the optimal capital stock. The domestic country does not invest abroad, and its capital stock neither depreciates nor can be augmented. As a consequence, the country’s current output net of interest payments is consumed in the current period.

The host country chooses an investment plan to maximize the present discounted utility of income denoted by

$$W_t = \sum_{s=t}^{\infty} \beta^{s-t} y_s$$

with $0 < \beta < 1$ and $y_s = f(k_s) - r(k_s - k^d)$. As is standard (cf., Yaari (1965)), $\beta$ is the discount factor with $\hat{\beta} = \theta \rho$, where $\theta$ is the probability of survival, an idiosyncratic factor which reflects the “patience” of decision-makers in a particular country, and $\rho = \frac{1}{r}$ is the common pure discount factor determined by the world market. As the world interest rate rises $\beta$ falls, indicating that all countries value the future less highly. In contrast, as $\theta$ falls the “country specific” $\beta$ falls, indicating that the country becomes more myopic. Following Barro and Sala-i-Martin (1995, p. 439), $\theta$ can be interpreted as a measure of country specific risk. See Asiedu and Villamil (1999) for measures of $\theta$ and $\beta$ for forty countries.\(^7\)

The host country faces a stationary problem, thus the optimal investment plan is time invariant. In any period $t$, the host has the option to renege on foreign investment agreements. The consequence of this action is that the host loses access to international capital markets in subsequent periods.\(^8\) If default occurs in any period $t$, for all future periods $s > t$, foreign investors abstain from the market (i.e., $k_s = k^d$ and $k^f = 0$). Clearly the need for international capital provides some incentive for borrowers to honor investment agreements in intertemporal problems, and the host will expropriate only if the gain exceeds the agent’s utility.

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\(^7\) The measure of country risk, $\theta$, is based on the average of five risk indicators: contract repudiation, expropriation risk, government corruption, the rule of law and the quality of the bureaucracy.

\(^8\) See Eaton and Fernandez (1995) and Klezler (1994) for a discussion on various forms of default penalties in international lending.
the cost. Since countries that expropriate eventually return to world capital markets, the punishment of complete exclusion in the future may seem severe. Interestingly, when agents discount the future highly even this severe punishment is not strong enough to deter default.

We now analyze the incentive constraint. The discounted present value from reneging on an agreement in period \( t \) and remaining autarkic thereafter is given by

\[
B(k_t) = f(k_t) + \sum_{s=t+1}^{\infty} \beta^{s-t} y_s = f(k) + \frac{\beta}{1-\beta} f(k^d)
\]

The discounted present value of not reneging on an agreement in period \( t \) and maintaining access to international capital markets is given by

\[
G(k_t) = \sum_{s=t}^{\infty} \beta^{s-t} y_s = \sum_{s=t}^{\infty} \beta^{s-t} [f(k_s) - r(k_s - k^d)] = \frac{1}{1-\beta} [f(k) - r(k - k^d)]
\]

As a consequence of stationarity, \( B(k) \) and \( G(k) \) are time invariant.

The problem solved by a benevolent social planner can now be specified. The planner chooses a stationary level of aggregate investment \( k \) to maximize \( W(k) \) subject to a repayment incentive constraint which ensures that the host will honor investment agreements, where \( k^d \) is given and \( k = k^d + k^f \).

**Problem 1.** Choose \( k \) to maximize \( W(k) = \frac{1}{1-\beta} [f(k) - r(k - k^d)] \) subject to:

\[
(1) \quad B(k) \leq G(k), \quad \forall \; k
\]

The host maximizes the discounted utility of output subject to incentive constraint (1). When (1) holds, contracts are self enforcing, i.e., honored voluntarily. Let \( \lambda \) be the LaGrange multiplier. The first order conditions are \( B(k) = G(k) \) and:

\[
(2) \quad f'(k) \cdot r = \frac{r\lambda (1 - \beta)}{1 + \lambda \beta}
\]

It is evident in Problem 1 that the host country's welfare depends on its domestic capital stock, \( k^d \), and its idiosyncratic discount factor \( \beta = \frac{\theta}{r} \), regardless of whether or not enforcement is a problem. When enforcement is imperfect, country specific characteristics \( k^d \) and \( \beta(\theta,r) \) play an additional role: they determine, in conjunction with \( r \), a country's ability to attract foreign capital, \( k^f \). In contrast when enforcement is perfect, \( r \) is sufficient to determine capital flows and each country receives the unconstrained optimal level of investment. This crucial observation motivates the analysis that follows.
Define the right hand side of (2) by \[ \Omega(\beta, \lambda, r) = \frac{r\lambda(1-\beta)}{1+\lambda\beta}. \]

\( \Omega \) is the wedge between the marginal product of capital and the return on capital, which is the efficiency loss due to inadequate contractual enforcement. When enforcement is imperfect and (1) binds, this causes \( \lambda > 0 \) and the well known under investment result follows immediately. \( \Omega(\beta, \lambda, r) \to 0 \) as:

(i) \( \beta \to 1 \): There is less inefficiency when agents care more about the future.

(ii) \( r \to 0 \): Low interest rates reduce inefficiency.

(iii) \( \lambda \to 0 \): There is less inefficiency when the constraint is weak.

The paper focuses on policy interventions that weaken the constraint.

The solutions to Problem 1 are described by one of three cases. Let \( k_u^* \) denote the optimal capital sequence when (1) does not bind (the unconstrained optimal plan) and \( k_c^* \) denote the plan when (1) binds (the constrained optimal plan). Then:

Case 1. \( B(k) > G(k) \) for all \( k > k^d \): The constraint set is empty and no foreign investment occurs.

Case 2. The constraint binds (i.e., \( \lambda > 0 \)): \( f'(k) > r \) and \( k = k_c^* \) is the optimal investment plan.

Case 3. The constraint does not bind (i.e., \( \lambda = 0 \)): \( f'(k) = r \) and \( k = k_u^* \) is the optimal investment plan.

In Case 1 the benefit from expropriation exceeds the gain from not expropriating for all levels of investment. As a result, the host will always expropriate. Foreign investors realize the incentive for the host to expropriate, hence in equilibrium no foreign investment occurs. In Case 2 the constraint binds and this leads to under investment relative to the unconstrained optimal plan (i.e., Case 3). Under investment is optimal given that (1) binds as this is a version of Kydland and Prescott’s (1977) time consistency result. The constrained equilibrium is Pareto inferior relative to the unconstrained equilibrium where there is full commitment to the ex-ante optimal plan, but no “commitment technology” exists which can induce agents to adhere to the “first best” intertemporal plan ex post (cf., Krasa and Villamil (1999)).

When there is no supra-national legal authority (e.g., court) that can enforce contracts across borders, contracts must be self-enforcing. As in Atkeson (1991), Eaton (1993), and Thomas and Worrall (1994), a contract is self-enforcing if it is incentive compatible for the agent to honor prior commitments (i.e., to not expropriate). Asiedu and Villamil (1999) characterize the set of self-enforcing investment plans as the self-enforcing interval \([k^d, \bar{k}]\), where \( B(\bar{k}) = G(\bar{k}) \) and \( B(k) < G(k) \) for \( k \in [k^d, \bar{k}] \).

(i) \( k_u^* \in [k^d, \bar{k}] \), then (1) does not bind and \( k_u^* \) is optimal.
(ii) If \( k^*_u \notin [k^d, k^-] \), then (1) binds, \( B(k) = G(k) \), and \( k^*_u = k^- \) is the maximum investment level for which \( k \) is self-enforcing.

The upper bound of the interval, \( k^- \), can be interpreted as a debt ceiling. If \( k^*_u > k^- \), the country faces a borrowing constraint. If \( k^*_u \leq k^- \), the country can borrow as much as it desires.\(^9\)

Asiedu and Villamil (1999) show that solutions to Problem 1 can be completely characterized by restrictions on the discount factor that segment the unit interval into three cases. The equilibrium is then completely determined by a comparison of \(  \beta \) and the relevant case on the unit interval. Define these two critical thresholds by:

\[\beta^*: The \ minimum \ discount \ factor \ for \ which \ plan \ k^*_u \ is \ self-enforcing; \ and \]

\[\beta: The \ minimum \ discount \ factor \ required \ to \ attract \ foreign \ investment.\]

These thresholds \( \beta(\theta, r) \) and \( \beta^*(\theta, r) \) segment the unit interval into three sub-intervals which correspond to Cases 1, 2, and 3 such that \( 0 < \beta < \beta^* < 1 \).\(^{10}\) A foreign agent’s investment decision is thus effectively a comparison of the host country’s idiosyncratic \( \beta(\theta, r) \) and the relevant sub-interval: Case 1 prevails when \( \beta \in (0, \beta) \), Case 2 prevails when \( \beta \in [\beta, \beta^*) \), and Case 3 prevails when \( \beta \in [\beta^*, 1) \). The three cases are depicted in Figure 1.

**Figure 1**

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(-------)</td>
<td>[--------)</td>
<td>[--------)</td>
</tr>
<tr>
<td>0</td>
<td>(\beta)</td>
<td>(\beta^*)</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 1 indicates that countries with low discount factors will have difficulty attracting foreign investment. Recall that \( \beta = \frac{\theta}{r} \), where \( \frac{1}{r} \) is the pure (market) discount factor and \( \theta \) is the probability of survival. By linking the discount factor, \( \beta \), to the country specific \( \theta \), the model provides a plausible explanation for the inability of some countries to attract foreign investment. When \( \beta(\theta, r) \) is very low, the

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\(^9\) Eaton and Gersovitz (1981) provide estimates of the probability that a country will face a borrowing constraint for 45 developing countries.

\(^{10}\) See Appendix B for a sketch of the proof for the derivation of \( \beta \) and \( \beta^* \).
country is more likely to be in the “low” Case 1 equilibrium with \( \beta(\theta, r) \in (0, \bar{\beta}) \). Barro and Sala-i-Martin’s (1995, p. 440) and Mauro’s (1995) empirical results indicate that effective legal and political institutions are statistically significant for explaining growth. These results are consistent with the model. Effective institutions will tend to raise \( \theta \), and therefore \( \beta(\theta, r) \), thus increasing the likelihood that a country is in Case 2 or 3. As a consequence, the country will be able to sustain some level of foreign investment. Similarly, it is well known that stability is correlated with successful development and that instability is associated with “growth disasters” (cf., Parente and Prescott (1986)). This result is also consistent with the predictions of the model.

3. External Assistance

Our model indicates that when enforcement is imperfect, a country’s fundamental characteristics affect its ability to borrow from abroad. In particular, when enforcement of private investment contracts is problematic, low discount factor countries may be unable to attract private foreign investment (i.e., Case 1) and investment in countries with a moderate discount factor may be constrained (i.e., Case 2). Thus some countries may be stuck in a “bad” equilibrium or poverty trap. Are there policies that can be used to move such economies to Pareto superior outcomes? To answer this question, we turn to an analysis of multilateral agencies, such as the World Bank, whose main objective is to promote long term development and poverty reduction in developing countries. We analyze two policies, technical assistance and loan subsidies, that are often used by multilateral organizations and examine their effect on private foreign investment flows. The class of policies that we consider affect the threshold discount factors, \( \bar{\beta} \) and \( \beta^* \) but does not affect the country specific discount factor \( \beta(\theta, r) \). We show that these policies shrink the intervals \((0, \bar{\beta})\) and \([\bar{\beta}, \beta^*]\) and hence make it is more likely that countries with low \( \beta(\theta, r) \) will be able to sustain some level of private foreign investment. In the final section we discuss the implication of our results for domestic policies by individual countries. Our concern in this section is how policies by a multilateral organization designed to affect a country’s gain and loss from expropriation can ameliorate the market failure arising from inadequate contractual enforcement. The goal is to determine how these policies affect private foreign capital flows ceteris paribus. We also provide estimates of policy parameters and numerical examples to illustrate our comparative static results. The examples are presented in Appendix A.

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11 See Asiedu and Villamil (1999) for a discussion of policies individual countries can follow in order to increase \( \beta \) and therefore increase the likelihood that a country will be in the good equilibrium.
Policy 1: Technical Assistance

Multilateral organizations often provide free technical assistance to developing countries in the form of technical cooperation grants. These grants take two forms: free-standing grants, which are intended to finance the transfer of technical and managerial skills for the purpose of building national capacity without reference to any specific investment project; and investment related technical assistance, which is provided to strengthen the capacity to execute specific investment projects. Over the period 1990-97, technical cooperation grants to low-income countries averaged $18 billion annually (cf., World Bank (1998)). Given the nature of technical assistance, it is reasonable to assume that the withdrawal of these services will result in an efficiency loss in production. We now modify the model to allow the host to have access to technical assistance. The penalty for default is two-fold: The host loses access to international capital markets and the multilateral agency ceases providing technical assistance to the host country. Finally, we assume that the host country loses a fraction of its output when technical assistance is withdrawn.

Let \( \delta \) be the fraction of output lost when technical assistance is withdrawn.\(^{12}\) The utility from not expropriating, \( G(k) \), remains the same. The utility from expropriating is now given by

\[
B(k) = f(k) + \frac{\beta (1-\delta)}{1-\beta} f(k^d)
\]

We first show that the provision of technical assistance eliminates the inefficient autarky equilibrium (i.e., Case 1). Next, we analyze the effect of a technical assistance policy on \( \beta^*, k^*_c \) and \( y^*_c \). The results are stated in Proposition 1 and Claim 1 respectively.

**Proposition 1.** Technical assistance eliminates the autarky equilibrium.

Proposition 1 indicates that countries that receive technical assistance will sustain some foreign investment although total investment may be constrained. This implies that \( \bar{\beta} = 0 \), suggesting that this type of policy may be especially useful in countries with poor enforcement, i.e., low \( \beta(\theta, r) \) countries. For completeness we note that when the constraint does not bind (i.e., Case 3), changes in \( \delta \) have no effect on total investment. In this equilibrium, \( \beta > \beta^* \). This indicates that high \( \beta(\theta, r) \) countries do not require external assistance of this type to sustain the unconstrained optimal level of investment. In this sense, the

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\(^{12}\) We provide estimates of \( \delta \) for 24 developing countries in Table 1.
critics of the World Bank are correct. Not all regions need assistance from multilateral agencies in order to attract foreign investment.

**Claim 1.** *When the incentive constraint binds, Technical Assistance has the following effects:*

\[
\begin{align*}
(a) \quad & \frac{d \beta^*}{d \delta} < 0; \quad (b) \quad \frac{d k^*_c}{d \delta} > 0; \quad \text{and} \quad (c) \quad \frac{d y^*_c}{d \delta} > 0 .
\end{align*}
\]

Claim 1 suggests that “constrained countries” that receive technical assistance from multilateral organizations will attract more private foreign investment. The reason is that the withdrawal of technical assistance, in the event of expropriation constitutes a bigger penalty. This higher penalty, $\delta$, lowers $\beta^*$ and raises both $k^*_c$ and $y^*_c$. Since utility increases with $\delta$, the equilibrium attained under technical assistance is Pareto superior to that attained when there is no technical assistance. Moreover, by decreasing $\beta^*$, technical assistance expands the “good” interval $(\beta^*, 1)$ and therefore increases the likelihood that low and moderate $\beta$ countries can sustain the unconstrained optimal level of investment. Thus when enforcement is problematic, the provision of technical assistance increases total investment and improves welfare. Further, the maximum $\delta$ that a multilateral organization should provide as a proxy for enforcement is well defined and given by $\hat{\delta} = y^* - y^*_c$.

We now present measures of $\delta$ for 24 countries. Further, example 1 in Appendix A provides a numerical example that clearly depicts Proposition 1 and Claim 1.
### Table 1

Technical Assistance Data for Selected developing Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Technical Assistance /GNP (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>0.06</td>
</tr>
<tr>
<td>Bangladesh</td>
<td>1.10</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.05</td>
</tr>
<tr>
<td>Burkina Faso</td>
<td>5.11</td>
</tr>
<tr>
<td>Cameroon</td>
<td>1.30</td>
</tr>
<tr>
<td>Chile</td>
<td>0.11</td>
</tr>
<tr>
<td>China</td>
<td>0.09</td>
</tr>
<tr>
<td>Colombia</td>
<td>0.20</td>
</tr>
<tr>
<td>Congo</td>
<td>1.22</td>
</tr>
<tr>
<td>Ghana</td>
<td>1.49</td>
</tr>
<tr>
<td>Haiti</td>
<td>3.01</td>
</tr>
<tr>
<td>India</td>
<td>0.14</td>
</tr>
<tr>
<td>Indonesia</td>
<td>0.29</td>
</tr>
<tr>
<td>Malaysia</td>
<td>0.20</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.03</td>
</tr>
<tr>
<td>Niger</td>
<td>5.08</td>
</tr>
<tr>
<td>Nigeria</td>
<td>0.35</td>
</tr>
<tr>
<td>Peru</td>
<td>0.36</td>
</tr>
<tr>
<td>Philippines</td>
<td>0.57</td>
</tr>
<tr>
<td>Russia</td>
<td>0.06</td>
</tr>
<tr>
<td>Sierra Leone</td>
<td>3.56</td>
</tr>
<tr>
<td>Swaziland</td>
<td>2.97</td>
</tr>
<tr>
<td>Togo</td>
<td>3.18</td>
</tr>
<tr>
<td>Zambia</td>
<td>4.38</td>
</tr>
</tbody>
</table>


We use the ratio of technical assistance to GNP as a measure of $\delta$, the fraction of output lost when technical assistance is withdrawn. Data for $\delta$ is averaged over the period 1989-96. As shown in Table 1, the cost of default, $\delta$, varies widely across country, taking on values as low as 0.1 percent for Argentina and as high as 4 percent and 5 percent for Zambia and Burkina Faso respectively.
Policy 2: Loan Subsidies

Official loans are an important source of finance for many developing countries. For instance, over 90 percent of foreign investment to Sub-Saharan Africa, the poorest region, are from official sources. Indeed, the primary role of the International Development Agency (IDA), an affiliate of the World Bank, is to provide credit to the Bank’s low-income members. Moreover, most official loans are made on concessional terms—they either carry below market interest rates and/or take the form of grants. Grants are funds for which there is no repayment requirement. For example, IDA credits have a 35-40 year maturity, are interest free, and carry a service charge of 0.75 percent. In this section we examine how the provision of subsidized loans affect private foreign investment and the incentive to expropriate. We also present data on private and official lending to 24 developing countries.

Suppose that in addition to borrowing from private international capital markets, the host can borrow from a multilateral agency at a below market rate. Assume further that the capital from official sources is limited in supply. The penalty for expropriation is that the host country permanently loses access to private and official loans. Let $\tau$ be the gross return on the subsidized loan, and $\gamma$ be the fraction of outside investment provided by the multilateral agency. The amount of investment provided by the multilateral organization at interest rate $\tau$ is $\gamma(k - k^d)$. The remaining outside investment, $(1 - \gamma)(k - k^d)$, is provided by private foreign investors. The utility from expropriation, $B(k)$, is unchanged. The utility from not expropriating, $G(k)$, is now given by

$$G(k) = \frac{1}{1 - \beta} [f(k) - r(1 - \gamma)(k - k^d) - \tau \gamma(k - k^d)]$$

We now report comparative static results for this policy. How do changes in $\gamma$ and $\tau$ affect $\beta$, $\beta^*$, $k^*_e$, and $y^*_e$?

Claim 2. When the incentive constraint binds, Loan Subsidies have the following effects:

(a) $\frac{d \beta}{d \gamma} < 0$ and $\frac{d \beta^*}{d \gamma} < 0$; (b) $\frac{d \beta}{d \tau} > 0$ and $\frac{d \beta^*}{d \tau} > 0$;

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13. Official loans refer to loans from multilateral and bilateral organizations.
14. If $\tau = 0$, the subsidized loan can be interpreted as a grant or debt forgiveness. Further, $\tau = 1$ implies an interest free loan and $\gamma = 0$ indicates no external assistance. Finally, we assume that all funds are used for investment. Fraudulent diversion of the funds for any other purpose would clearly alter the results.
Claim 2 shows that the threat of losing access to subsidized loans in the event of expropriation provides an incentive for borrowers to honor investment contracts. As the fraction of investment from multilateral organizations, $\gamma$, increases, the threshold discount factor, $B$, decreases. A similar result holds for $\beta^*$. Utility increases with $\gamma$ and decreases with $\tau$, thus Policy 2 improves welfare. Further, an increase in $\gamma$ increases the level of foreign investment. This result is consistent with empirical facts. Based on data for 45 developing countries, Eaton and Gersovitz (1981) conclude that countries that borrow from official sources attract more private foreign investment. They provide two explanations for these results. First, official loans send a signal to private investors that the country is stable. Second, official lending may indirectly imply a “commitment by these institutions to act as lenders of last resort” (p. 303) should a default occur. Our model provides an alternative explanation for this empirical fact. The threat of losing access to official loans in the event of expropriation increases the penalty of a default, and as a result decreases the expropriation risk (i.e., weakens the constraint). Private investors realize the reduction in risk and therefore invest more. Our analysis is also consistent with the empirical results of Nunnenkamp and Picht (1989). Using data for 53 developing countries, they conclude that countries that received aid from multilateral agencies were less likely to engage in willful default. Burton and Inoue (1987) document this result as well. They conclude that countries that relied heavily on official assistance treated foreign investment more favorably.

A number of economists and policy-makers have recently argued that multilateral loans should carry above market rates. Our model suggests that such a policy will depress investment flows. An increase in multilateral interest rate, $\tau$, raises $B$ and thereby increases the likelihood that more countries will be in the autarky equilibrium. Furthermore, it leads to a decrease in the level of investment. The reason is that as $\tau$ rises, the net gain to the host from default increases. Rational investors anticipate the increase in expropriation risk (i.e., the tightness of the incentive constraint) and therefore invest less.

In Table 2 we present estimates of our policy parameters for 24 developing countries. We use the percentage of loans from official sources and the interest rates on official loans as measures of $\gamma$ and $\tau$ respectively.

(c) $\frac{dk^*_e}{d\gamma} > 0$ and $\frac{dy^*_e}{d\gamma} > 0$;  (d) $\frac{dk^*_e}{d\tau} < 0$ and $\frac{dy^*_e}{d\tau} < 0$.

15 Recently the U.S. congress considered a proposal requiring the IMF to charge above market rates for its loans (e.g., The Wall Street Journal, 10/8/98, Section A).
of loans from official sources, proxied by autarky equilibrium, i.e., did not receive any private loans over the period 1990-96. Further, the fraction

In Asiedu and Villamil (1999) using indicators from the

For example, about 93 percent of Niger’s multilateral debt is concessional (i.e., has a grant element of at

Table 2

Loan Information for Selected Developing Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Discount Factor, β</th>
<th>Distribution of Debt (%)</th>
<th>Distribution of Multilateral Debt (%)</th>
<th>Interest Free Loans (IDA credits)</th>
<th>% of total loans</th>
<th>Interest Rate</th>
<th>% of total loans</th>
<th>Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>.62</td>
<td>12</td>
<td>15</td>
<td>73</td>
<td>0</td>
<td>0</td>
<td>43</td>
<td>6.99</td>
</tr>
<tr>
<td>Brazil</td>
<td>.67</td>
<td>9</td>
<td>17</td>
<td>74</td>
<td>0</td>
<td>0</td>
<td>40</td>
<td>6.99</td>
</tr>
<tr>
<td>Burkina Faso</td>
<td>.47</td>
<td>80</td>
<td>19</td>
<td>1</td>
<td>88</td>
<td>56</td>
<td>100</td>
<td>1.27</td>
</tr>
<tr>
<td>Cameroon</td>
<td>.55</td>
<td>22</td>
<td>56</td>
<td>22</td>
<td>30</td>
<td>22</td>
<td>93</td>
<td>4.29</td>
</tr>
<tr>
<td>Chile</td>
<td>.65</td>
<td>23</td>
<td>5</td>
<td>72</td>
<td>1</td>
<td>0</td>
<td>68</td>
<td>6.13</td>
</tr>
<tr>
<td>China</td>
<td>.66</td>
<td>16</td>
<td>20</td>
<td>64</td>
<td>47</td>
<td>46</td>
<td>33</td>
<td>5.44</td>
</tr>
<tr>
<td>Colombia</td>
<td>.57</td>
<td>33</td>
<td>13</td>
<td>54</td>
<td>3</td>
<td>0</td>
<td>39</td>
<td>7.03</td>
</tr>
<tr>
<td>Congo</td>
<td>.42</td>
<td>24</td>
<td>67</td>
<td>9</td>
<td>71</td>
<td>56</td>
<td>100</td>
<td>5.36</td>
</tr>
<tr>
<td>Ghana</td>
<td>.59</td>
<td>64</td>
<td>25</td>
<td>11</td>
<td>87</td>
<td>79</td>
<td>81</td>
<td>1.73</td>
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<tr>
<td>Haiti</td>
<td>.23</td>
<td>77</td>
<td>17</td>
<td>6</td>
<td>100</td>
<td>65</td>
<td>100</td>
<td>1.42</td>
</tr>
<tr>
<td>India</td>
<td>.58</td>
<td>33</td>
<td>33</td>
<td>34</td>
<td>60</td>
<td>59</td>
<td>67</td>
<td>3.88</td>
</tr>
<tr>
<td>Indonesia</td>
<td>.62</td>
<td>22</td>
<td>33</td>
<td>45</td>
<td>9</td>
<td>5</td>
<td>69</td>
<td>5.29</td>
</tr>
<tr>
<td>Malaysia</td>
<td>.61</td>
<td>9</td>
<td>14</td>
<td>77</td>
<td>0</td>
<td>0</td>
<td>21</td>
<td>5.47</td>
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<tr>
<td>Mexico</td>
<td>.67</td>
<td>17</td>
<td>11</td>
<td>72</td>
<td>0</td>
<td>0</td>
<td>37</td>
<td>7.58</td>
</tr>
<tr>
<td>Nigeria</td>
<td>.43</td>
<td>55</td>
<td>30</td>
<td>15</td>
<td>93</td>
<td>66</td>
<td>82</td>
<td>1.64</td>
</tr>
<tr>
<td>Nigeria</td>
<td>.47</td>
<td>47</td>
<td>49</td>
<td>36</td>
<td>6</td>
<td>0</td>
<td>97</td>
<td>6.16</td>
</tr>
<tr>
<td>Peru</td>
<td>.49</td>
<td>49</td>
<td>51</td>
<td>33</td>
<td>6</td>
<td>0</td>
<td>97</td>
<td>6.16</td>
</tr>
<tr>
<td>Philippines</td>
<td>.46</td>
<td>25</td>
<td>40</td>
<td>35</td>
<td>10</td>
<td>2</td>
<td>64</td>
<td>5.08</td>
</tr>
<tr>
<td>Russia</td>
<td>NA</td>
<td>1</td>
<td>42</td>
<td>57</td>
<td>0</td>
<td>0</td>
<td>72</td>
<td>6.46</td>
</tr>
<tr>
<td>Swaziland</td>
<td>NA</td>
<td>53</td>
<td>46</td>
<td>1</td>
<td>45</td>
<td>5</td>
<td>100</td>
<td>4.83</td>
</tr>
<tr>
<td>Togo</td>
<td>.43</td>
<td>55</td>
<td>41</td>
<td>4</td>
<td>94</td>
<td>75</td>
<td>100</td>
<td>0.90</td>
</tr>
<tr>
<td>Zambia</td>
<td>.42</td>
<td>36</td>
<td>57</td>
<td>7</td>
<td>67</td>
<td>45</td>
<td>89</td>
<td>1.84</td>
</tr>
</tbody>
</table>


Notes: Loans from multilateral organizations include loans and credits from the World Bank, regional development banks (African Development Bank, Asian Development Bank, and Inter-American Development Bank) and other multilateral agencies. Bilateral loans are loans from governments and their agencies, loans from autonomous bodies, and direct loans from official credit agencies. Private credits include bonds, commercial banks and other private creditors. Grants are funds for which there is no repayment requirement. Concessional loans refer to loans with a grant element of at least 25 percent. In terms of the parameters of our model, the percentage of loans from official sources and the interest rates on official loans proxy for \( \gamma \) and \( \tau \) respectively. Estimates for \( \beta \) are constructed in Asiedu and Villamil (1999) using indicators from the International Country Risk Indicators, averaged over the period 1990-95.

The data indicates that four of the countries (Burkina Faso, Haiti, Swaziland and Togo) are in the autarky equilibrium, i.e., did not receive any private loans over the period 1990-96. Further, the fraction of loans from official sources, proxied by \( \gamma \), is very high for most of the countries, averaging about 93 percent for countries in Sub-Saharan Africa. Moreover, the interest rates on official loans are very low. For example, about 93 percent of Niger’s multilateral debt is concessional (i.e., has a grant element of at
least 25 percent) and 66 percent of the loans are interest free. In addition, official loans to Niger carry an interest rate of about 2 percent (proxy for $\tau$) compared to 12 percent for private loans (proxy for $r$).

4. Conclusion

This paper examines the role of international organizations in directing private capital flows when contractual enforcement is problematic. When enforcement is imperfect, a country’s ability to attract foreign investment is determined by the country specific discount factor $\beta(\theta, r)$, and two thresholds, $\beta^*(r, k^d)$ and $\beta(r, k^d)$, that segment the interval $(0, 1)$ into three disjoint sub-intervals on which $\beta$ is defined. If $\beta \in (0, \beta)$ autarky occurs, if $\beta \in [\beta, \beta^*)$ investment is constrained optimal, and if $\beta \in [\beta^*, 1)$ investment is unconstrained optimal. We consider how two policies which are frequently used by multilateral organizations can affect the thresholds (and hence the length of the sub-intervals). A loan subsidy policy $(\tau, \gamma)$ affects the right-hand-side of the incentive repayment constraint and a technical assistance policy $\delta$ affects the left-hand-side. In this case the thresholds are determined by $\beta^*(r, k^d, \omega)$ and $\beta(r, k^d, \omega)$ where $\omega = (\delta, \tau, \gamma)$ is a vector of policy variables at the disposal of the multilateral organization.

Our analysis shows that by the appropriate choice of policy, a multilateral organization can lengthen the “good” sub-interval $[\beta^*, 1)$ where unconstrained investment occurs (Case 3) and reduce the “bad” sub-interval $(0, \beta)$ where autarky occurs (i.e., Case 1). This policy increases the number of countries that can access private capital markets and/or sustain the unconstrained optimal level of investment. Further, it increases the constrained optimal level of investment, $k^*_c$, and welfare. We thus show that the services provided by multilateral organizations can proxy for enforcement and thereby promote higher private foreign investment. In this precise sense, a case can be made for external assistance since it can increase welfare and promote private investment when it is designed appropriately. These policies are not a substitute for private investment, but rather remedy a market failure (inadequate enforcement). Some type of intervention is necessary in order for private markets to operate efficiently. The maximum gains and costs of the policies are well defined in the model, thus making the results comparable with other policy proposals.

Finally, the multilateral policies we consider do not directly affect $\beta(\theta, r)$ or $k^d$ (where $k^d$ is also a determinant of $\beta$ and $\beta^*$). As a consequence, the potentially beneficial role of multilateral organizations as providers of a type of implicit enforcement does not render “country level” policy irrelevant. Our results indicate that country level policies designed to increase $k^d$ and $\theta$ are important because they increase welfare directly (cf., Problem 1) and affect foreign capital flows. Thus such “country level” policies are also clearly desirable. Our results can be thought of as a benchmark analysis for formulating policies for multilateral lending agencies. Specifically, even when $k^d$ and $\theta$ are fixed, appropriate policies by a
multilateral organization can proxy for imperfect enforcement and hence mitigate the effects of this market failure.
Appendix A

We present three numerical examples to illustrate the implications of multilateral policy on private foreign investment. Example 1 looks at the effect of technical assistance policy (changes in $\delta$), while examples 2 and 3 examine the effect of a loan subsidy program on the threshold discount factors, $\beta$ and $\beta^*$, and private foreign investment. Example 2 considers changes in the multilateral interest rate, $\tau$ and example 3 analyzes changes in the fraction of multilateral loans, $\gamma$. For each policy variable, we compute $\beta$ and $\beta^*$ and present a graph of private foreign investment flows. The parameter values we assume are displayed in Table 3.

Table 3

Parameter Values. Production Function, $f(k) = k^\alpha$

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital’s Share of Output, $\alpha$</td>
<td>.3</td>
</tr>
<tr>
<td>Gross Return on Capital, $r$</td>
<td>1.05</td>
</tr>
<tr>
<td>Domestic Capital Stock</td>
<td>.1002</td>
</tr>
</tbody>
</table>

Notes: We assume that the domestic capital stock is equal to 60 percent of the unconstrained optimal capital stock, i.e., $k^d = .6k^*_u$ where $f'(k^*_u) = r$.

Example 1

To illustrate how changes in $\delta$ affect the threshold discount factors and optimal investment flows, we consider three values of $\delta$: $\delta = 0$ (i.e., no technical assistance), $\delta = 0.05$, and $\delta = 0.1$. Table 4 presents the threshold discount factors for each value of $\delta$ and Figure 2 clearly illustrates the impact of $\delta$ on private foreign investment flows.
Table 4

Technical Assistance and Threshold Discount Factors

<table>
<thead>
<tr>
<th>Output lost from Withdrawal of Technical Assistance, δ</th>
<th>Threshold Discount Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β</td>
</tr>
<tr>
<td>0</td>
<td>.6994</td>
</tr>
<tr>
<td>.05</td>
<td>0</td>
</tr>
<tr>
<td>.1</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 2

Technical Assistance and Private Foreign Investment

Notes: Each curve shows, for different values of default penalty, δ, the level of private foreign investment as a function of β. δ = 0 means no technical assistance.
Note that in the absence of technical assistance (i.e., $\delta = 0$), countries with $\beta < \bar{\beta} = .6994$ are in the autarky equilibrium (i.e., Case 1) and unable to access the international capital market. Further, the minimum $\beta$ required to sustain the unconstrained optimal level of investment is $\beta^* = .8446$ (i.e., Case 3). This compares with $\bar{\beta} = 0$ and $\beta^* = .5266$ when the default penalty is 10 percent.

Example 2 (Changes in $\tau$)

We consider three values of $\tau$ (the interest rate charged by multilateral organizations): $\tau = 0$ (i.e., free funds), $\tau = .75$ and $\tau = 1.03$. The results are presented in Table 5 and Figure 3. We use the parameter values specified in Table 3 and assume that 50 percent of foreign loans are provided by the multilateral agency, i.e., $\gamma = 0.5$.

Table 5

Subsidized Loans and Threshold Discount Factors

<table>
<thead>
<tr>
<th>Multilateral Interest Rate, $\tau$</th>
<th>Threshold Discount Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta$</td>
</tr>
<tr>
<td>0</td>
<td>.3497</td>
</tr>
<tr>
<td>.75</td>
<td>.5995</td>
</tr>
<tr>
<td>1.03</td>
<td>.6927</td>
</tr>
</tbody>
</table>
Notes: Each curve shows, for different values of the multilateral interest rate, $\tau$, the level of private foreign investment as a function of $\beta$. $\tau = 0$ means multilateral assistance takes the form of a grant, i.e., free money.

Example 3 (Changes in $\gamma$)

We consider three values of $\gamma$ (the fraction of loans provided by a multilateral organization): $\gamma = 0$ (i.e., no multilateral assistance), $\gamma = 0.3$, $\gamma = 0.7$. We use the parameter values specified in Table A and assume that multilateral loans are in the form of grants, i.e., $\tau = 0$. The results are presented in Table 5 and Figure 4.
Table 5
Subsidized Loans and Threshold Discount Factors

<table>
<thead>
<tr>
<th>Fraction of Multilateral loans, γ</th>
<th>Threshold Discount Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β</td>
</tr>
<tr>
<td>0</td>
<td>.6994</td>
</tr>
<tr>
<td>.3</td>
<td>.4896</td>
</tr>
<tr>
<td>.7</td>
<td>.2098</td>
</tr>
</tbody>
</table>

Figure 4
Loan Subsidy and Private Foreign Investment

Notes: Each curve shows, for different values of the fraction of multilateral investment, γ, the level of private foreign investment as a function of β. γ = 0 means no multilateral assistance.
Appendix B

A Sketch of the Proof for the Derivation of $\beta$ and $\beta^*$: In the autarky equilibrium, $B(k)$ cuts $G(k)$ at $k^d$ from below (see Figure 5). This implies $B'(k^d) < G'(k^d)$ and $\beta f'(k^d) < r$. Define $\hat{\beta}(k^d, r) = \frac{r}{f'(k^d)}$.

Then, when $\beta < \hat{\beta}$, autarky occurs hence $\hat{\beta}$ is the minimum discount factor required to attract foreign investment.

An investment plan, $k$, is self-enforcing if and only if $B(k) \leq G(k)$, i.e.,

\begin{equation}
0 < \frac{r(k - k^d)}{f(k) - f(k^d)} \leq \beta < 1.
\end{equation}

Substitute $k = k^*_u$ and $r = f'(k^*_u)$ in (3) and define $\beta^*$ as $\beta^*(k^*_u, r) = \frac{(k^*_u - k^d)f'(k^*_u)}{f(k^*_u) - f(k^d)}$. Then $k^*_u$ is self-enforcing if and only if $\beta(\theta, r) \geq \beta^*$, and $\beta^*$ is the minimum discount factor required to attract foreign investment to the unconstrained level of investment, $k^*_u$.

Proof of Proposition 1. In the autarky equilibrium (Case 1), $\delta = 0$ and $B(k)$ cuts $G(k)$ at $k^d$ from below. An increase in $\delta$ shifts $B(k)$ downward while $G(k)$ remains unchanged (see Figure 5). The proof follows from the fact that for $\delta > 0$, there exists a $k^* > k^d$ such that $B(k^*) = G(k^*)$ and $B(k) < G(k)$ for $k \in (k^d, k^*)$. This implies that for $k^d < k < k^*$, no expropriation occurs.
Figure 5
Equilibrium under Technical Assistance

Proof of Claim 1. Under Technical Assistance, an investment plan, $k$, is self-enforcing if and only if $B(k) \leq G(k)$. That is,

\[(4) \quad 0 < \frac{r(k - k^d)}{f(k) - (1 - \delta)f(k^d)} \leq \beta < 1.\]

Substitute $k = k_u^*$ and $r = f'(k_u^*)$ in (4). Then $k_u^*$ is self-enforcing if and only if $\beta \geq \beta^*$, where $\beta^*$ is defined by:

\[(5) \quad \beta^*(k^d, r, \delta) = \frac{r[k_u^*(r) - k^d]}{f(k_u^*(r)) - (1 - \delta)f(k^d)}.\]

When the incentive constraint binds, $k^*_c$ satisfies
(6) \[ \beta [f(k^*_c) - (1 - \delta) f(k^d)] - r(k^*_c - k^d) = 0 \]

and welfare, \( y^*_c \) is given by

(7) \[ y^*_c = W(k^*_c) = \frac{1}{1 - \beta} [f(k^*_c) - r(k^*_c - k^d)] \]

Differentiating (5), (6) and (7) with respect to \( \delta \) yields:

\[
\frac{d\beta}{d\delta} = -\frac{\delta f'(k^*_c)(k^*_c - k^d)f(k^d)}{[f(k^*_c) - (1 - \delta) f(k^d)]^2}
\]

\[
\frac{dk^*_c}{d\delta} = -\frac{\beta f(k^d)}{\beta f'(k^*_c)} - r
\]

\[
(1 - \beta) \frac{dy^*_c}{d\delta} = [f'(k^*_c) - r] \frac{dk^*_c}{d\delta}
\]

Since \( B(k) \) cuts \( G(k) \) at \( k^*_c \) from below, \( \beta f'(k^*_c) - r < 0 \) and the results follow.

**Proof of Claim 2.** Under Loan Subsidy, \( B(k) \) and \( G(k) \) are given by

\[
G(k) = \frac{1}{1 - \beta} [f(k) - r(1 - \gamma)(k - k^d) - \tau \gamma(k - k^d)]
\]

\[ B(k) = f(k) + \frac{\beta}{1 - \beta} f(k^d) \]

The autarky equilibrium occurs if and only if \( B(k) \) cuts \( G(k) \) at \( k^d \) from below, i.e., \( B'(k^d) < G'(k^d) \).

This implies \( \beta f'(k^d) < r(1 - \gamma) + \gamma \tau \). Thus the smallest discount factor required to attract foreign investment, \( \bar{\beta} \), is given by

(8) \[ \bar{\beta}(k^d, r, \gamma, \tau) = \frac{r(1 - \gamma) + \gamma \tau}{f'(k^d)} \]

Under loan subsidy, the unconstrained optimal investment plan, \( k^*_u \), is self-enforcing if and only if \( B(k^*_u) \leq G(k^*_u) \). This implies \( k^*_u \) is self-enforcing if and only if \( \bar{\beta} \geq \beta^* \), where \( \beta^* \) is defined by:
When the incentive constraint binds, $k^*_c$ satisfies

$$\beta\left[f(k^*_c) - f(k^d)\right] - (k^*_c - k^d)[r(1 - \gamma) + \tau\gamma] = 0$$

and welfare, $y^*_c$ is given by

$$y^*_c = W(k^*_c) = \frac{1}{1 - \beta}[f(k^*_c) - r(1 - \gamma)(k^*_c - k^d) - \tau\gamma(k^*_c - k^d)]$$

Differentiating (8), (9), (10) and (11) with respect to $\gamma$ and $\tau$ yields:

$$\frac{d\beta}{d\gamma} = -\frac{(r - \tau)}{f^*(k^d)}$$

$$\frac{d\beta^*}{d\gamma} = -\frac{[f'(k^*_c) - \tau](k^*_c - k^d)}{f(k^*_c) - f(k^d)}$$

$$\frac{dk^*_c}{d\gamma} = -\frac{(r - \tau)(k^*_c - k^d)}{\beta f'(k^*_c) - [r(1 - \gamma) + \tau\gamma]}$$

$$(1 - \beta)\frac{dy^*_c}{d\gamma} = \left[(f'(k^*_c) - r) + \gamma(\tau + r)\right]\frac{dk^*_c}{d\gamma} + (k^*_c - k^d)(r - \tau)$$

$$\frac{d\beta}{d\tau} = \frac{\gamma}{f'(k^d)}$$

$$\frac{d\beta^*}{d\tau} = \frac{\gamma(k^*_c - k^d)}{f(k^*_c) - f(k^d)}$$
\[ \frac{dk_c^*}{d\tau} = \frac{\gamma (k_c^* - k^d)}{\beta f'(k_c^*) - [r(1 - \gamma) + \tau\gamma]} \]

\[ (1 - \beta) \frac{dy^*}{d\tau} = [(f'(k_c^*) - r) + \gamma (\tau + r)] \frac{dk_c^*}{d\tau} - \gamma (k_c^* - k^d) \]

Since \( B(k) \) cuts \( G(k) \) at \( k_c^* \) from below, \( [\beta f'(k_c^*) - (r(1 - \gamma) + \tau\gamma)] < 0 \) and the results follow.
References


