Constructing Divisia Monetary Aggregates for Singapore

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June 20, 2021

This paper is the original working paper version of an article invited for a Topic Collection, "Feature Papers of JRFM," with Michael McAleer serving as Collection Editor. The collection is to be published by the MDPI *Journal of Risk and Financial Management*.

Abstract

Since Barnett (1978) derived the user cost price of money, the economic theory of monetary services aggregation has been developed and extended into a field of its own with solid foundations in microeconomic theory. Divisia monetary aggregates have repeatedly been shown to be strictly preferred to their simple-sum counterparts, which have no competent foundations in microeconomic aggregation or index number theory. However, most central banks in the world, including that of Singapore, the Monetary Authority of Singapore (MAS), still report their monetary aggregates as simple summations. Recent DSGE macroeconomic models often ignore aggregate quantities of money as possible instruments or targets of monetary policy. In the case of a small open economy like Singapore's, exchange rates are often targeted to achieve goals for inflation and output gap. See, e.g., McCallum (2006). Is that because quantities of money are irrelevant to economic activity? To examine the relevance of Divisia monetary aggregates in predicting real economy activity in Singapore, we construct monetary services indices for Singapore using the recent credit-card-augmented Divisia monetary aggregates formula. We produce those state-of-the-art monetary services indexes for Jan 1991 to Mar 2021. In future work, we plan to use our data to explore central bank policy in Singapore and to propose improvements in that policy. By making our data available to the public, we encourage others to do the same.

Keywords: Divisia index, Divisia monetary aggregates, credit card augmented Divisia, open-economy macroeconomics, monetary policy analysis, Singapore.

JEL Classification: E32, E40, E41, E47, E50, E51, E52, E58

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1. Introduction

Since Irving Fisher (1922) published his classic book, *The Making of Index Numbers*, statistical indexes have been extensively applied in economic measurement. For instance, to measure real GDP, no one would today add apples and automobiles, since one apple is not a perfect substitute for one automobile. For the same reason, we cannot impute the same weight to percentage changes in the price of automobiles as to the percentage changes in the price of apples, when measuring inflation. Although widely used in economic measurement since the appearance of Fisher's book, statistical index theory has not been applied in financial and monetary aggregation until recent decades.

Up until the 1980s, economists throughout the world measured different levels of monetary aggregation, such as M0/MB (monetary base), M1 (narrow money), M2 (broad money), and M3 and M4 (financial liquidity), by simply adding up the quantities of component assets. Simple summation assigns the same weights to different monetary assets and thereby implicitly assumes that all monetary assets are perfect substitutes. In modern economies, in which monetary assets possess different levels of liquidity and yield different interest rates, simple sum measures are misleading and can damage inferences about economic behavior and the economy. Chrystal and MacDonald (1994) coined the now well-known term "Barnett critique" to designate the resulting distortions of economic inferences.

To properly aggregate components in monetary service aggregation, we need both their quantities and prices. But how to measure monetary service prices was not known to economists until the 1980s. Monetary asset services are not analogous to perishable consumer good services, such as apples, but to capital goods or durable goods, such as houses or automobiles. Hence, we need to measure their service prices in term of their user cost prices.

The concept of user cost pricing of durable services was first introduced by Jorgenson (1963). He introduced user cost theory applicable to durable and capital goods for which perfect rental markets do not exist. When perfect rental markets exist for a good, the user cost price equals the market rental price. When a perfect rental market does not exist for a durable, the theoretically computed user cost price is sometimes called the "equivalent rental price" or shadow rental price.

The theory of monetary aggregation was originated by Barnett in the 1980s, following his derivation of the user cost price of monetary services in Barnett (1978, 1980). Using the resulting user cost pricing, Barnett (1980, 1981, 1987) applied existing index number and aggregation theory to construct the Divisia index for monetary service aggregation. The famous Divisia index, originated by Francois Divisia (1925), measures the growth rate of a quantity (or price) aggregate as the weighted average of the growth rates of the quantities (or prices) of the component goods over which the index aggregates. The weights are the component expenditure shares.

Since the economic theory of monetary aggregation became available, the theory has been developed and extended substantially. Barnett, Liu, and Jensen (2000) extended the theory to risk, based upon the consumption capital assets pricing model (CCAPM). That result extended Barnett's perfect certainty theory to the case of risk, when consumers of monetary services are risk-averse and interest rates are not known at the beginning of the period. Barnett (2007) extended the theory to multilateral monetary aggregation over different countries. More recently, Barnett, Chauvet, Leon, and Su (2016) and Barnett and Su (2016, 2017, 2018) have taken credit-card transactions into account and produced the theoretical framework for the new credit-card-augmented Divisia monetary aggregates. Other extensions have included measurement of the economic capital stock of money, based on the expected discounted flow of monetary services, and extension of the risk adjustment to the case of intertemporal non-separability.

Hundreds of empirical papers from throughout the world have compared Divisia monetary aggregates with their simple sum counterparts.[1] Since simple sum monetary aggregation is theoretically inadmissible, having no competent theoretical foundations, it should be no surprise that in almost all cases, the Divisia monetary index has proven to be strictly preferable to its simple sum counterpart, relative to all available empirical tests. See, for example, Barnett, Offenbacher, and Spindt (1984), Barnett (2011), Belongia and Ireland (2014), and Ellington (2018). Belongia and Ireland (2015) are critical of the omission of monetary quantities in recent mainstream macroeconomic models. These omissions are largely a result of empirical findings in such papers as Bernanke and Blinder (1988), who argue that the demand for money function has become unstable; but all such findings use simple sum monetary aggregates. Recent DSGE models often

[1] Key articles, books and works on the topic can be found at the library of Center for Financial Stablity (CFS) http://www.centerforfinancialstability.org/amfm_library.php

include an interest rate feedback rule as a basis for monetary policy, while totally ignoring monetary services in the economy.[2] Replacing the traditional simply sum measure of money supply by Divisia measures, Belongia and Ireland (2015), among many other researchers, have shown that money still shares a strong relationship with aggregate economic activity, and the demand for money function still exhibits stability. This simple solution has been found to be true in hundreds of publications throughout the world, since Divisia monetary aggregates became available.

Faced with all this theoretical and empirical evidence, central banks such, as the Federal Reserve (FED) in the US, the Bank of England (BOE) in the UK, the European Central Bank (ECB), the Bank of Japan (BoJ), the National Bank of Poland, and the Bank of Israel, among others, have, at various times and in diverse ways, produced and maintained Divisia indexes for monetary aggregation. Some central banks choose to make it available to the public on an official basis, such as the BOE. Others choose to make those aggregates available only for internal use. However, the availability of the simple sum aggregates has continued. For good reasons, those incompetent simple-sum aggregates are declining in usage by central banks and by the economics profession.

Many other central banks in the world, including the Monetary Authority of Singapore (MAS), continue to report their money supplies solely as simple sum measures. Empirical work using Divisia monetary aggregates in Singapore is limited, with the only related work in the literature being Habibullah (1999). The focus of that paper was not primarily the case of Singapore, but rather the monetary policies in many Asian countries, including Indonesia, Malaysia, Singapore, Philippines, South Korea, Taiwan, and Thailand among others. The data used in that research were mostly before the Asian financial crisis and did not use credit-card-augmented Divisia monetary aggregates, which were not yet known at that time.

Our paper constructs Divisia monetary aggregates for Singapore based on monthly data from Jan 1991 to Mar 2021. We find that the major contributions to the growth rates of Divisia monetary service flows come from demand deposits, fixed deposits, and savings (and other) deposits in commercial banks. Fixed deposits and savings deposits in finance companies provide

[2] The most common one in the literature is the Taylor Rule, based on Taylor (1993).

moderate contributions, while the weights of other components such as negotiable CDs, repurchase agreement, and Treasury bills are negligible.

Although credit card transactions are augmented into our monetary aggregates, their weights are small. Therefore, we find their contributions at this time to the growth rate of Divisia monetary services in Singapore to be minor, although this could change in the future as money market institutional innovations continue. Another finding is that during the period before 2000, when interest rates were high and more volatile, Divisia monetary aggregates behaved significantly differently from the simple sum measures, while during the period after 2000, when interest rates on monetary assets have become close to each other at very low levels, Divisia monetary aggregates have behaved almost identically to the simple sum measures.

In future work, we plan to use the constructed data to examine monetary policy in Singapore, and we would encourage others to do the same. Our planned first direction will be to examine the cyclical correlations and Granger causality relations between different measures of money and real economic variables. We also plan to build a New Keynesian model for a small open economy to be used to examine the potential role of money aggregates as a policy target in Singapore, in comparison with the central bank's current policy rule, targeting a trade-weighted exchange rate index.

2. Methodology

2.1. Conventional Divisia Monetary Aggregates

Barnett (1978) derived the user cost of monetary asset services from an intertemporal consumer utility maximization problem. Let $\mathbf{m}_t^* = (m_{1,t}^*, m_{2,t}^*, ..., m_{n,t}^*)'$ be the solution for period *t*'s monetary assets. Barnett (1980,1981) showed that $\mathbf{m}_t^*$ is also the solution to the current period conditional decision

$$\text{maximize u}(\mathbf{m}_t) \text{ subject to } \pi_t \bullet m_t = y_t,$$

where y_t is total expenditure allocated to the the portfolio of n monetary assets during the intertemporal decision, and $\boldsymbol{\pi}_t = (\pi_{1,t}, \pi_{2,t}, ..., \pi_{n,t})'$ is the vector of user costs of monetary asset services. To assure existence of a current period monetary services aggregate, the category utility function, u, is assumed to be blockwise weakly separable within intertemporal tastes.

The resulting nominal user cost price of each monetary asset is

$$\pi_{i,t} = \frac{p_t^*(R_t - r_{i,t})}{1 + R_t}, \tag{1}$$

where p_t^* is the true cost of living index, used to deflate nominal quantities to real quantities, R_t is the expected one-period holding yield on the benchmark asset, and $r_{i,t}$ is the rate of return on the i-th monetary asset. In theory, the benchmark rate is the rate of return on a pure capital investment held solely for its investment rate of return, and thereby providing no other services. As emphasized in Barnett (1978, 2011), the user-cost price of a monetary asset is not its interest rate but its opportunity cost, consisting of the interest rate forgone by consuming the services of the asset. For example, if the asset is currency, having interest rate of zero, the forgone interest rate is the benchmark rate itself.

The corresponding real user cost price is

$$\frac{\pi_{i,t}}{p_t^*} = \frac{R_t - r_{i,t}}{1 + R_t}. \tag{2}$$

With availability of the user cost prices of monetary assets, both their quantities and their prices are well-defined. The economic theory of aggregation over monetary assets becomes available. Barnett (1987) proved that the exact monetary quantity aggregate, $M_t = M(m_t)$, can be tracked without error in continuous time by the Divisia index, defining the growth rate of aggregate monetary services to be

$$\frac{d \log M_t}{dt} = \sum_{i=1}^{n} s_{i,t} \frac{d \log m_{i,t}^*}{dt}, \tag{3}$$

where

$$s_{i,t} = \frac{\pi_{i,t} m_{i,t}^*}{y_t} = \frac{\pi_{i,t} m_{i,t}^*}{\sum_{j=1}^{n} \pi_{j,t} m_{j,t}^*}. \tag{4}$$

The weight $s_{i,t}$ of monetary asset i is its share in the total expenditure on the portfolio. Since economic data are in discrete time, an approximation is needed. The Tornqvist-Theil approximation (often called the Tornqvist index or just the Divisia in index in discrete time) is a second order approximation to the continuous Divisia index,

$$\log M_t - \log M_{t-1} = \sum_{i=1}^{n} \bar{s}_{i,t} (\log m_{i,t}^* - \log m_{i,t-1}^*), \tag{5}$$

where the discrete-time share weights are approximated by

$$\bar{s}_{i,t} = \frac{1}{2}(s_{i,t} + s_{i,t-1}). \tag{6}$$

In short, the growth rate of a Divisia monetary quantity index is the share weighted average of the growth rates of its components.

Barnett (1987) showed that the discrete time Divisia index is accurate to within three decimal places for monthly or weekly data. As a result, the remainder term in the Tornqvist approximation is less than the roundoff error in the available component data. Equation (4) can equivalently be written as

$$\frac{M_t}{M_{t-1}} = \prod_{i=1}^{n} \left(\frac{m_{i,t}^*}{m_{i,t-1}^*} \right)^{\bar{s}_{i,t}}. \tag{7}$$

The growth rate of the dual Divisia user-cost price aggregate, $\Pi_t = \Pi(\boldsymbol{\pi}_t)$, in continuous time, is derived in a similar manner to be

$$\frac{d\log \Pi_t}{dt}=\sum_{i=1}^{n} s_{i,t}\frac{d\log \pi_{i,t}}{dt}, \tag{8}$$

with the corresponding Tornqvist discrete-time approximation being

$$\log \Pi_t - \log \Pi_{t-1} = \sum_{i=1}^{n} \bar{s}_{i,t}(\log \pi_{i,t} - \log \pi_{i,t-1}). \tag{9}$$

An alternative way to derive the dual user cost price aggregate is from Fisher's factor reversal test, in accordance with which

$$\Pi_t = \frac{\boldsymbol{\pi}_t \bullet \mathbf{m}_t^*}{M_t} = \frac{\sum_{i=1}^{n} \pi_{i,t} m_{i,t}^*}{M_t}. \tag{10}$$

so that $\Pi_t M_t = \boldsymbol{\pi}_t \bullet \mathbf{m}_t^* = \sum_{i=1}^{n} \pi_{i,t} m_{i,t}^*$. See Barnett (1982) for a rigorous discussion.[3]

2.2. Credit-Card-Augmented Divisia Monetary Aggregates

In recent years, credit card payments have become increasingly common in modern economies worldwide. By accounting conventions, liabilities cannot be added to assets. Since credit card balances are liabilities, they cannot be added to monetary assets. Hence credit cards cannot be included in simple sum monetary aggregates. But in economic theory, aggregation over services is possible, regardless of whether the services are produced from assets or liabilities. The deferred payments services of credit card transactions can be augmented into the Divisia monetary service aggregates.

The theoretical framework is provided in Barnett, Chauvet, Leon, and Su (2016) and Barnett and Su (2016, 2017, 2018). Accordingly, the user cost price of a credit card's services is

[3] In continuous time, the two methods of computing the dual user cost aggregate produce identical results. In discrete time, the two methods produce slightly different results, with the difference being less than the roundoff error in the component data and thereby negligible.

$$\tilde{\pi}_{j,t} = \frac{p_t^*(e_{j,t} - R_t)}{1 + R_t}, \tag{11}$$

where $e_{j,t}$ is the interest rate charged by credit card type j, where $j = 1, \ldots k$. The volume of purchases of goods and services during period t with credit card type j is $c_{j,t}$.[4] The consumer's utility maximizing solution for the transaction services of the k credit card types is $\mathbf{c}_{i,t}^* = (c_{1,t}^*, c_{2,t}^*, \ldots, c_{k,t}^*)'$. The growth-rate weight of monetary asset i's services is

$$w_{i,t} = \frac{\pi_{i,t} m_{i,t}^*}{\boldsymbol{\pi}_t \bullet \mathbf{m}_t^* + \tilde{\boldsymbol{\pi}}_t \bullet \mathbf{c}_t^*}, \tag{12}$$

while the growth-rate weight of credit card j's services is

$$\tilde{w}_{j,t} = \frac{\tilde{\pi}_{j,t} c_{j,t}^*}{\boldsymbol{\pi}_t \bullet \mathbf{m}_t^* + \tilde{\boldsymbol{\pi}}_t \bullet \mathbf{c}_t^*}. \tag{13}$$

The credit-card-services-augmented Divisia monetary aggregate becomes

$$\frac{d \log M_t}{dt} = \sum_{i=1}^{n} w_{i,t} \frac{d \log m_{i,t}^*}{dt} + \sum_{j=1}^{k} \tilde{w}_{j,t} \frac{d \log c_{j,t}^*}{dt}. \tag{14}$$

The Tornqvist discrete time approximation is analogous to that for the conventional Divisia index.

3. Data and Construction

To construct Divisia monetary aggregates for money services, we need data on both quantities and interest rates of each monetary asset. This section describes the data we use and the construction results.

3.1. Data Description

[4] The central bank of Singapore calls credit card transaction volumes "total card billings."

Table 1 provides basic description for the data set. Data on levels and rates of return on monetary assets are monthly from Jan 1991 to Mar 2021, provided by the Monetary Authority of Singapore (MAS). The true cost of living index is measured by the consumer price index (CPI), which is from the Singapore department of statistics. For the United States, the Federal Reserve reports interest rates charged on credit card deposits averaged over all credit card users, including those who do not pay interest on their credit card balances, since they do not carry forward unpaid balances. That average interest rate is the one to use in modeling the decisions of the representative consumer, aggregated over all consumers. See Barnett and Su (2017).

But unfortunately, the central bank in Singapore (MAS) does not report those interest rates. However, ValueChampion in Singapore does report that interest rate averaged over time. Experiments with the United States data show negligible differences in the credit-card-augmented monetary aggregates, if that interest rate is averaged over the sample period and then treated as a constant, rather than being used as the actual interest rate each month. As a result, with our Singapore data, we use the interest rate averaged over time, as reported at the surprisingly high level of 25% per year by ValueChampion. Perhaps that high interest rate may partially explain why the share of credit card deferred payment services in Singapore is relatively low.

The central bank of Singapore (MAS) categorizes the primary components of M1 as currency in circulation and demand deposits in banks. The MAS simple sum M2 includes M1 and the banking sector components, fixed deposits (CDs), savings (and other) deposits, and negotiable certificates of deposits (NCDs). Simple sum M3 incorporates the non-banking sector by including net deposits in finance companies. Post Office Saving Bank (POSB) deposits existed in Singapore before Nov 1998. Up to Oct 1998, POSB was included by MAS in its non-banking sector data and included in M3, but not in M2. However, from Nov 1998, with the acquisition of POSB by the Development Bank of Singapore (DBS), POSB's data have been incorporated as part of the banking system in M1 and thereby also in M2 and M3.

Nesting components: In this paper, we follow Barnett *et al* (2013) in clustering and nesting components of monetary assets. Accordingly, our Divisia M1 (DM1) has the same components as its counterpart simple sum aggregate, M1. Our Divisia M2 (DM2) aggregate includes components in DM1 along with fixed deposits (CDs) and savings (and other) deposits, both in the banking sector and the non-banking sector. Our Divisia M3 (DM3) includes the components of DM2 along with NCDs and Repurchase Agreements (Repos). Finally, our Divisia M4 (DM4) incorporates Treasury bills into Divisia M3. Table 2 summarizes the components in the MAS simple sum aggregates and our Divisia aggregates.

Data on levels: The data on levels of components are in current values, and the units are in millions of Singapore dollars. Net deposits in finance companies are included in M2, but the break-down components, fixed deposits and saving (and other) deposits, are not separately reported. We directly investigated the Assets and Liabilities of Finance Companies to get the break-down components. We recovered the fixed deposits and saving and other deposits in finance companies by using the proportion of net deposits in total deposits.

Since there were no available interest rates for deposits in POSB, we assume that these rates are the same as those in the banking sector. The data on the POSB level (up to Oct 1998) are incorporated into the banking sector. We calculate the proportion of fixed deposits and savings (and other) deposits out of total deposits in both the banking sector and the non-banking sector. We then use those proportions to split the net deposits in POSB into fixed deposits and saving (and other) deposits and then add them into the banking-sector.

In the series for the level of NCDs during the period from Aug 2009 to Jun 2010, the quantities reported are zeros, which does not seem credible. We smooth the data on NCDs for the above period by approximating the zero-quantities by the average of the quantities 12 months before and 12 months after that period.

The benchmark rate: In theory, the benchmark rate is the rate of return on pure capital. Its rate of return cannot be less than the rates of return on any monetary assets that provide services to depositors along with investment yield. In this paper, we follow Barnett *et al* (2013) in choosing the short-term lending rate as the benchmark rate. Banks cannot be expected to pay higher rates of interest to their depositors than they earn on their investments. Indeed, in the case of Singapore, the prime lending rate is always higher than the interest rates on the component monetary assets. See Figure 1.

Data on rates of return: Since currency and demand deposit do not yield interest rates, we set their interest rates at zero. Short-term fixed deposits typically include 3-month CDs, 6-month CDs and 12-month CDs. We do have data on their interest rates; however, we do not have the corresponding interest rates on fixed deposits in banks and finance companies. Hence, we use 3-month CD interest rates for banks and finance companies to represent the rates of return on fixed deposits in banks and non-banking institutions. For T-bills, we add together all T-bills and impose the 12-month T-bill yield as their rate of return.

Since the rate of return on NCDs is not reported, we use the rate on 3-month commercial paper as a proxy. The rates of return series for 3-month commercial paper and repurchase agreements were discontinued from Jan 2014. The missing observations are estimated by a regression of each series on the 12-month T-bill yield.

User Cost and Interest Rate Aggregation

Divisia user-cost price aggregates can be computed in a manner similar to the Divisia monetary quantity aggregates, using equation (9) with the weights computed by equations (4) and (6). However, in this paper, given that we already constructed the Divisia quantity index, M_t, the corresponding user cost price aggregate is derived from Fisher's factor reversed test as in equation (10). Credit-card-augmented user-cost price aggregates are also computed accordingly.

For interest rate aggregation, this paper follows Barnett *et al* (2013) using accounting principles in computing the interest rate aggregates. Accordingly, the aggregated interest rate on a portfolio is the rate of return on the portfolio,

$$r_t = \frac{\sum_{i=1}^{n} r_{i,t} m_{i,t}}{\sum_{i=1}^{n} m_{i,t}}. \tag{15}$$

3.2. Data Construction and Results

As explained above, our clustering of components into different levels of aggregation prior to computing the Divisia aggregates is different from that of the simple sum measures reported by the MAS. See Table 2. We computed the growth rates of the Divisia monetary aggregates and their credit-card-augmented variants at the levels of aggregation we have chosen. Corresponding interest rate aggregates and user cost aggregates are also computed.

Divisia M1 (DM1) contains the same two components as its simple sum counterpart, and their corresponding interest rates are equal to zero. Hence the user cost prices for these components are the same. Under those conditions, the Divisia quantity index becomes the simple sum. The growth rate of DM1 and its level (normalized to equal 100 in Jan 1991) are the same as those of M1. However, the credit card augmented DM1A behaves somewhat differently. See Figure 3.

DM2, DM3, and DM4 have almost identical growth rates but are substantially different from the growth rate of DM1. See Figures 2 and 3. To explain this difference, see Tables 3 and 4 for the growth-rate weights of the components in the Divisia monetary aggregates. While DM1 contains only two components, currency and demand deposits, their weights for the latest months in our sample (Mar 2021) are 20.66% and 79.34% respectively. However, DM2, DM3, and DM4 assign different weights to those two components, 7.62% and 29.28% respectively. The major components that contribute to the growth rates of DM2, DM3, and DM4 are three components of the banking sector: demand deposits, fixed deposits in commercial banks, and saving (and other) deposits in commercial banks. These three components account for 60% of the fluctuation in the growth rates of DM2, DM3, and DM4.

Finance companies provide a moderate contribution to the growth rates of those Divisia monetary aggregates. Although DM3 and DM4 incorporate additional components into DM2, namely NCDs, repos, and T-bills, their weights are almost negligible. The credit-card-augmented Divisia aggregates behave very similarly to the conventional Divisia monetary aggregates, since the volume of credit card transactions in Singapore is currently relatively small compared to other sources of monetary services. Hence, the growth-rate weight of credit card transaction volumes is currently small, about 9.5% at the M1 level of aggregation and 3.2% at broader levels of aggregation. See Table 5. However, the role of credit card deferred payment services may grow in the future, as financial services innovations continue to evolve.

For a comparison of the Divisia and simple sum aggregates, see Figures 2 and 4. Simple sum M2 experiences a sudden peak in 1999, while simple sum M3 does not, because of the acquisition of POSB into the banking sector. This is not an expansion of the money supply, but rather a structural change in nesting of components into different levels of aggregation. The Divisia monetary aggregates do not experience this sudden misleading spike. After 2000, the growth rates of the Divisia monetary aggregates and the simple sum versions are close to each other, but not before 2000. The reason is the behavior of interest rates in Singapore. See Figures 1 and 6. Before 2000, interest rates are higher and more volatile compared to the later period. After 2000, interest rates for fixed deposits and saving deposits in both commercial banks and finance companies are at very low levels, almost zero, and thereby very similar to each other. As a result, the user cost prices for those assets are almost identical to each other, so that the Divisia indices are close to their simple sum counterparts during that time period.

The period before 2000 is particularly interesting. There is a huge contraction of money supply during the Asian financial crisis, 1997-1998, as displayed very clearly in the DM3 growth rates, but simple sum M3 does not show it. See Figure 4.

Interest rate aggregates and real user cost price aggregates are plotted in Figures 6 and 7. Different levels of interest rate aggregation produce almost identical results and follow the common trend of interest rates in the world, with interest rates becoming very low after 2000.

4. Conclusion

Although aggregation theory and index number theory have been extensively applied in economic measurement for more than a century, monetary aggregation theory has appeared and been applied more recently. Large numbers of theoretical studies as well as empirical studies have repeatedly shown that Divisia monetary aggregates are superior to their simple sum counterparts, which have no competent foundations in economic theory. Nevertheless, many central banks in the world, including the Monetary Authority of Singapore, continue reporting money supply as a simple sum. This paper provides the construction of Divisia monetary aggregates for Singapore and thereby will serves as the first step for research on the role of monetary services in the important Singapore economy.

We encourage others to use our Divisia Singapore data in their studies. We ourselves plan to use the data to examine cyclical correlations and Granger causality relation between different measures of money and real economic variables. Furthermore, we hope to build a New Keynesian model for a small open economy with a banking sector to examine the role of money aggregates as a possible policy target in Singapore, as opposed to the current trade-weighted exchange rate target.

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Appendix

A.1. Tables

Table 1 Monetary Asset Components

m	Asset	Units of measurement	Rates of Return Used
1	Currency	million $S	0%
2	Demand Deposits	million $S	0%
3	Fixed Deposits in Commercial Banks	million $S	3-month CDs in banks
4	Negotiable CDs in Commercial Banks	million $S	3-month Commercial Bills
5	Saving and Other Deposits in Commercial Banks	million $S	saving rate in banks
6	Fixed Deposits in Finance Companies	million $S	3-month CDs in finance companies
7	Saving and Other Deposits in Finance Companies	million $S	saving rate in finance companies
8	Deposits in Post Office Saving Bank*	million $S	NA
9	Overnight and Term Repurchases	million $S	Repos rate
10	Treasury Bills (all T-Bills and SGSs)	million $S	12-month T-bill yield
11	Credit Card Transaction Volumes**	million $S	Average Singapore credit card interest rate

Data source: Monetary Authority of Singapore (MAS). The monthly dataset covers the period from January 1991 to March 2021.
*Post Office Saving Bank deposits were acquired by the Development Bank of Singapore from Nov 1998.
**In Singapore, those volumes are called "Total Credit Card Billings."

Table 2 Nesting Components in Monetary Aggregates

m	Asset	M1	M2	M3	DM1	DM2	DM3	DM4
1	Currency	1	1	1	1	1	1	1
2	Demand Deposits	1	1	1	1	1	1	1
3	Fixed Deposits in Commercial Banks	0	1	1	0	1	1	1
4	Negotiable CDs in Commercial Banks	0	1	1	0	0	1	1
5	Saving and Other Deposits in Commercial Banks	0	1	1	0	1	1	1
6	Fixed Deposits in Finance Companies	0	0	1	0	1	1	1
7	Saving and Other Deposits in Finance Companies	0	0	1	0	1	1	1
8	Deposits in Post Office Saving Bank*	0	0	1	0	1	1	1
9	Overnight and Term Repurchases	0	0	0	0	0	1	1
10	Treasury Bills (all T-Bills and SGSs)	0	0	0	0	0	0	1

*Since the details on interest rates for POSB are not available, we split the quantities into fixed deposits and saving deposits and incorporated them into banking sector.

Table 3 Growth-rate Weights in the Last Month (Mar 2021), Percentages

m	Asset	DM1	DM2	DM3	DM4
1	Currency	20.6578	7.6221	7.5551	7.5263
2	Demand Deposits	79.3422	29.2808	29.0237	28.9133
3	Fixed Deposits in Commercial Banks	0	26.2904	26.0594	25.9601
4	Negotiable CDs in Commercial Banks	0	0	0.0183	0.0182
5	Saving and Other Deposits in Commercial Banks	0	35.0375	34.7298	34.5976
6	Fixed Deposits in Finance Companies	0	1.7080	1.6930	1.6865
7	Saving and Other Deposits in Finance Companies	0	0.0612	0.0607	0.0605
9	Overnight and Term Repurchases	0	0	0.8601	0.8568
10	Treasury Bills (all T-Bills)	0	0	0	0.3806
		100	100	100	100

Table 4 Growth-rate Weights in the Last 12 Months (Apr 2020 - Mar 2021), Percentages

m	Asset	DM1	DM2	DM3	DM4
1	Currency	21.3403	7.5333	7.4647	7.4345
2	Demand Deposits	78.6597	27.8111	27.5577	27.4472
3	Fixed Deposits in Commercial Banks	0	29.0457	28.7815	28.6632
4	Negotiable CDs in Commercial Banks	0	0	0.0146	0.0145
5	Saving and Other Deposits in Commercial Banks	0	33.7368	33.4296	33.2948
6	Fixed Deposits in Finance Companies	0	1.8234	1.8068	1.7994
7	Saving and Other Deposits in Finance Companies	0	0.0496	0.0492	0.0490
9	Overnight and Term Repurchases	0	0	0.8960	0.8924
10	Treasury Bills (all T-Bills)	0	0	0	0.4051
		100	100	100	100

Table 5 Credit Card Augmented Growth-rate Weights (Jun 2019), Percentages

m	Asset	DM1a	DM2a	DM3a	DM4a
1	Currency	21.9790	7.3206	7.2777	7.2606
2	Demand Deposits	68.5413	22.8294	22.6955	22.6422
3	Fixed Deposits in Commercial Banks	0	33.3545	33.1589	33.0810
4	Negotiable CDs in Commercial Banks	0	0	0.0045	0.0045
5	Saving and Other Deposits in Commercial Banks	0	31.3550	31.1711	31.0979
6	Fixed Deposits in Finance Companies	0	1.9372	1.9258	1.9213
7	Saving and Other Deposits in Finance Companies	0	0.0459	0.0456	0.0455
9	Overnight and Term Repurchases	0	0	0.5820	0.5806
10	Treasury Bills (all T-Bills)	0	0	0	0.2348
11	Credit Card Transaction Volumes*	9.4798	3.1574	3.1389	3.1315
		100	100	0	100

* In Singapore, those volumes are called “Total Credit Card Billings.”

A.2. Figures

Figure 1. Benchmark Rate versus Other Interest Rates

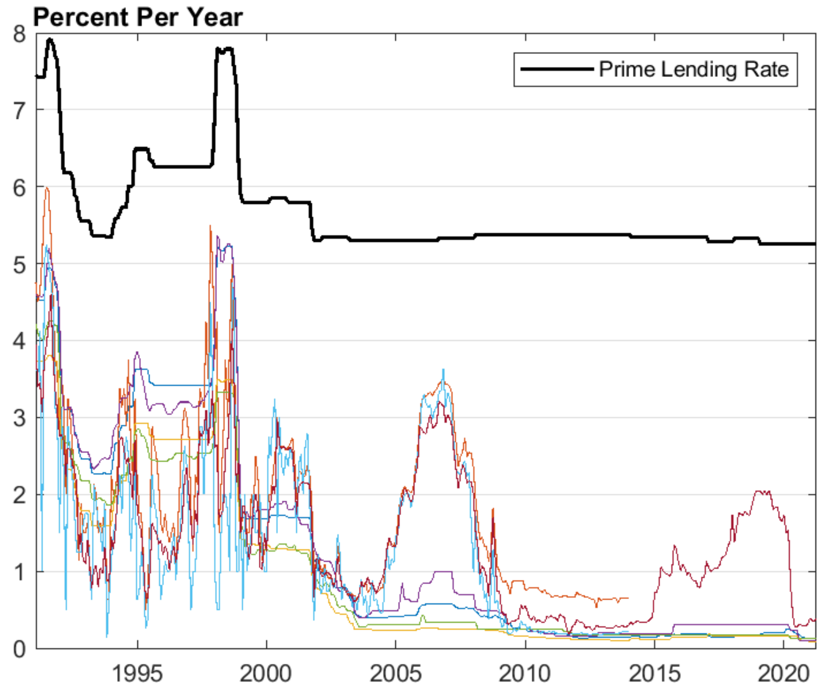

Figure 2. Divisia versus Simple Sum Aggregates, 1991-2021

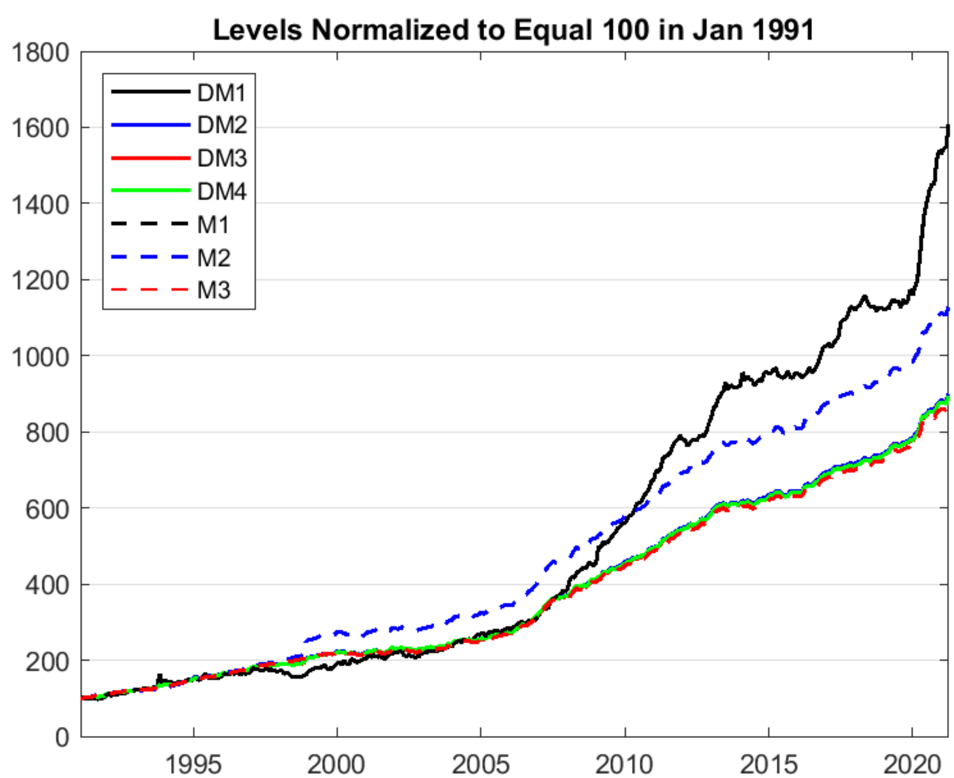

Figure 3. Divisia and Credit Card Augmented Divisia Monetary Aggregates, 1991-2021

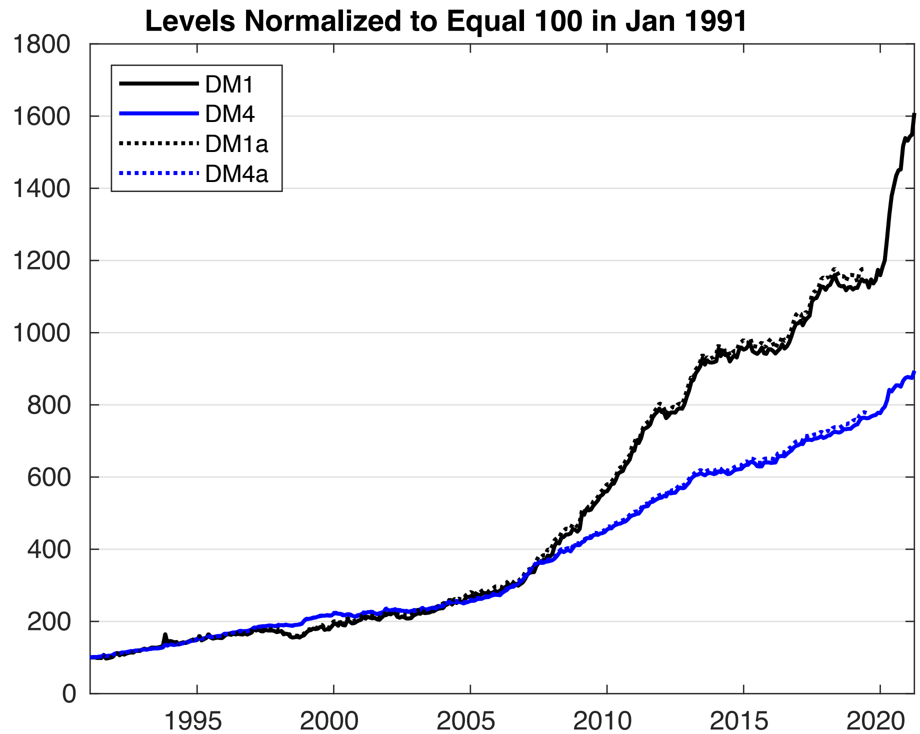

Figure 4. Divisia M3 versus Simple Sum M3 Growth Rates 1992-2021

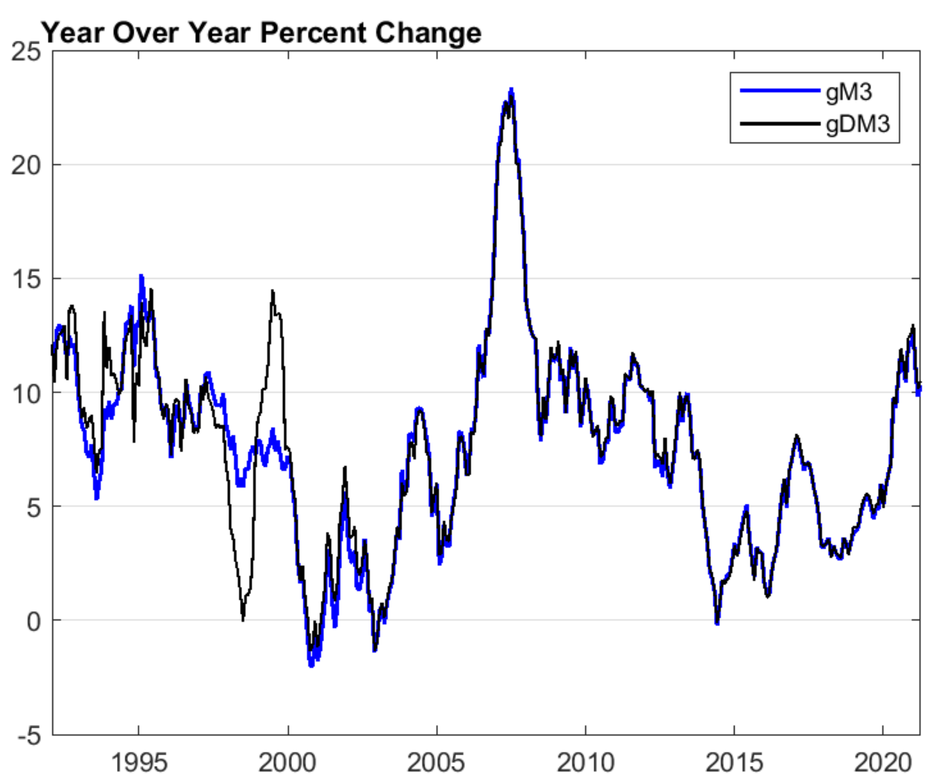

Figure 5. Divisia M1 and M4 and Augmented Divisia M1 and M4 Growth Rates, 1992-2021

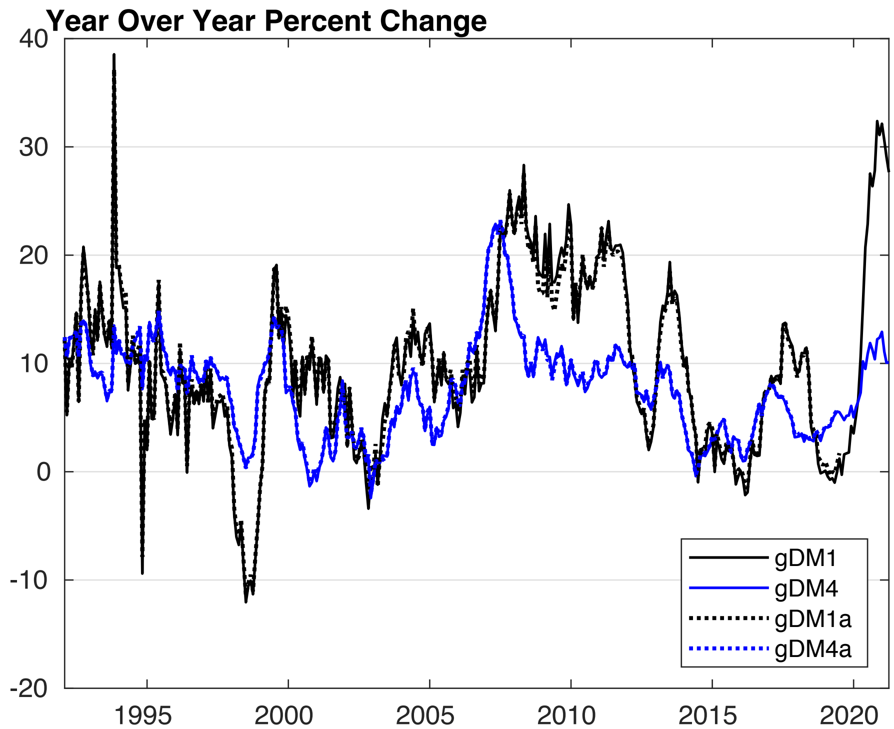

Figure 6. Interest Rate Aggregates 1991-2021

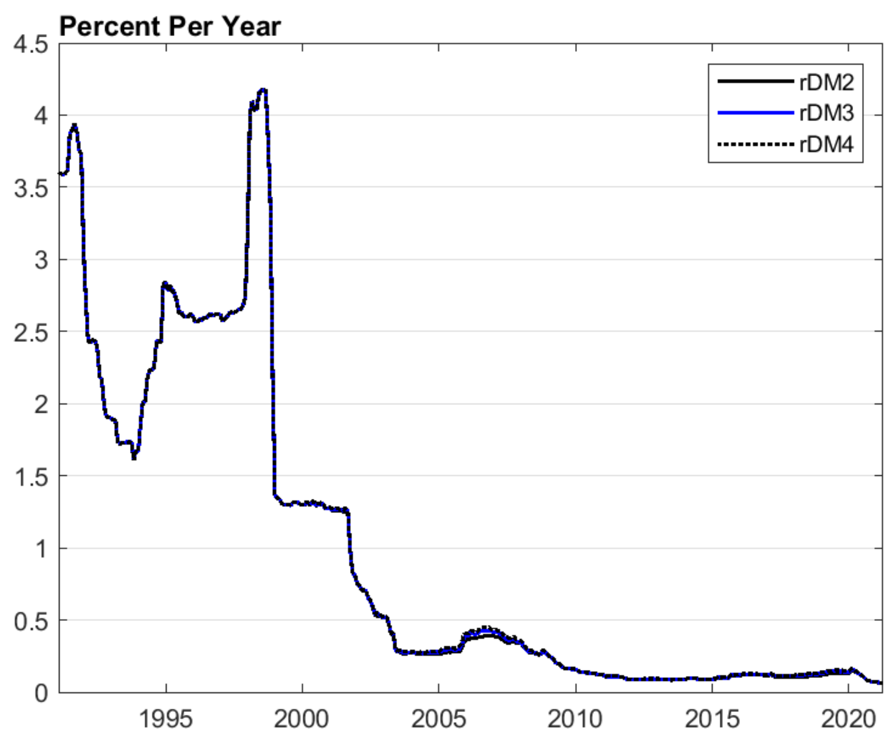

Figure 7. Real User Cost Aggregates 1991-2021