MONETARY POLICY AND DETERMINACY:

AN INQUIRY IN OPEN ECONOMY NEW KEYNESIAN FRAMEWORK

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January 11, 2022

ABSTRACT

We analyze determinacy in the baseline open-economy New Keynesian model developed by Gali and Monacelli (2005). We find that the open economy structure causes multifaceted behaviors in the system creating extra challenges for policy making. The degree of openness significantly affects determinacy properties of equilibrium under various forms and timing of monetary policy rules. Conditions for the uniqueness and local stability of equilibria are established. Determinacy diagrams are constructed to display the regions of unique and multiple equilibria. Numerical analyses are performed to confirm the theoretical results. Limit cycles and periodic behaviors are possible, but in some cases only for unrealistic parameter settings. Complex structures of open economies require rigorous policy design to achieve optimality.

Keywords: bifurcation; determinacy; dynamic systems; New Keynesian; stability; open economy; Taylor Principle

JEL-codes: C14, C22, C52, C61, C62, E32, E37, E61, L16

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1. Introduction and Review of Literature

Determinacy concerns the existence of a unique equilibrium path of a dynamic system. Equilibrium of a macroeconomic model is called determinate, as defined by McCallum (2009b), if it is locally unique and dynamically stable under relevant specifications of policy tools. Studying the determinacy issues in a wide range of models has been an important subject among macroeconomists. One of the reasons for high focus on the subject is the fact that uniqueness of the solution path plays an important role for policy makers and researchers. Determinacy is important to the monetary authority in determining policy to manage inflationary expectations and preventing self-fulfilling economic fluctuations. Indeterminacy permits existence of multiple solutions, which could be fundamental equilibria or nonfundamental sunspot equilibria, as suggested by McCallum (2003) and Bullard (2006), among other authors.

McCallum (2003) further divides the indeterminacy cases into two categories: failure of the model to determine the values of nominal variables (nominal indeterminacy) or of real variables (real indeterminacy). But Woodford (2003b) finds the distinction to be insignificant. He argues that both types of indeterminacy are quantitatively indifferent. Models with indeterminate solutions, as stressed by Bullard and Mitra (2002), are considered undesirable for macroeconomic analysis and policy design. In the presence of indeterminacy, as pointed out by McCallum (2003), Gauthier and Guesnerie (2005), Beyer and Farmer (2007) and others, non-fundamental shocks trigger extra variance and fluctuations in the economy. Then decision makers, unable to acquire full information, encounter unforeseen problems in monetary policy design.

Following Clarida, Gali, and Gertler's (2000) seminal work, interest has been growing in the determinacy issue associated with certain types of monetary policy rules in the context of different model settings, with inflation being a primary concern. Studying determinacy of macroeconomic models, as suggested by Coibion and Gorodnichenko (2011), requires attention to trend inflation, and to the monetary authority's policy responses to inflation rate, price level, and output gap or output growth. In particular, consideration must be given to forward or backward looking approaches and complementary policy tools such as interest rate smoothing. In addition, macroeconomic models must consider sticky or flexible prices, closed or open economy, and other variables and structure potentially affecting the overall results.

McCallum (2003, 2007, 2009b) argues that learnability is a more essential criterion than determinacy as a necessary condition for a plausible rational expectations equilibrium. Learnability suggests that households and firms update their information sets and re-formulate their expectations in accordance with evolving economic conditions. Furthermore, McCallum (2003) and Bullard (2006) suggest that even in the case of multiple stable equilibria, if one of the equilibrium points is least squares learnable, the unique learnable solution path will suffice for monetary policy design following a Taylor rule. Those authors, among others, maintain that learnability of the solution path is more important than determinacy in monetary policy conduct, so long as only one of the nonunique paths is learnable by the private sector. While learnable equilibria are normally determinate, the converse is not necessarily true.

Bullard and Mitra (2002), using the methodology of Evans and Honkapohja (1999, 2001), evaluate the Taylor-type monetary policy rules based on determinacy, expectational stability, and learnability. They find that monetary policy rules satisfying the Taylor Principle usually produce both determinate and learnable equilibria. They conclude that the determinacy settings of the equilibrium depend not only upon the monetary policy rule but also on the overall configuration of the economic structure. Different model configurations provide valuable insights into possible indeterminacy problems. Carlstrom, Fuerst, and Ghironi (2006) have found that regardless of what price index the monetary authority considers, the Taylor Principle holds in a multi-sector economy in which sectors differ in price stickiness.

Taylor Principle has often been considered the right policy tool to achieve determinacy in equilibrium. But under certain circumstances, such as the presence of trend inflation, the Taylor Principle fails to guarantee determinacy. Giannoni (2014) and Ambler and Lam (2015) argue that interest rate rules that respond to fluctuations in price-level are less prone to equilibrium indeterminacy than interest rate rules, such as Taylor rules, which respond to fluctuations in inflation rate. Coibion and Gorodnichenko (2011), Huang and Thurston (2012), and Hirose et al (2017) argue that the Taylor Principle does not guarantee determinacy of New Keynesian models, especially in the presence of positive trend inflation. Hirose et al (2017), in their study of Great Inflation of 1970's in the US, conclude that to achieve determinacy within a Generalized New Keynesian framework, an active interest rate policy should be run along with a lower trend inflation (lower inflation target), or with diminished policy response to the output gap, or with firmer response to output growth. Kiley (2007) shows that as trend inflation increases, the determinacy region shrinks. He suggests a moderately active interest rate policy accompanied by a slightly positive response to output gap to ensure determinacy.

Gerko and Sossounov (2015) studied the effects on determinacy of positive trend inflation within a new Keynesian model with Calvo-type price setting and capital accumulation. While they verify the previous research findings on active monetary policy not necessarily guaranteeing determinacy of equilibrium, they also find that the parameters of monetary policy which ensure determinacy mainly depend upon the level of trend inflation. As the level of trend inflation increases, the regions of determinacy under Taylor–type rules shrink. While the Taylor-type rule might lead to indeterminacy within a large region of the parameter space, Gerko and Sossounov (2015) advocate employing strict price level targeting along with responses to current output gap to ensure determinacy of equilibrium. Fanelli (2012), on the other hand, argues that if the monetary authority does not react to inflation by an aggressive increase in the nominal interest rate, a unique equilibrium may not be achieved. Adding to the previous conclusions, Coibion and Gorodnichenko (2011) show that in the presence of positive trend inflation, an endogenous monetary policy based on the Taylor Principle may not be sufficient. Components of the policy, such as interest rate smoothing along with price-level targeting, can be needed to ensure determinacy.

Dupor (2001) shows that including endogenous investment in the neoclassical imperfect competition model with sticky prices reverses the effects of interest rate rules on determinacy. While passive policy rules lead to locally unique equilibria, active rules do not. Carlstrom and Fuerst (2005) explore a Calvo-type sticky price model including capital and investment spending. They find that monetary authority aggressive response to the current inflation rate is the only way to achieve local determinacy. Under forward-looking interest rate rules, the determinacy region shrinks considerably, and local indeterminacy arises. Tesfaselassie and Schaling (2016) explore Blanchard and Gali's (2010) new Keynesian model incorporating labor market frictions. They find that determinacy depends not only on the policy rule but also on inflation and unemployment expectations as well as hiring costs. Under the policy rules reacting to current inflation and unemployment, the indeterminacy region of the parameter space widens together with hiring costs. Under policy rules based on inflation and unemployment expectations, the indeterminacy region shrinks with hiring costs, while too much or too little reaction may still lead to indeterminacy. Assuming that the steady state is known, lack of enough reaction to inflation and unemployment can also lead to indeterminacy.

We explore the determinacy conditions of New Keynesian open economy models, which have rarely been considered in macroeconomics research. The open economy framework makes the determinacy analysis substantially more complicated. De Fiore and Liu (2005) and Karagiannides (2020) find that whether a policy rule leads to unique equilibria depends upon the degree of openness to trade. Karagiannides (2020) find that as trade openness increases, Taylor-rule based monetary policies should put more weight on output gap and less on price stability. But as openness decreases, price stability should take priority. In open economies, De Fiore and Liu (2005) explain transmission mechanisms which could lead to determinacy and transmission mechanisms that would not lead to determinacy. Unlike closed economies, in which the transmission mechanism operates through the substitution effect between consumption and leisure or savings, in open economies a rise in real interest rates affects the exchange rates and thus term of trades between foreign and domestic goods. Improving terms of trade, depending on the level of openness, creates incentives for the household to substitute consumption for leisure or saving.

In the open economy case, as the real interest rate is increased by the monetary authority, the domestic currency appreciates. The domestic goods' price index rises relative to the consumer price index. Domestic inflation then differs from CPI growth, producing complications for policy making. Llosa and Tiesta (2008) investigate under which conditions rule-based policies could generate a determinate and learnable rational expectational equilibrium in a New Keynesian open economy model. Similar to the findings by Zanna (2003), De Fiori, and Liu (2005) and Bullard and Schaling (2006), they find that the effects of trade openness depend on the elasticity of substitution between tradable domestic and imported goods.

Moreover, with contemporaneous data, the Taylor principle satisfies necessary and sufficient conditions for determinate and learnable equilibria. The monetary authority could achieve unique equilibrium by targeting either the CPI or domestic inflation, while a managed exchange rate policy could ease the extent of response to inflation. With forecasted data on the other hand, openness makes it harder to achieve a determinate and learnable equilibrium. Then the monetary authority should employ an aggressive response to domestic inflation to prevent indeterminacy. Nevertheless, substantial additional analytical and empirical research remains to be done in exploring determinacy issues of New Keynesian open economy models.

Our study seeks to enlighten some aspects of the determinacy problem in the New Keynesian open economy literature. For that purpose, we employ Gali and Monacelli's (2005) model of a small open economy in the New Keynesian tradition. Gali and Monacelli's (2005) model represents a small open economy as part of the world economy, which is itself a continuum of small open economies, identical in terms of preferences, technology, and Calvo-type staggered price setting. Under various policy regimes, the model seamlessly reveals the trade-offs between the stabilizations of inflation, output gap, and exchange rate. Since its publication, the Gali and Monacelli (2005) model has attracted the attention of many researchers and policy makers and has become one of the most influential models in macroeconomic analysis. It has been used as a baseline model in a wide range of research and policy analysis.[1]

We investigated the determinacy conditions under a variety of alternative monetary policy rules. We have found that in a broad class of open economy New Keynesian models, the degree of openness has a significant role in equilibrium determinacy under various forms and timing of monetary policy rules. The open economy framework creates substantial complications within the dynamic structure of the system. The resulting broad range of qualitative behaviors requires rigorous policy responses.

The conditions for the uniqueness and local stability of the equilibria are established for each model and are evaluated using numerical analysis. We reestablish the determinacy conditions in the open economy New Keynesian model structures. Determinacy diagrams are constructed to show the regions of unique and multiple equilibria. Numerical analyses are performed to confirm the theoretical results. The numerical simulations show that limit cycles and periodic behaviors are possible, but on some occasions, only at unlikely parameter settings.

[1]For example, the model has been used in research about monetary policy (e.g., Faia and Monacelli (2008); Dai, Sidiropoulos and Spyromitros (2009); Ferrero and Seneca (2019); and Kitano and Takaku (2015)), macroeconomic dynamics, trade, and uncertainty (e.g., Milani and Park (2015) and Caldara, Iacoviello, Molligo, Prestipino, and Raffo (2020)), bifurcation of macroeconometric models (Barnett and Chen (2015) and Barnett and Eryilmaz (2013, 2016)), and financial or fiscal shocks (e.g., Boscá, Doménech, Ferri, Méndez, and Rubio-Ramírez (2020)). The model has also been used to investigate learnability of equilibria under sticky nominal wages (Araújo (2016)) and under different sets of expectation-based policy rules (Llosa and Tuesta (2008)).

2. Model

In this study, we use Gali and Monacelli's (2005) model of a small open economy in the New Keynesian tradition. The model consists of the following three equations: the IS curve, which represents the demand side; the aggregate supply curve, often called the New Keynesian (NK) Phillips curve; and a simple (i.e., non-optimized) monetary policy rule.

The IS curve is:

$$x_t = E_t x_{t+1} - \frac{1+\alpha(\omega-1)}{\sigma}\left(r_t - E_t \pi_{t+1} - \overline{r}_t\right), \qquad (1)$$

where x_t is the gap between actual output and flexible-price equilibrium output, π_t is the inflation rate, r_t is the nominal interest rate, $\overline{r_t}$ is the small open economy's natural rate of interest, and β is the discount factor. Then $\sigma_\alpha = \sigma(1-\alpha+\alpha\omega)^{-1}$ and $\omega = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$ are composite parameters, while E_t is the expectation operator. Lowercase letters denote the logs of the respective variables.

The aggregate supply Phillips curve is:

$$\pi_t = \beta E_t \pi_{t+1} + \mu\left(\frac{\sigma}{1+\alpha(\omega-1)} + \varphi\right)x_t, \qquad (2)$$

where π_t is the inflation rate, $\mu = \frac{(1-\beta\theta)(1-\theta)}{\theta}$, and $\omega = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$.

The monetary policy rule is:

$$r_t = \overline{r}_t + \phi_\pi \pi_t + \phi_x x_t, \qquad (3)$$

where the coefficients $\phi_x > 0$ and $\phi_\pi > 0$ measure the sensitivity of the nominal interest rate in the central bank's response to changes in output gap and inflation rate, respectively. The policy rule, (3), is a version of the Taylor rule (Taylor 1993).

The first two equations are derived from the optimization of consumers and firms' objective functions. Both equations are in log-linearized form. As for the monetary policy rule, we consider a variety of such simple policy rules.[2]

[2] As Bullard and Mitra (2002) and Rotemberg and Woodford (1999) suggest, this formulation is not subject to the Lucas Critique, since the parameters of the structural equations defining the economy do not depend upon the parameters of the policy rule.

The open economy is isomorphic to the closed economy version. Nevertheless, unlike the closed economy, the Gali and Monacelli model depends upon the open economy parameters, such as the degree of openness, the terms of trade, the substitutability among goods of different origin, and the world output, which is exogenously determined. Therefore, it is important to identify any influence of the open economy parameters on the determinacy of the model.

3. Determinacy Analysis:

Following Barnett and Eryilmaz (2016), we consider varying the timing of the monetary policy rule to consider contemporaneous, forward and backward looking policy rules as well as their hybrid combinations. We evaluate each model based on the determinacy criterion and establish the conditions for the determinacy of equilibria for each model. We derive analytical results and present numerical simulations. We use methodology based on the number of eigenvalues inside the unit circle, given the number of predetermined variables of the model.

By rearranging the equations (1), (2), and (3), we first write the system in the form $E_t y_{t+1} = C y_t$. For a two-equation first order stochastic difference equation system in terms of domestic inflation and output gap, the eigenvalues, λ_1 and λ_2, of the Jacobian matrix C are computed by setting $\det(C - \lambda I) = 0$. This gives a second-order characteristic polynomial, $p(\lambda) = \lambda^2 - a_1\lambda + a_0 = 0$. The determinacy of the system, following Blanchard and Kahn (1980), requires that both eigenvalues of the coefficient matrix C are outside the unit circle, so that the eigenvalues have modulus greater than one. This condition can be met, if and only if $|a_0| < 1$ and $|a_1| < 1 + |a_0|$. Then, following Bullard and Mitra's (2002) methodology, we construct the propositions which establish the necessary and sufficient conditions for the matrix C to have both eigenvalues outside the unit circle.[3]

Following Bullard and Mitra's (2002) approach, we use the calibrated values of the parameters as given in Gali and Monacelli (2005). Those values are $\beta = 0.99$, $\alpha = 0.4$, $\sigma = \omega = 1$, $\varphi = 3$, and $\mu = 0.086$. For the three equation case including policy equation (3), we set the policy parameters at $\phi_x = 0.125$, $\phi_\pi = 1.5$, and $\phi_r = 0.5$.[4]

[3] For a detailed discussion of the methodology, see Blanchard and Khan (1980) and Bullard and Mitra (2002).

[4] Bullard and Mitra (2002) suggest $0 < \phi_x < 4$ and $0 < \phi_\pi < 10$ for policy analysis.

3.1 Under Current-Looking Taylor Rule:

Consider the following model, in which the first two equations explain the economy, while the third equation is the monetary policy rule tracked by the central bank:

$$\pi_t = \beta E_t \pi_{t+1} + \mu \left(\frac{\sigma}{1+\alpha(\omega-1)} + \varphi \right) x_t , \tag{4}$$

$$x_t = E_t x_{t+1} - \frac{1+\alpha(\omega-1)}{\sigma} \left(r_t - E_t \pi_{t+1} - \overline{r}_t \right), \tag{5}$$

$$r_t = \overline{r}_t + \phi_\pi \pi_t + \phi_x x_t , \tag{6}$$

where x_t denotes the output gap, π_t is the inflation rate, and r_t is the nominal interest rate. Equation (6) describes the policy rule as a current looking Taylor rule, in which the interest rate is set according to the current inflation rate and the current output gap.

Rearranging the terms, the system can be written in the form $E_t y_{t+1} = C y_t$ as

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 + \frac{\mu}{\beta} + \left(1+\alpha(\omega-1)\right)\left(\frac{\beta\phi_x + \varphi\mu}{\beta\sigma} \right) & \frac{(\beta\phi_\pi - 1)\left(1+\alpha(\omega-1)\right)}{\beta\sigma} \\ -\frac{\mu}{\beta}\left(\varphi + \frac{\sigma}{1+\alpha(\omega-1)} \right) & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix}. \tag{7}$$

To confirm that $x_t = \pi_t = 0$ is the only solution, we need to check the determinacy properties of the system (7). Following Blanchard and Kahn (1980), determinacy requires that both eigenvalues of the coefficient matrix C are outside the unit circle. Following Bullard and Mitra (2002), Proposition (1) establishes the necessary and sufficient conditions for the matrix C to have both eigenvalues outside the unit circle..

Proposition 1. Given monetary policy based on the current-looking Taylor rule, the open economy New Keynesian model described by system (7) has a unique stationary equilibrium, if and only if

$$-\frac{\sigma(1-\beta)}{1+\alpha(\omega-1)} < \phi_x + \mu \left(\frac{\sigma}{1+\alpha(\omega-1)} + \varphi \right) \phi_\pi \tag{9}$$

and

$$(1-\beta)\phi_x + (\phi_\pi - 1)\left(\frac{\sigma}{1+\alpha(\omega-1)} + \varphi \right) \mu > 0 . \tag{10}$$

Since $\phi_x, \phi_\pi > 0$ by assumption, the determinacy condition (10) holds if $\phi_\pi > 1$, although the condition $\phi_\pi > 1$ can be relaxed a little, if ϕ_x is large enough. Hence, a unique, stationary equilibrium can be achieved through an active interest rate policy satisfying the Taylor Principle, as defined by Woodford (2001, 2003b) and Bullard and Mitra (2002). The Taylor Principle requires that the nominal interest rate must be raised by more than the increase in inflation rate, so that the real interest rate increases.

The open economy framework has no impact on the determinacy condition under the current-looking Taylor rule. For any values of α and ω, an active monetary policy is sufficient for equilibrium determinacy. However, Llosa and Tuesta (2008) argue that the determinacy region widens as openness to international trade increases. This positive relationship operates through the terms of trade's influence on inflation and output gap. An increase in terms of trade creates an expenditure shift from domestic goods to foreign ones. Thus fluctuations in output gap have a lower effect on domestic inflation. Therefore, the monetary authority could be less concerned about output fluctuations and could focus on inflation.

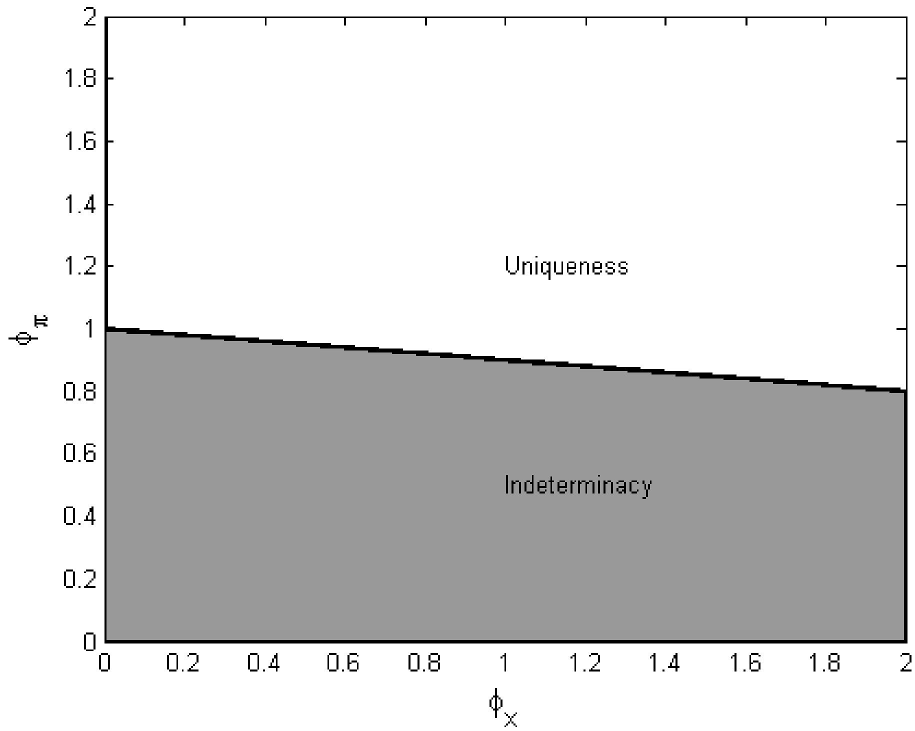
The uniqueness of the equilibrium can be checked by computing the eigenvalues of the Jacobian matrix. For the baseline values of the parameters, the Jacobian matrix of the system (7) is

$$C = \begin{bmatrix} 1.4684 & 0.4899 \\ -0.3434 & 1.0101 \end{bmatrix},$$

with eigenvalues $\lambda_1 = 1.2393 + 0.3402\text{i}$ and $\lambda_2 = 1.2393 - 0.3402\text{i}$ and with modulus $R = \sqrt{(1.2393)^2 + (0.3402)^2} = 1.2851$. The system has a pair of complex conjugate eigenvalues with modulus greater than one. Since the number of eigenvalues outside the unit circle is equal to the number of forward-looking variables, there exists a unique solution to the system.

Figure (1) illustrates the regions of the determinate and of the indeterminate equilibria in (x_t, π_t)-space, as implied by condition (10). Geometrically, as shown in Figure (1), the determinacy region is illustrated by an upper bound and a lower bound for ϕ_π as a function of ϕ_x in accordance with Proposition (1).

Figure 1: Determinacy region under the current looking Taylor rule.



Ascari and Ropele (2009) argue that trend inflation contracts the determinacy region of a standard New Keynesian DSGE model, when monetary policy is run by a current-looking interest rate rule, and hence the Taylor principle is not a sufficient condition for local determinacy of equilibrium. But Bullard and Mitra (2002) show that determinacy can be achieved easily by incorporating current-looking policy rules setting the interest rate as a response to present values of both inflation and output gap. Carlstrom and Fuerst (2005) explore a Calvo-type sticky price model including capital and investment spending. In that model, the monetary authority's aggressive response to current inflation is the only way to achieve the local determinacy.

The main drawback of such policy rules, as stated by McCallum (1999), is that they are unrealistic, since it is not possible concurrently to observe the current values of the model variables and set the interest rate accordingly. The reason is that policy makers do not immediately have all information about the existing status of economy. Barnett and Eryilmaz (2012b), using the open economy New Keynesian model suggested by Walsh (2003) and grounded on Clarida, Gali, and Gertler (2001, 2002), establish the conditions for a Hopf bifurcation to occur, when a current-looking Taylor rule using domestic inflation is conducted by the central bank.

3.2 Under Pure Current-Looking Inflation Targeting Rule:

Consider the model consisting of equations (4) and (5), together with the following policy rule:

$$r_t = \overline{r}_t + \phi_\pi \pi_t . \tag{11}$$

Equation (11) implies that the nominal interest rate is determined according to the changes in current inflation rate. Here, monetary policy rule (11) does not include an interest rate response to the output gap, unlike the standard Taylor rule.

Substituting (11) into (5) for $r_t - \overline{r}_t$, and rearranging the terms, the system can be written in the form $E_t y_{t+1} = C y_t$,

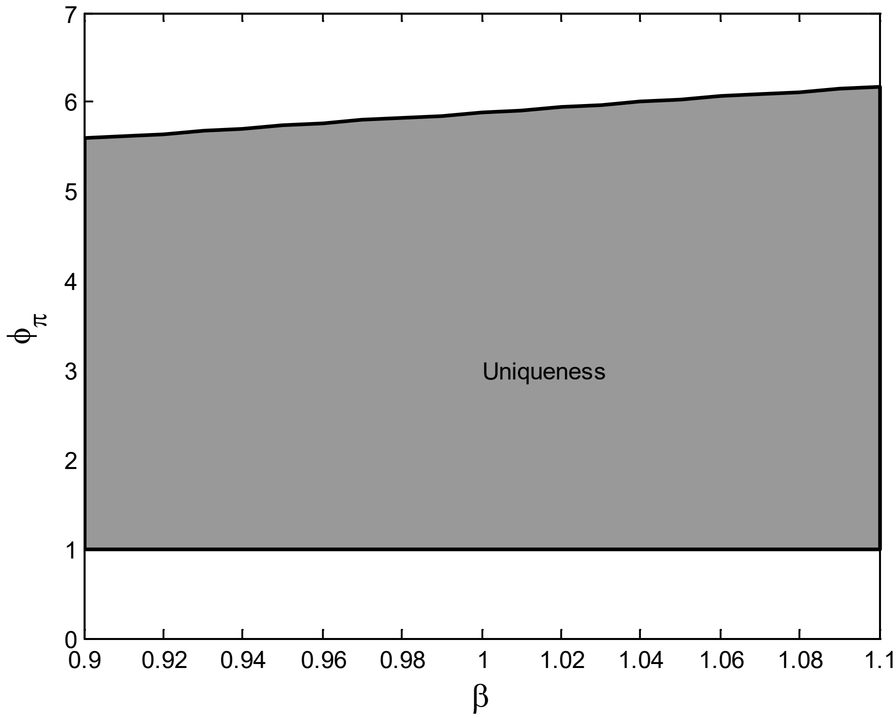
$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \frac{1}{\beta} \begin{bmatrix} \left(\mu + \beta + \frac{1+\alpha(\omega-1)}{\sigma} \mu\varphi \right) & -\frac{1+\alpha(\omega-1)}{\sigma}(1-\beta\phi_\pi) \\ -\mu\left(\frac{\sigma}{1+\alpha(\omega-1)} + \varphi \right) & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} . \tag{12}$$

Notice that there are two endogenous variables: inflation rate, π_t, and output gap, x_t. There is no predetermined variable in the model. Following Blanchard and Kahn (1980), the system (12) has a unique equilibrium solution for the output gap and the inflation rate, if and only if the number of the matrix *C*'s eigenvalues that are outside the unit circle is equal to the number of forward looking (non-predetermined) variables. In this case, the number of those variables, $E_t x_{t+1}$ and $E_t \pi_{t+1}$, is two. Then, following Bullard and Mitra (2002), Proposition (2) characterizes the necessary and sufficient conditions for the determinacy, as illustrated in Figure 2.

Proposition 2: Given monetary policy based on pure current-looking inflation targeting, the open economy New Keynesian model described by system (12) has a unique stationary equilibrium, if and only if

$$1 < \phi_\pi < \frac{(\beta-1)\sigma}{\mu\sigma + \mu\varphi(1+\alpha(\omega-1))}. \tag{13}$$

Figure 2: Determinacy region under pure current-looking inflation targeting



Regarding determinacy, the condition $\phi_\pi > 1$ requires an active monetary policy, so that the central bank adjusts nominal interest rates more than one-for-one in response to a deviation in inflation rate from its target level. On the other hand, Minford and Srinivasan (2010) argue that with $\phi_\pi > 1$, explosive solutions are also possible within the model, just as multiple solutions are possible with $\phi_\pi < 1$. But the upper boundary implies that the policy should not react too aggressively, since that would also lead to indeterminacy. Thus, the upper boundary prevents overreaction of the monetary authority to changes in inflation, since such overreaction might result in explosive solutions. However, as the degree of openness (captured by α) increases, the upper bound gets lower, and the determinacy region shrinks.

Ball (1998) argues that pure inflation targeting brings some risks in an open economy environment by giving rise to large fluctuations in exchange rate and output. He suggests following long-run inflation targeting to avoid such problems.

As illustrated in Figure (2), the pure current-looking inflation-targeting monetary policy yields a unique equilibrium for a feasible set of parameter values. Given the baseline values of the parameters, the Jacobian matrix of the system is

$$C = \begin{bmatrix} 1.3434 & 0.5051 \\ -0.3434 & 1.0101 \end{bmatrix},$$

with complex conjugate eigenvalues $\lambda_1 = 1.1768 + 0.3817\mathrm{i}$ and $\lambda_2 = 1.1768 - 0.3817\mathrm{i}$, having modulus $R = \sqrt{1.1768^2 + 0.3817^2} = 1.2372$. With a radius greater than unity, both eigenvalues are outside the unit circle. Since the number of eigenvalues outside the unit circle and the number of forward-looking variables are equal, the system (12) has a unique, stationary equilibrium solution.

3.3 With Credibility Gap under the Current-Looking Inflation Targeting:

We now modify the policy rule to evaluate the effects of a credibility gap, which shows to what extend agents discount the central bank's decisions, as described in Gali (2008). As before, we assume that the economy is described by equations (4) and (5), while the central bank follows the pure current-looking inflation targeting rule, so that

$$r_t = \overline{r}_t + \phi_\pi \pi_t . \tag{14}$$

Suppose that the public, on the other hand, believes that the monetary policy rule is given by

$$r_t = \overline{r}_t + \phi_\pi (1 - \delta) \pi_t , \tag{15}$$

where $\phi_\pi > 1$ and $\delta \in \mathbb{R}$ measures the credibility gap. We consider the system consisting of the equations (4), (5), and (15).

Substituting (15) into (5) for $r_t - \overline{r}_t$, and rearranging the terms, the system can be written in the form $E_t y_{t+1} = C y_t$ as follows

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \frac{1}{\beta} \begin{bmatrix} \left(\mu + \beta + \frac{1+\alpha(\omega-1)}{\sigma} \mu\varphi \right) & -\left(\beta\phi_\pi (1-\delta) + 1 \right) \frac{1+\alpha(\omega-1)}{\sigma} \\ -\mu \left(\frac{\sigma}{1+\alpha(\omega-1)} + \varphi \right) & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix}. \tag{16}$$

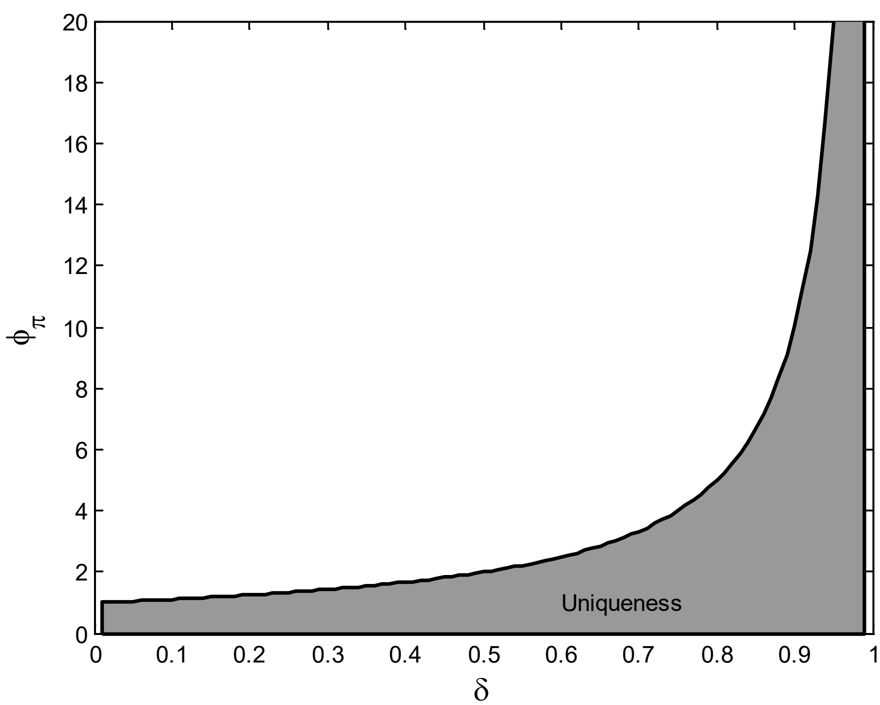
Following Bullard and Mitra (2002), the proposition below characterizes the necessary and sufficient conditions for the determinacy of system (16).

Proposition 3: Given the monetary policy based on current-looking inflation targeting with credibility gap, the open economy New Keynesian model described by the system (16) has a unique stationary equilibrium, if and only if

$$1 < \phi_\pi (1-\delta) < \frac{\sigma(\beta-1)}{\mu\sigma + \mu\varphi(1+\alpha(\omega-1))}. \quad (17)$$

Since $\phi_\pi > 0$ by assumption, the lower boundary of the determinacy condition (17) equivalently holds, if $\phi_\pi > \frac{1}{1-\delta}$. This resembles the Taylor rule, with the exception of the credibility gap. As the credibility gap rises, the determinacy region shrinks, giving rise to less room for existence of a unique solution. When $\delta = 0$, the model collapses to the model in Section 3.2. Therefore, in case of a credibility gap, the monetary policy authority has to compensate for the lack of credibility by pursuing a more aggressive policy. The more aggressive policy requires raising the nominal interest rate much more than the deviation of the inflation rate from its target level. This policy results in a larger increase in the real interest rate. Figure (3) illustrates the determinacy region as a function of the credibility parameter δ. The credibility gap dramatically narrows the region of unique equilibrium.

Figure 3: Determinacy diagram under current-looking inflation targeting with credibility gap



At the point where $\left(\phi_\pi(1-\delta)-1\right)\left(\frac{\mu\sigma}{1+\alpha(\omega-1)}+\mu\varphi\right)=0$

ϕ_π , we obtain $\phi_\pi = 2$. Thus, the branching point occurs at $\phi_\pi = 2$. In the open economy framework, the occurrence of a branching point requires that the monetary policy instrument responds to changes in the inflation rate by twice as much in the presence of a credibility gap.

3.4 Under Current-Looking Taylor Rule with Interest Rate Smoothing:

It has been shown empirically that the lagged interest rate usually receives a statistically significant coefficient estimate, when the interest rate is regressed on inflation and output gap. Some authors conclude that the lag coefficient reflects inertial behavior, while others argue that the lag represents gradual adjustment policy by the monetary authority. Parameter uncertainty, imperfect information, and pursuit of financial stability are considered some of the motivations leading the policy maker to adopt such a precautionary policy. See, for example, Sack (2000), Rudebusch (2005), and Walsh (2003) for further discussion of the subject.

Consider the model in which equations (4) and (5) describe the economy, while the following equation is the monetary policy rule:

$$r_t = \overline{r_t} + \phi_\pi \pi_t + \phi_x x_t + \phi_r r_{t-1}, \tag{18}$$

where ϕ_r is the degree of interest rate smoothing, while ϕ_π and ϕ_x are the central bank's relative policy weights assigned to the inflation rate and the output gap, respectively. Equation (18) states that the nominal interest rate is determined by the central bank in response to the current values of the inflation rate and output gap, as well as the policy rate in the previous period.

Woodford (2003a) finds that the interest rate inertia coefficient ϕ_r is equal to 0.46, implying that interest rates should be adjusted roughly half of the way toward the target level within a quarter.

Moving the equation (18) one period forward, adding expectations, rearranging terms, and defining $y_t = [x_t, \pi_t, r_t]'$, we can write the system of equations in the form $E_t y_{t+1} = Cy_t + D$ as follows:

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ E_t r_{t+1} \end{bmatrix} = C \begin{bmatrix} x_t \\ \pi_t \\ r_t \end{bmatrix} + \begin{bmatrix} -\frac{1-\alpha+\alpha\omega}{\sigma}\overline{r_t} \\ 0 \\ E_t \overline{r}_{t+1} - \phi_x \overline{r_t}\frac{1-\alpha+\alpha\omega}{\sigma} \end{bmatrix}, \tag{19}$$

where

$$C = \begin{bmatrix} \frac{\mu}{\beta}\left(1+\varphi\frac{1-\alpha+\alpha\omega}{\sigma}\right)+1 & -\frac{1-\alpha+\alpha\omega}{\beta\sigma} & \frac{1-\alpha+\alpha\omega}{\sigma} \\ -\frac{\mu}{\beta}\left(\frac{\sigma}{1+\alpha\left(\omega-1\right)}+\varphi\right) & \frac{1}{\beta} & 0 \\ \phi_x+\frac{\mu}{\beta}\left(1+\varphi\frac{1-\alpha+\alpha\omega}{\sigma}\right)\left(\phi_x\frac{1-\alpha+\alpha\omega}{\sigma}-\phi_\pi\right) & -\frac{1}{\beta}\left(\phi_x\frac{1-\alpha+\alpha\omega}{\sigma}-\phi_\pi\right) & \phi_r+\phi_x\frac{1-\alpha+\alpha\omega}{\sigma} \end{bmatrix}.$$

Note that there are three endogenous variables: the rate of inflation, π_t, the output gap, x_t, and the nominal interest rate, r_t. Following Blanchard and Kahn (1980), the system (19) has a unique, stationary equilibrium solution, if and only if the number of eigenvalues of the 3x3 coefficient matrix C outside the unit circle is equal to the number of forward-looking (non-predetermined) variables. There are three such variables, $E_t x_{t+1}$, $E_t \pi_{t+1}$ and $E_t r_{t+1}$. Consequently, we should have all the eigenvalues to be outside the unit circle for uniqueness.

Proposition 4: Given monetary policy based on current-looking Taylor rule with interest rate smoothing, the open economy New Keynesian model described by the system (19) leads to indeterminacy.

We can numerically verify whether Proposition (4) holds for the given values of the parameters in Gali and Monacelli (2005). Their Jacobian matrix is

$$C = \begin{bmatrix} 1.3434 & -1.0101 & 1 \\ -0.3434 & 1.0101 & 0 \\ -0.3472 & 1.3889 & 0.6250 \end{bmatrix},$$

with eigenvalues $\lambda_1 = 1.3743 + 0.5546i$, $\lambda_2 = 1.3743 - 0.5546i$, and $\lambda_3 = 0.23$. Note that one solution is real, positive, and inside the unit circle, while the other two solutions are complex conjugate with radius greater than one. Since the number of eigenvalues outside the unit circle (which is two) is less than the number of forward-looking variables (which is three), there is no unique solution to the system. The indeterminacy result suggests that there are other stationary equilibrium solutions to the system (19) under the current-looking Taylor rule with interest rate smoothing.

3.5 Under Forward-Looking Taylor Rule:

Rational expectations based policy rules have been studied by many economists, such as Evans and Honkapohja (2003a, b) and Branch and McGough (2009, 2010). Evans and

Honkapohja (2003a) argue that expectations-based rules can give rise to determinate and stable equilibria under learning. As Bullard and Mitra (2002) point out, forward-looking rules, together with backward-looking policy rules, have been considered as alternatives to account for McCallum's (1999) criticism of unrealistic dependence upon only partially available contemporaneous information. Batini and Haldane (1999) argue that inflation-forecasting based rules perform well in comparison with other simple rules. The forward-looking approach lets the policy makers evaluate the time lag between performing a certain policy and observing its impacts on economy, while evaluating the future conditions of the economy in a realistic setting based on the available information set.

Using a Calvo-type sticky price model with a capital and investment spending component, Carlstrom and Fuerst (2005) conclude that the determinacy region, under forward-looking interest rate rule, shrinks dramatically and eventually leads to indeterminacy. McCallum (2003) argues that multiple solutions may arise, when the policy rule responds to the expected future inflation, rather than current inflation. This is called "Woodford warning" by McCallum (2003) and Svensson (1997a,b), since it was first pointed out by Woodford (1994). In this section, we derive the conditions to ensure a unique equilibrium solution.

Consider the model in which equations (4) and (5) describe the economy, while equation (20) represents the monetary policy rule followed by the central bank:

$$r_t = \overline{r}_t + \phi_\pi E_t \pi_{t+1} + \phi_x E_t x_{t+1}, \qquad (20)$$

where x_t denotes the output gap, π_t is the inflation rate, and r_t is the nominal interest rate. E_t is the expectation operator. The policy parameters, ϕ_π and ϕ_x, represent the magnitude of the central bank's responses to the next period's expected inflation rate and expected output gap, respectively. As before, there is no exogenous shock.

Note that the policy rule, (20), nests the standard Taylor rule as a special case. In this specification, the actual inflation and output gap are replaced by the expected inflation and expected output gap. The policymaker, however, looks only one quarter ahead, while adjusting the nominal interest rate. Clarida, Gali, and Gertler (2000) employ this approach in estimating the reaction function of the Federal Reserve for the postwar US economy.

Substituting (20) into (5) for $r_t - \overline{r}_t$ and rearranging the terms, we have the reduced system in normal form, $E_t y_{t+1} = C y_t$:

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \dfrac{\beta\sigma - (\mu\sigma + \mu\varphi(1+\alpha(\omega-1)))(\phi_\pi - 1)}{\beta\sigma - \beta\phi_x(1+\alpha(\omega-1))} & \dfrac{(\phi_\pi - 1)(1+\alpha(\omega-1))}{\beta\sigma - \beta\phi_x(1+\alpha(\omega-1))} \\ -\dfrac{\mu\sigma + \mu\varphi(1+\alpha(\omega-1))}{\beta + \alpha\beta(\omega-1)} & \dfrac{1}{\beta} \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix}. \tag{21}$$

As before, we begin our analysis by examining the determinacy conditions of the system (21). Note that there exist two free endogenous variables, x_t and π_t. Then, following Blanchard and Kahn (1980), the equilibrium solution would be unique, if and only if both eigenvalues are outside the unit circle. The following Proposition characterizes the necessary and sufficient conditions for the determinacy of the system (21).

Proposition 5: Under the monetary policy based on forward-looking interest rate rule, the open economy New Keynesian model described by system (21) has a unique stationary equilibrium, if and only if

$$\phi_x < \frac{\sigma(1-\beta^{-1})}{1+\alpha(\omega-1)}, \tag{22}$$

$$(1-\beta)\phi_x + \mu\left(\frac{\sigma}{1+\alpha(\omega-1)} + \varphi\right)(\phi_\pi - 1) > 0, \tag{23}$$

and

$$(1+\beta)\phi_x + \mu\left(\frac{\sigma}{1+\alpha(\omega-1)} + \varphi\right)(\phi_\pi - 1) < \frac{2\sigma(1+\beta)}{1+\alpha(\omega-1)}. \tag{24}$$

The conditions (23) and (24) provide lower and upper boundaries, respectively, for the monetary policy to yield a unique stationary equilibrium. Therefore, conditions (23) and (24) are the necessary and sufficient conditions for the Jacobian matrix C to have both eigenvalues outside the unit circle.

For the baseline values of the parameters, this upper bound requires

$$(1+\beta)\phi_x + \mu\left(\frac{\sigma}{1+\alpha(\omega-1)} + \varphi\right)(\phi_\pi - 1) \geq 2.99$$

$$(1-\beta)\phi_x + \mu\left(\frac{\sigma}{1+\alpha(\omega-1)} + \varphi\right)(\phi_\pi - 1) = 0$$

$$\phi_\pi$$

parameters α and ω, the lower the upper boundary. Consequently, the range of determinacy is smaller in the open economy framework and gets smaller as the values of the parameters α and ω increase.

It seems that indeterminacy is more likely to happen in the open economy framework and becomes a serious issue as the degree of openness increases. McKnight (2007) finds similar results for an open economy model with the scope of indeterminacy increasing as the degree of openness increases under forward-looking monetary policy rules. Llosa and Tuesta (2008) also support our findings. They argue that targeting the expected rate of future inflation would shrink the determinacy region significantly, due to the interaction between trade openness and policy actions. Increasing nominal exchange rates causes an increase in expected inflation. Monetary authority increases in interest rates produce distortion in expectations, boosting expectations of even higher domestic inflation.

Using the calibration values of the parameters given in Gali and Monacelli (2005) and solving the equation $(1-\beta)\phi_x + \mu\left(\frac{\sigma}{1+\alpha(\omega-1)} + \varphi\right)(\phi_\pi - 1) = 0$ for ϕ_π, we find $\phi_\pi = 1$, approximately. That means the system (21) will have a branching point at around $\phi_\pi = 1$. Therefore, we can say that in an open economy framework, the monetary policy instrument, the short-term interest rate, should respond slightly more than in a closed economy to changes in the expected inflation rate, accompanied by a small but positive response to the expected output gap.

Figure 4: Determinacy and indeterminacy regions under forward looking Taylor rule.

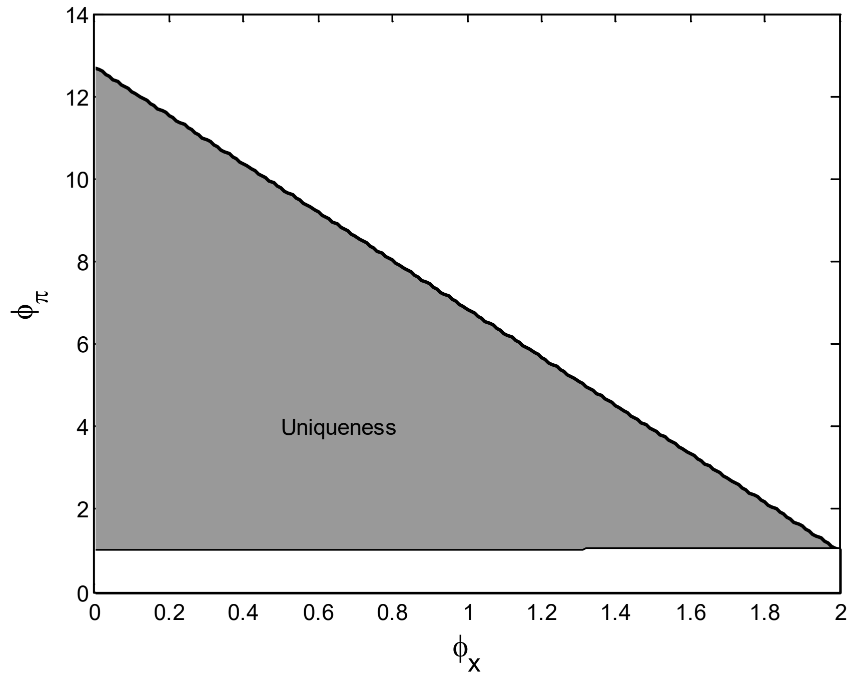


Figure (4) depicts the regions of determinacy and indeterminacy in (ϕ_x, ϕ_π)-space, given the baseline values of the parameters. High values of ϕ_π and/or ϕ_x cause the indeterminacy problem. Contrary to the current-looking policy rule case, uniqueness of equilibrium under the forward-looking policy rule requires a mild reaction of the monetary authority to shifts in inflation rate or in the output gap. Thus, the monetary authority should react neither too strongly nor too weakly to changes in the expected inflation or the expected output gap. Rules with $\phi_\pi > 1$ accompanied by a moderate reaction to expected output gap would be enough to acquire a unique equilibrium.

Given the benchmark values of the parameters, the Jacobian matrix is

$$C = \begin{bmatrix} 0.9466 & 0.5772 \\ -0.3434 & 1.0101 \end{bmatrix},$$

having eigenvalues $\lambda_1 = 0.9784 + 0.4441i$ and $\lambda_2 = 0.9784 - 0.4441i$ with radius R=1.0745. Figure (5) illustrates three trajectories constructed for different parameter settings, indicating the occurrence of a Hopf bifurcation within the system (21).[5]

Figure 5: Three trajectories of the system (21)

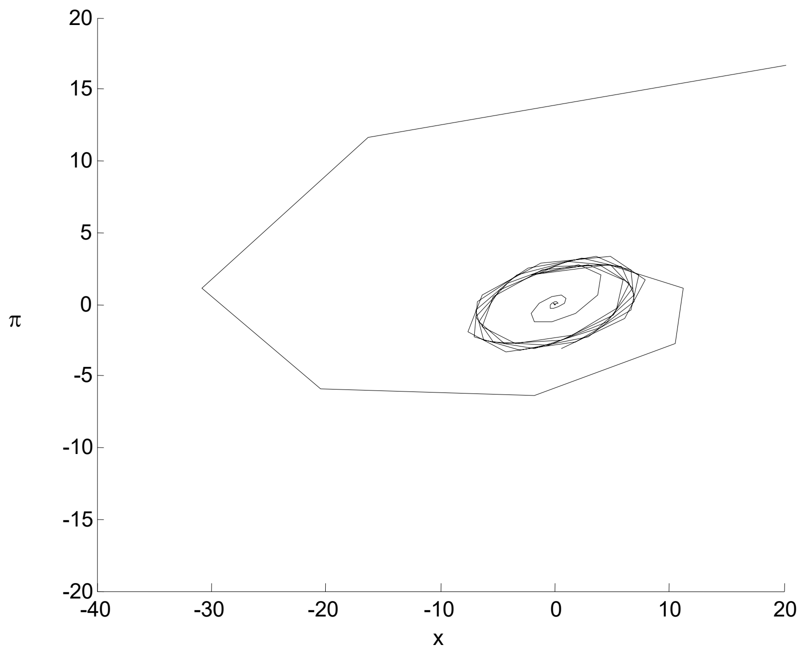
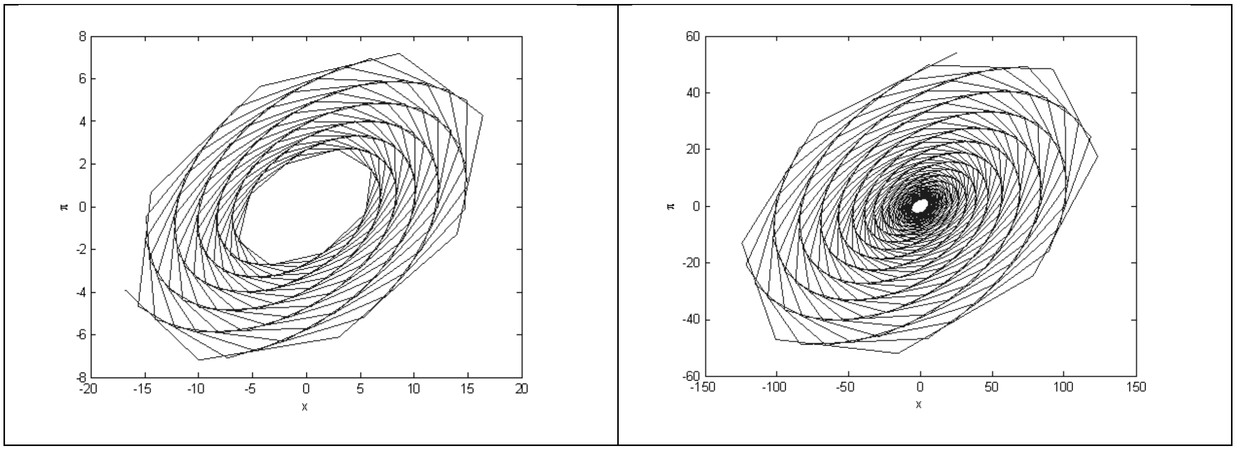


Figure (6) displays phase diagrams for two different numbers of iterations at $\phi_\pi = 2.8$ and $\phi_x = 0$. As also observed by Barnett and Eryilmaz (2016), the system has a periodic solution at these parameter values. The origin is a stable spiral point. Any solution that starts near the origin in phase plane will eventually spiral towards the origin. Because the trajectories spiral inward, the origin is a stable sink.

[5] Diverging trajectory (black) drawn at $\phi_\pi = 2.8$ and $\phi_x = 0.4$, limit cycle (red) drawn at $\phi_\pi = 2.8$ and $\phi_x = 0$, converging trajectory (blue) at $\phi_\pi = 2.8$ and $\phi_x = -0.4$.

Figure 6: Phase diagrams displaying periodic solutions, using two different numbers of iterations at $\phi_\pi = 2.8$ and $\phi_x = 0$ in system (21).



Considering the same policy rule, Barnett and Eryilmaz (2012) found a Hopf bifurcation within the Gali and Monacelli (2005) functional structure in the open economy framework, when $\phi_x^* = \frac{(\beta - 1)}{\beta} \frac{\sigma}{1 + \alpha(\omega - 1)}$, if $\Delta < 0$ is also satisfied.

3.6 Under Pure Forward-Looking Inflation Targeting:

Most major countries have been pursuing inflation targeting, sometimes accompanied by an output gap target, to reduce the high and volatile inflation risk. Therefore, in recent years, rules that set the policy rate in response to the forecasted rate of inflation have been widely consistent with "inflation-averse" monetary policies.

Consider the model in which equations (4) and (5) describe the economy, while equation (25) represents the monetary policy rule followed by the central bank:

$$r_t = \overline{r_t} + \phi_\pi E_t \pi_{t+1}, \tag{25}$$

where x_t denotes the output gap, π_t is the inflation rate, and r_t is the nominal interest rate. E_t is the expectation operator. Equation (3.2.11) describes a pure forward-looking interest rate rule, in which the policy parameter ϕ_π measures the extent of the policy rate's response to the next period's expected inflation. The nominal interest rate is determined by looking at the changes in next period's expected inflation. As before, there is no exogenous shock.

Substituting (25) into (5) for $r_t - \overline{r}_t$, and rearranging the terms, we have the following reduced system in normal form, $E_t y_{t+1} = C y_t$:

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 - \left(\frac{\mu}{\beta} + \frac{\varphi\mu\left(1+\alpha(\omega-1)\right)}{\beta\sigma} \right)(\phi_\pi - 1) & \frac{(\phi_\pi - 1)\left(1+\alpha(\omega-1)\right)}{\beta\sigma} \\ -\frac{\mu}{\beta}\left(\frac{\sigma}{1+\alpha(\omega-1)} + \varphi \right) & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix}. \quad (26)$$

Figure 7: Phase space plot for $\beta = 1$ and $\phi_\pi = 8$ in system (26)

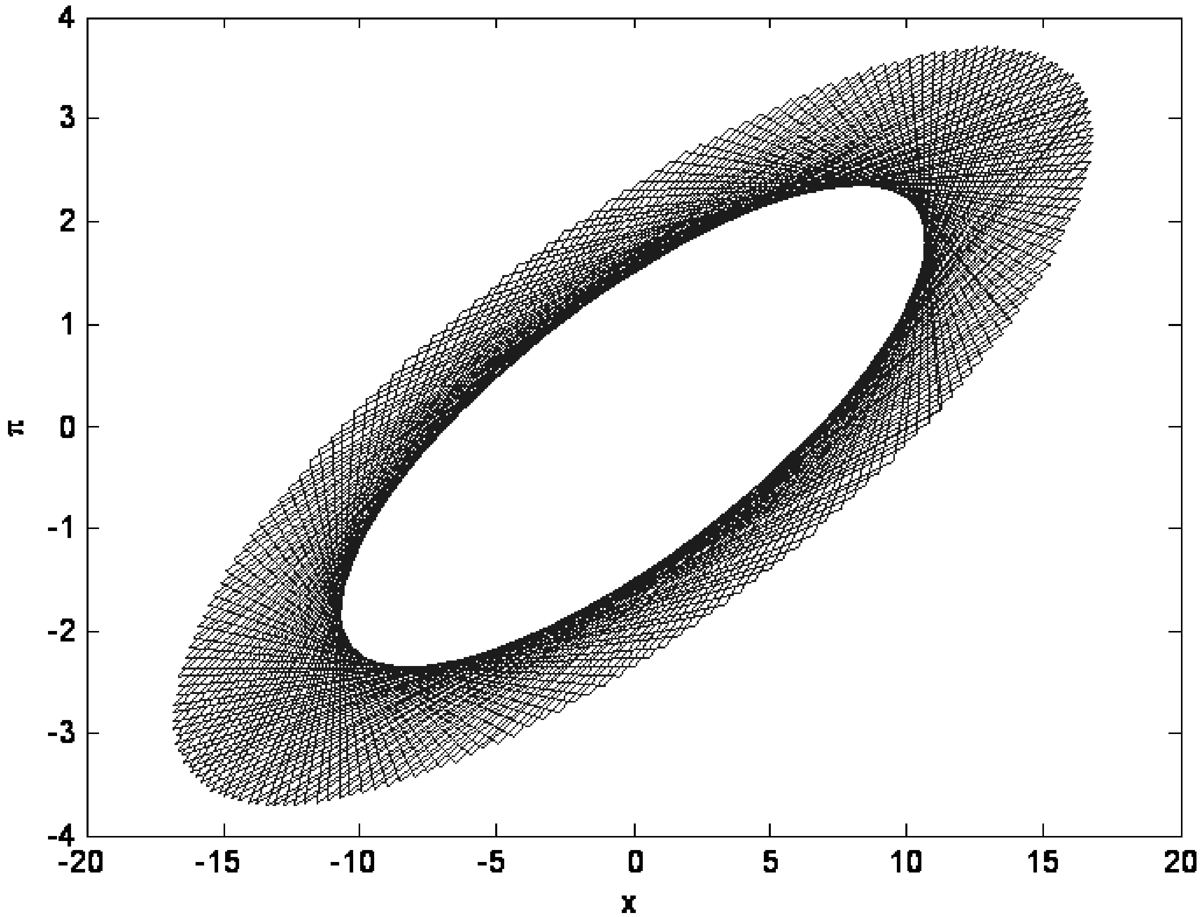


Figure (7) illustrates a solution path for $\beta = 1$ $\phi_\pi = 8$

existence of bifurcations in open economy framework, found that the system (26) undergoes a Hopf bifurcation, when $\beta^* = 1$, if $\Delta < 0$ is also satisfied.

Note that there exist two free endogenous variables, x_t and π_t. Then, following Blanchard and Kahn (1980), the equilibrium solution would be unique, if and only if both eigenvalues are outside the unit circle. The following Proposition characterizes the necessary and sufficient conditions for having a determinate equilibrium solution to the system (26).

Proposition 6: Under the pure forward looking inflation targeting rule, the open economy New Keynesian model specified by system (26) has a unique stationary equilibrium, if and only if

$$\beta > 1 \tag{27}$$

and

$$\mu\left(\frac{\sigma}{1+\alpha\left(\omega-1\right)}+\varphi\right)\left(\phi_\pi - 1\right) > 0 . \tag{28}$$

Notice that the condition (27) is not satisfied, since $\beta \in \left(0,1\right)$. Hence, for the given parameter values in Gali and Monacelli (2005), the system (26) does not guarantee the uniqueness of the equilibrium solution path. If monetary policy is irresponsive to output, so that $\phi_x = 0$, then controlling the parameter value of inflation has an insufficient effect on the determinacy. In this case, no value of ϕ_π can ensure a determinate equilibrium unless $\beta > 1$. That said, a discount factor, β, greater than unity is required to fix the indeterminacy problem, but this would be questionable from an empirical perspective. Besides, as we also verified numerically, $\beta = 1$ is a branching point separating a unique equilibrium from multiple equilibria.

As suggested by Proposition 6, one major drawback of pure forward-looking inflation targeting is that it often causes equilibrium indeterminacy. As a means to prevent such policy-induced instability problem, some authors suggest that another endogenous variable, such as output gap, should be targeted along with expected inflation. For example, De Fiore and Liu (2005) and Carlstrom and Fuerst (2006) argue that the New Keynesian model with pure forward-looking inflation targeting produces instability. The failure of pure inflation targeting emphasizes the importance of policy response to output. Those authors also argue that with nominal rigidities, equilibrium indeterminacy cannot be solved just by having the nominal rate respond to both inflation and output gap. Huang and Meng (2007) show that interest rate policy rules that are unresponsive to output usually give rise to equilibrium indeterminacy. They argue that increasing the degree of price stickiness or allowing for policy response to current output can produce determinacy of equilibrium. But the first method has a quantitatively negligible effect, while the second method's success is sensitive to the elasticity of labor supply and the degree of stickiness.

Without determinacy, a multiplicity of stable equilibria exists for Model 3.6. Theoretically, any of these solution paths could be realized. In such cases, as Cochrane (2009) argues, the New Keynesian model has nothing to say about inflation, other than that anything can happen. That is a reason the non-uniqueness problem is important in modeling. Bernanke and Woodford (1997) argue that forecasted inflation targeting is inconsistent with rational expectation equilibrium and prone to indeterminacy. They suggest that the monetary authority should develop a structural model and monitor some other variables besides the inflation target.

Batini and Haldane (1999) argue that even though the forward-looking dimension makes the policy rule perform better than the standard Taylor rule, longer forecast horizons (longer than 3-6 quarters) risk macroeconomic stability. Giannoni (2014) argues that the presence of indeterminacy in a sticky price model under inflation targeting is possible for a reasonable subset of parameter values. He also shows that the indeterminacy vanishes when the central bank targets a price level. Dittmar and Gavin's (2004) findings support this argument in a flexible-price model by advocating price level targeting instead of inflation rate targeting.

As we determined numerically, one of the two real eigenvalues of the Jacobian matrix lies inside the unit circle, while the other is outside. Given that both x_t and π_t are non-predetermined, the existence of an eigenvalue inside the unit circle implies the existence of multiple equilibria. Hence, there is no guarantee that $x_t = \pi_t = 0$ will be the equilibrium solution.

Gali (2008), on the other hand, states that the following condition

$$1 < \phi_\pi < 1 + \frac{2\sigma_\alpha\left(1+\beta\right)}{\kappa_\alpha}, \tag{29}$$

would be necessary and sufficient for determinacy. This condition suggests that, besides satisfying the Taylor principle ($\phi_\pi > 1$

$\phi_\pi = 1$, a

smooth and non-converging sunspot equilibrium emerges. When $\phi_\pi = 1 + \frac{2\sigma_\alpha(1+\beta)}{\kappa_\alpha}$, on the other hand, a cyclical and non-converging sunspot equilibrium appears. Nakagawa (2009) supports the same argument and states that an aggressive response to expected inflation would lead to equilibrium indeterminacy. The current economy would fluctuate, even though expectations for the future economy would be stabilized.

Given the values of the parameters by Gali (2008), $\sigma_\alpha = 1$, $\beta = 1$, $\kappa_\alpha = 0.1275$, and $\phi_\pi = 32.2157$, Nakagawa (2009) finds sunspot equilibrium dynamics such that sunspot equilibria under $\phi_\pi > 32.2$ are oscillatory convergent. The sunspot equilibria under $\phi_\pi < 1$ smoothly approach the steady state. If $\phi_\pi = 1$ or $\phi_\pi = 32.2$, sunspot dynamics stop converging.

3.7 Under Backward-Looking Taylor Rule:

A backward-looking approach enables policy makers to consider lagged information on output gap and inflation, while determining the current period's policy rate. This approach is often considered a more realistic assumption than making decisions based on contemporaneous information. Carlstrom and Fuerst (1999, 2000) advocate a backward-looking policy rule to reach a unique stationary equilibrium, while also arguing that the conditions for determinacy in a small open economy are not different from in a closed economy. However, De Fiore and Liu (2005) argue that as the openness to trade increases, the region of determinacy shrinks. See, for example, McCallum (1999) and Bullard and Mitra (2002) for further discussion of lagged data use in monetary policy rules.

Consider the model in which equations (4) and (5) describe the economy, while equation (30) is the interest rate rule employed by the central bank for monetary policy:

$$r_t = \overline{r}_t + \phi_\pi \pi_{t-1} + \phi_x x_{t-1}. \tag{30}$$

Moving equation (30) one period forward, adding expectations, rearranging the terms, and defining $y_t = [x_t, \pi_t, r_t]'$, we can write the system in the standard form, $E_t y_{t+1} = Cy_t$:

$$E_t y_{t+1} = Cy_t + \begin{bmatrix} -\frac{1+\alpha(\omega-1)}{\sigma}\overline{r}_t \\ 0_t \\ E_t \overline{r}_{t+1} \end{bmatrix}, \tag{31}$$

$$\text{where } C = \begin{bmatrix} \frac{\mu}{\beta}\left(1+\frac{\varphi(1+\alpha(\omega-1))}{\sigma}\right)+1 & -\frac{1+\alpha(\omega-1)}{\beta\sigma} & \frac{1+\alpha(\omega-1)}{\sigma} \\ -\frac{\mu}{\beta}\left(\frac{\sigma}{1+\alpha(\omega-1)}+\varphi\right) & \frac{1}{\beta} & 0 \\ \phi_x & \phi_\pi & 0 \end{bmatrix}.$$

Note that there are three endogenous variables (the rate of inflation, π_t, the output gap, x_t, and the nominal interest rate, r_t) and two predetermined variables (x_{t-1} and π_{t-1}). Following Blanchard and Kahn (1980), the system (31) has a unique, stationary equilibrium solution, if and only if the number of eigenvalues outside the unit circle is equal to the number of forward looking (non-predetermined) variables. In this cases, there are two such variables, $E_t x_{t+1}$ and $E_t \pi_{t+1}$. Accordingly, two of the eigenvalues must be outside the unit circle for uniqueness.

With the backward-looking Taylor rule and using Descartes' rule of signs theorem, we have the following Proposition, characterizing the necessary and sufficient conditions for system (31) to have a unique stationary equilibrium.

Proposition 7: Under the backward-looking Taylor rule, the open economy New Keynesian model specified by the system (31) has a unique stationary equilibrium, if and only if

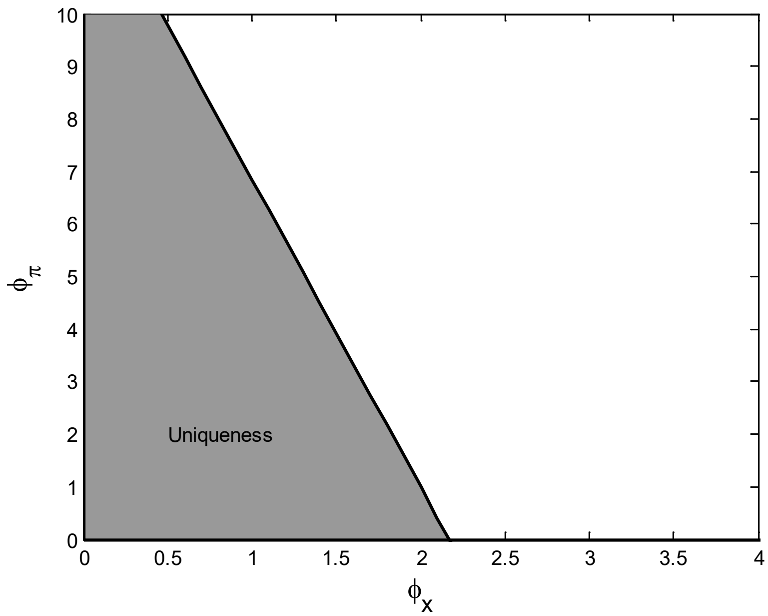
$$\mu\left(\frac{\sigma}{1+\alpha(\omega-1)}+\varphi\right)(\phi_\pi-1)+(1-\beta)\phi_x>0, \tag{32}$$

and

$$\mu\left(\frac{\sigma}{1+\alpha(\omega-1)}+\varphi\right)(\phi_\pi-1)+(1+\beta)\phi_x<\frac{2\sigma(1+\beta)}{1+\alpha(\omega-1)}. \tag{33}$$

Conditions (32) and (33) together imply that a sufficiently active policy rule with $\phi_\pi > 1$, accompanied by a small response to the output gap, is sufficient for a unique equilibrium. Eusepi (2005) argues that contrary to the forecast-based Taylor rules, the backward-looking Taylor rule stabilizes the economy by leading to a uniquely learnable equilibrium. Figure (8) illustrates the regions of unique and multiple solutions.

Figure 8: Determinacy diagram for the backward-looking Taylor rule



Using the calibrated values of the parameters given by Gali and Monacelli (2005), determinacy of the equilibrium can be checked by computing the eigenvalues of the Jacobian matrix

$$C = \begin{bmatrix} 1.3434 & -1.0101 & 1 \\ -0.3434 & 1.0101 & 0 \\ 0.1250 & 1.5 & 0 \end{bmatrix}$$

with eigenvalues $\lambda_1 = 1.3518 + 0.0658\text{i}$, $\lambda_2 = 1.3518 - 0.0658\text{i}$, and $\lambda_3 = -0.3502$. Note that one solution is real and inside the unit circle in absolute value, while the radius of the two complex conjugate solutions is greater than one with $R = 1.3534$. The number of eigenvalues outside the unit circle is equal to the number of forward-looking variables, which is two. Hence, there exists a unique solution of the system.

3.8 Under Pure Backward-Looking Inflation Targeting Rule:

Consider the model in which equations (4) and (5) describe the economy, while equation (34) is the interest rate rule employed by the central bank:

$$r_t = \overline{r_t} + \phi_\pi \pi_{t-1} \,. \tag{34}$$

Equation (34) is pure backward-looking inflation targeting, in which the nominal interest rate is set according to the inflation rate realized in the previous period, $t-1$

$y_t = [x_t, \pi_t, r_t]'$, we can write the system in normal form, $E_t y_{t+1} = C y_t$, as follows,

$$E_t y_{t+1} = C y_t + \begin{bmatrix} -\frac{1+\alpha(\omega-1)}{\sigma}\overline{r}_t \\ 0_t \\ E_t\overline{r}_{t+1} \end{bmatrix} \tag{35}$$

where

$$C = \begin{bmatrix} \mu\left(\frac{1}{\beta}+\varphi\frac{1+\alpha(\omega-1)}{\beta\sigma}\right)+1 & -\frac{1+\alpha(\omega-1)}{\beta\sigma} & \frac{1+\alpha(\omega-1)}{\sigma} \\ -\frac{\mu}{\beta}\left(\frac{\sigma}{1+\alpha(\omega-1)}+\varphi\right) & \frac{1}{\beta} & 0 \\ 0 & \phi_\pi & 0 \end{bmatrix}.$$

Note that there are three endogenous variables (the rate of inflation, π_t, the output gap, x_t, and the nominal interest rate, r_t) and one pre-determined variable (π_{t-1}). Following Blanchard and Kahn (1980), the system has a unique, stationary equilibrium solution, if and only if the number of eigenvalues outside the unit circle is equal to the number of forward looking (non-predetermined) variables. In this case, there are two such variables, $E_t x_{t+1}$ and $E_t \pi_{t+1}$.

Following Bullard and Mitra (2002) and using Descartes' rule of signs theorem, we have Proposition (8), characterizing the necessary and sufficient conditions for determinacy.

Proposition 8: Under pure backward-looking inflation targeting rule, the open economy New Keynesian model described by the system (35) has a unique stationary equilibrium, if and only if

$$\mu\left(\frac{\sigma}{1+\alpha(\omega-1)}+\varphi\right)(\phi_\pi-1) > 0, \tag{36}$$

and

$$\mu\left(\frac{\sigma}{1+\alpha(\omega-1)}+\varphi\right)(\phi_\pi-1) < \frac{2(1+\beta)\sigma}{1-\alpha+\alpha\omega}. \tag{37}$$

Conditions (36) and (37) show that a sufficiently active policy rule with $\phi_\pi > 1$ leads to a determinate equilibrium. We can numerically verify whether Proposition (8) holds for the given values of the parameters in Gali and Monacelli (2005). The uniqueness of a solution can be easily checked by computing the Jacobian matrix, which is

$$C = \begin{bmatrix} 1.3434 & -1.0101 & 1 \\ -0.3434 & 1.0101 & 0 \\ 0 & 1.5 & 0 \end{bmatrix}$$

with eigenvalues $\lambda_1 =$ 1.3217 + 0.1720i , $\lambda_2 =$ 1.3217 - 0.1720i , and $\lambda_3 =$ -0.2900 . Note that one solution is real and inside the unique circle in absolute value, while the radius of the two complex conjugate solutions are outside the unit circle with $R = 1.3534$. Recalling Blanchard and Kahn (1980), the system (35) has a unique, stationary equilibrium solution, since the number of eigenvalues outside the unit circle is equal to the number of forward-looking variables.

3.9 Under Backward-Looking Taylor Rule with Interest Rate Smoothing:

Consider the model in which the equations (4) and (5) describe the economy, while equation (38) is the interest rate rule followed by the central bank:

$$r_t = \overline{r}_t + \phi_\pi \pi_{t-1} + \phi_x x_{t-1} + \phi_r r_{t-1} . \qquad (38)$$

Equation (38) describes the policy rule as a backward-looking policy rule, in which the nominal interest rate is set according to the previous period's inflation rate, output gap, and policy rate. Moving the equation (38) one period forward, adding expectations, rearranging the terms, and then defining $y_t = [x_t, \pi_t, r_t]'$, the system can be written in the form, $E_t y_{t+1} = C y_t$,

$$E_t y_{t+1} = C y_t + \begin{bmatrix} -\dfrac{1-\alpha+\alpha\omega}{\sigma}\overline{r}_t \\ 0 \\ E_t \overline{r}_{t+1} \end{bmatrix}, \qquad (39)$$

where $C = \begin{bmatrix} \mu\left(\dfrac{1}{\beta} + \varphi\dfrac{1-\alpha+\alpha\omega}{\beta\sigma}\right)+1 & -\dfrac{1-\alpha+\alpha\omega}{\beta\sigma} & \dfrac{1-\alpha+\alpha\omega}{\sigma} \\ -\mu\left(1+\varphi\dfrac{1-\alpha+\alpha\omega}{\sigma}\right) & \dfrac{1}{\beta} & 0 \\ \phi_x & \phi_\pi & \phi_r \end{bmatrix}$.

Following Farebrother (1973) and Gandolfo (1996), a third-order dynamical system whose characteristic polynomial is

$$\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0,$$

where $a_i \in \mathbb{R}$ for all $i = 1,2,3$, is stable if and only if

$$1 + a_2 + a_1 + a_0 > 0,$$
$$1 - a_2 + a_1 - a_0 > 0,$$
$$1 - a_1 + a_2 a_0 - a_0^2 > 0.$$

Using the baseline values of the parameters given in Gali and Monacelli (2005), we numerically find the system (39) to be stable, if the third condition satisfies $\phi_r > 2.7795$. Otherwise, the system (39) is not stable.

3.10 Under Hybrid Taylor Rule:

In this specification, the current rate of inflation in the standard Taylor rule is replaced by next period's forecasted rate of inflation. Barnett and Duzhak (2008, 2010) and Barnett and Eryilmaz (2012) examine this rule in their bifurcation analysis of New Keynesian models. Clarida, Gali, and Gertler (2000) employ this version of the policy rule to analyze the pre-Volcker and Volcker-Greenspan era. Thurston (2010), however, argues that this modification has only minor effects through an additional condition on uniqueness. Bofinger and Mayer (2006), on the other hand, argue that the hybrid Taylor rule lacks the simplicity of simple policy rules and should be rejected for practical reasons.

Consider the following model, in which equations (4) and (5) describe the economy, while equation (40) represents the monetary policy rule:

$$r_t = \overline{r}_t + \phi_\pi E_t \pi_{t+1} + \phi_x x_t. \tag{40}$$

Equation (40) describes the policy rule such that the nominal interest rate is set according to expected inflation rate and current output gap. Substituting (40) for $r_t - \overline{r}_t$ into (5), we obtain a reduced system of first order difference equations in terms of inflation and output gap. That reduced system can be written in normal form, $E_t y_{t+1} = C y_t$, as follows:

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = C \begin{bmatrix} x_t \\ \pi_t \end{bmatrix}, \tag{41}$$

where

$$C=\begin{bmatrix} \dfrac{\beta\phi_x+\mu\left(\dfrac{\sigma}{1+\alpha(\omega-1)}+\varphi\right)(1-\phi_\pi)}{\dfrac{\beta\sigma}{1+\alpha(\omega-1)}}+1 & \dfrac{(\phi_\pi-1)(1+\alpha(\omega-1))}{\beta\sigma} \\ -\dfrac{\mu}{\beta}\left(\dfrac{\sigma}{1+\alpha(\omega-1)}+\varphi\right) & \dfrac{1}{\beta} \end{bmatrix}.$$

Clearly, $x_t = \pi_t = 0$ for all *t* constitutes an equilibrium solution to the system (41).

Following Bullard and Mitra (2002), Proposition 9 characterizes the necessary and sufficient conditions for determinacy.

Proposition 9: Under the Hybrid Taylor Rule, as specified in (40), the open economy New Keynesian model described by the system (41) has a unique stationary equilibrium, if and only if

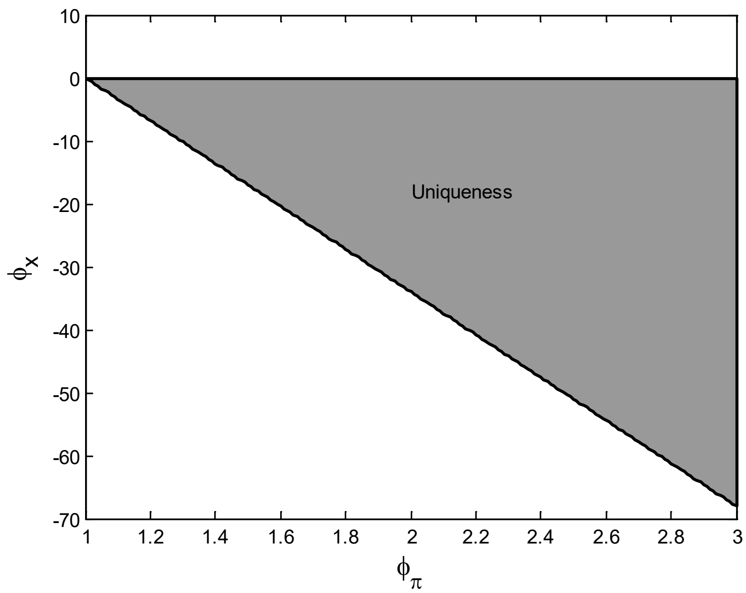
$$\phi_x > \frac{\sigma(\beta-1)}{1+\alpha(\omega-1)} \tag{42}$$

and

$$(1-\beta)\phi_x+\mu\left(\frac{\sigma}{1+\alpha(\omega-1)}+\varphi\right)(\phi_\pi-1)>0\,. \tag{43}$$

The policy maker can reach the uniquely determined stationary equilibrium by choosing feasible values for the policy parameters. Since $\beta\in(0,1)$, the condition (42) can be easily satisfied for positive values of the parameter ϕ_x. Hence, condition (43) is the critical one regarding determinacy. Any value of the inflation parameter greater than unity, that is $\phi_\pi > 1$, accompanied by a non-negative output parameter, ϕ_x, would be sufficient to satisfy condition (43). Nevertheless, as Thurston (2010) points out, a negative ϕ_x may sometimes be consistent with uniqueness and optimality, even though a negative value is not necessary for that purpose. Figure (9), constructed based on the condition (43), shows the regions of unique and multiple equilibria.

Figure 9: Determinacy diagram for the system (41) with Hybrid Taylor rule



Since condition (43) is the critical one regarding determinacy, both eigenvalues will be outside the unit circle, if and only if $(1-\beta)\phi_x + (\phi_\pi - 1)\left(\frac{\sigma}{1+\alpha(\omega-1)} + \varphi\right)\mu > 0$. Since $\phi_x, \phi_\pi > 0$ by assumption, it follows that $\phi_\pi > 1$ would be sufficient for condition (43) to hold. When inflation increases, if the central bank raises the nominal interest rate more than one-for-one, the real interest rate also increases. That would be sufficient to achieve a uniquely determined stationary equilibrium.

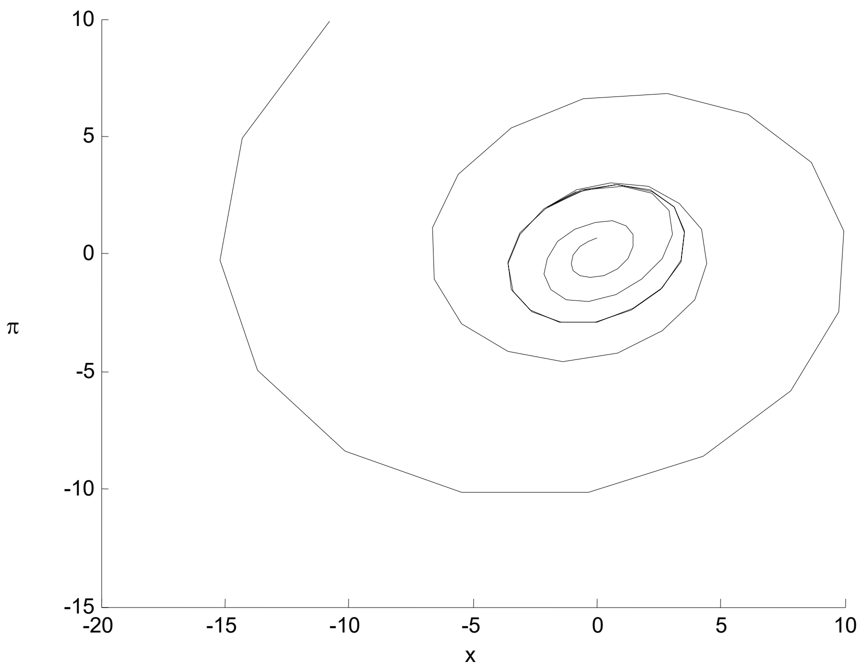
Given the benchmark values of the parameters*,* we numerically obtain the Jacobian matrix as

$$C = \begin{bmatrix} 0.9533 & 0.5051 \\ -0.3434 & 1.0101 \end{bmatrix},$$

having eigenvalues $\lambda_1 = 0.9817 + 0.4155\text{i}$ and $\lambda_2 = 0.9817 - 0.4155\text{i}$, and with modulus $R = \sqrt{(0.9817)^2 + (0.4155)^2} = 1.0660$. The Jacobian matrix *C* has complex conjugate eigenvalues with a radius greater than unity, implying that the system (41) has a unique, stationary equilibrium.

Figure (10) illustrates different solution paths with different stability properties indicating a Hopf bifurcation, consistent with Barnett and Eryilmaz (2012). The inner spiral trajectory is converging to the equilibrium point, whereas the outer spiral is diverging. The limit cycle, thus, is unstable.

Figure 10: Phase diagram showing a Hopf bifurcation under the hybrid Taylor rule.[6]



3.11 Under Hybrid Monetary Policy Rule and Interest Rate Smoothing:

Based on empirical studies, there is a general consensus that the monetary policy rule taking the lagged nominal interest rate into account performs well in estimating the actual policy rule employed by the central bank. The coefficient of the lagged nominal interest rate is found statistically significant and large, when that rate is regressed on inflation and output gap. This result suggests that the monetary policy authority adjusts the policy rate gradually to changes in output gap and inflation rate. On the other hand, Taylor (1999) maintains that policy rules with a lagged interest rate work poorly in models without rational expectations. For a discussion of the significance of lagged interest rate in the estimation of the monetary policy rules, see e.g., English, Nelson, and Sack (2003).

Consider the model in which equations (4) and (5) describe the economy, while equation (44) represents the monetary policy rule:

$$r_t = \overline{r}_t + \left(1-\phi_r\right)\left(\phi_\pi \pi_{t+1} + \phi_x x_t\right) + \phi_r r_{t-1}. \tag{44}$$

[6] Limit cycle for $\phi_\pi = 1.5$ and $\phi_x = -0.01$, inner spiral for $\phi_\pi = 1.5$ and $\phi_x = -0.1$, outer spiral for $\phi_\pi = 1.5$ and $\phi_x = 0.1$.

Equation (3.4.10) describes the policy rule as a hybrid version of the Taylor rule, in which the nominal interest rate is set according to the expected inflation rate, the current output gap, and the previous period's nominal interest rate. Rearranging the terms and defining $y_t = [x_t, \pi_t, r_t]'$, we can write the system in normal form, $E_t y_{t+1} = C y_t$,

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ E_t r_{t+1} \end{bmatrix} = C \begin{bmatrix} x_t \\ \pi_t \\ r_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \overline{r}_t, \tag{45}$$

where $C = \begin{bmatrix} \frac{\mu}{\beta}\left(1+\varphi\frac{1-\alpha+\alpha\omega}{\sigma}\right)+1 & -\frac{1-\alpha+\alpha\omega}{\beta\sigma} & \frac{1-\alpha+\alpha\omega}{\sigma} \\ -\left(\frac{\sigma}{1+\alpha(\omega-1)}+\varphi\right)\frac{\mu}{\beta} & \frac{1}{\beta} & 0 \\ (1-\phi_r)(\phi_x-\phi_\pi)\left(\frac{\sigma}{1+\alpha(\omega-1)}+\varphi\right)\frac{\mu}{\beta} & \frac{(1-\phi_r)\phi_\pi}{\beta} & \phi_r \end{bmatrix}$.

Following Farebrother (1973) and Gandolfo (1996), a third-order dynamical system whose characteristic polynomial is

$$\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0,$$

where $a_i \in \mathbb{R}$ for all $i = 1,2,3$, is stable, if and only if

$$1 + a_2 + a_1 + a_0 > 0,$$
$$1 - a_2 + a_1 - a_0 > 0,$$
$$1 - a_1 + a_2 a_0 - a_0^2 > 0.$$

Using the benchmark values of the parameters given in Gali and Monacelli (2005) and taking $\phi_r = 0.5$, we find that the third condition is not satisfied. On the other hand, if $\phi_r = 1$, the first condition fails. Thus, for the given values of the parameters, the system (45) is found unstable.

4. Conclusion

We analyze the determinacy conditions under various monetary policy rules in the open economy New Keynesian structure proposed by Gali and Monacelli (2005), widely used as a respected baseline model for analysis of policy in an open economy framework. We find that the open economy framework has no impact on determinacy under the current-looking

Taylor rule. Following an active monetary policy is sufficient for equilibrium determinacy. Under the pure current-looking inflation targeting rule, a unique equilibrium for a feasible set of parameter values is possible, but the determinacy region shrinks as the degree of openness increases. We also consider the effects of a credibility gap, showing the extent to which agents discount the central bank's decisions. We find that as the credibility gap rises the determinacy region shrinks, leaving less room for the existence of a unique solution. Therefore, in the case of a credibility gap, the monetary authority has to compensate for the lack of credibility by pursuing a more aggressive policy.

Under current-looking Taylor rule with interest rate smoothing, on the other hand, there is no unique solution to the system. Under forward-looking Taylor rule, the range of determinacy is smaller in the open economy framework. In this case, the indeterminacy is more likely to happen in the open economy framework and becomes a serious issue as the degree of openness increases. The short-term interest rate should be more responsive to changes in expected inflation rate and should be accompanied by a small but positive response to the expected output gap. Under pure forward-looking inflation targeting without an output gap response, the system does not guarantee the uniqueness of the equilibrium solution path. Under backward-looking Taylor rule, a sufficiently active policy rule accompanied by a small response to the output gap is sufficient to lead to a unique equilibrium and determinacy. Under pure backward-looking inflation targeting without response to output, a sufficiently active policy rule can itself leads to a determinate equilibrium. However, with interest rate smoothing, the system is found to be stable, only if the parameter of lagged nominal interest rate exceeds a certain value, which may not be the case.

Lastly, under hybrid Taylor rule, with the current rate of inflation in the standard Taylor rule replaced by next period's forecasted rate of inflation, any value of the inflation parameter greater than unity accompanied by a non-negative output parameter would be sufficient to achieve determinacy. But, when adding the lagged nominal interest rate into the model, the system is found be unstable within the plausible range of the parameters.

However, our analysis is restricted to cases closely resembling the Gali and Monacelli (2005) New Keynesian open economy model. We cannot be certain of the degree to which our conclusions apply to a more general environment, although our results are suggestive of concerns that should be taken seriously in policy design, since the Gali and Monacelli (2005) model is widely respected and used as a baseline in analysis of policy in an open economy framework.

Despite the importance of determinacy in policy analysis, some authors argue that determinacy is neither necessary nor sufficient for an equilibrium solution to be considered plausible for policy analysis. Evans and Honkapohja (2003b), Bullard and Mitra (2002), and McCallum (2009a), among others, argue that even though determinacy of an equilibrium is desirable, the crucial criterion for selecting the most plausible solution is learnability. Evans and Honkapohja (2003b) studied optimal policy rules in terms of E-stability and argue that policy maker should take into account private expectations, in order to prevent magnified deviation from rational expectation equilibrium caused by small expectational errors by private agents with adaptive learning. They note that while determinacy of the solution implies E-stability, the converse does not necessarily hold in all cases. Although more than one stable equilibrium might exist, only one of them may be learnable. That is the solution which uniquely should be considered for policy analysis.

Furthermore, determinacy analysis is unable to rule out explosive solutions involving nominal variables, as transversality conditions can do for real variables. Providing that the Taylor Principle is satisfied, determinacy and E-stability imply each other. Then the differences between their policy implications disappear, as observed by Bullard and Mitra (2002), who argue that determinacy criteria should be replaced by learnability criteria in policy consideration of rational equilibrium solutions. Thus, it would be a useful topic for future research to explore our results in terms of learnability criteria as well. Another relevant topic for future research could be the possibility of Shilnikov chaos in the Gali and Monacelli (2005) model. Using a different New Keynesian model, Barnett at al. (2021) found that similar policy challenges can be produced without nonuniqueness by Shilnikov chaos.[7] Of particular interest is their finding of unintentional downward drift of interest rates within the Shilnikov fractal attractor set, closely resembling the interest rate drift in the U. S. during the past 3 decades.

[7] While the Shilnikov chaos solution paths are unique, they are not stable. With determinacy often defined to require both uniqueness and stability, the solutions in Barnett et al. (2021) are not determinate in that most general sense. In addition, the chaotic solution paths are sensitive to initial conditions.

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