Solving the Price Puzzle Via A Functional Coefficient Factor-Augmented VAR Model

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Abstract: Effects of monetary policy shocks on large amounts of macroeconomic variables are identified by a new class of functional-coefficient factor-augmented vector autoregressive (FAVAR) models, which allows coefficients of classical FAVAR models to vary with some variable. In the empirical study, we analyze the impulse response functions estimated by the newly proposed model and compare our results with those from classical FAVAR models. Our empirical finding is that our new model has an ability to eliminate the well-known price puzzle without adding new variables into the dataset.

Keywords: Factor-augmented vector autoregressive; Functional coefficient models; Impulse response functions; Nonparametric estimation; Price puzzle

JEL Classification: C14, C32, E30, E31.

1 Introduction

Linear vector autoregression (VAR) models and their extensions such as vector autoregressive moving average (VARMA) models and VARs with exogenous variables (VARX) were well developed in last century for studying the effects of monetary policy shocks on macroeconomic variables and modeling the dynamic interdependences among them. These models mainly arise as powerful tool-kits for macroeconomists but only impose minimal restrictions on the identification of large-scale macroeconomic models (Sims, 1980). However, growing numbers of literature in both theoretical and empirical fields documented nonlinear features of the data that were frequently used to feed linear VAR models; see, for instance, Stock and Watson (1996), Tsay (1998), Hansen (2001), Sims and Zha (2006), and references therein. To introduce more flexibilities in linear VAR models, various nonlinear VAR models have been proposed. For example, the early and recent works include the nonlinear VAR models with heteroskedasticity as in Härdle, Tsybakov and Yang (1998), the threshold and regime switches VAR models in Tsay (1998) and Sims and Zha (2006), respectively, the VAR models with randomly evolved parameters as in Primiceri (2005), Giraitis, Kapetanios and Yates (2014), and Giraitis, Kapetanios and Yates (2018), and among others.

Despite the popularity, both linear and nonlinear VAR models suffer from criticisms that center around the relatively small information set used by low-dimensional VARs. As pointed out by Bernanke, Boivin and Eliasz (2005), "To conserve degrees of freedom, standard VARs rarely employ more than six to eight variables". Indeed, if the dimensions of vector of macroeconomic variables in VAR models are too small, large amount of information used by actual central bank may not be considered in the econometric models. Consequently, the policy innovations are likely to be measured inaccurately and the results of forecasting or impulse response analysis may be severely contaminated. To incorporate more variables in the VAR systems, recent studies explored statistical theories of VAR models under high-dimensional settings. For example, Hsu, Hung and Chang (2008) proposed a subset selection method for VAR models based on the least absolute shrinkage and selection operator (LASSO) penalization. In addition, Guo, Wang and Yao (2016) reduced the dimension of the transition matrices in VAR models by imposing banded structure and established the convergence rates of the least squares estimators. Finally, Zhu, Pan, Li, Liu and Wang (2017) embedded network structure into the VAR models to analyze large-scale social network data.

Although aforementioned methods work fairly well, they assume that the variables entered in econometric models are observable. Nevertheless, Bernanke et al. (2005) claimed that the assumption that both the central bank and the econometrician observe all the elements for estimating VAR model is too strong. As an alternative, Stock and Watson (2002) seminally introduced the method of factor-augmented forecasts (also known as "diffusion index forecasts") in the VAR literature to exploit the information in a large set of macroeconomic variables. After this work, factor-augmented methods are being used by an increasing number of researchers and begin to fuse with linear VAR models and their variants. Pioneering contributions include the factor-augmented vector autoregressions (FAVAR) proposed in Bernanke et al. (2005) and the asymptotic theory for the estimated parameters of the factor-augmented regressions in Bai and Ng (2006). In the further extensions, Dufour and Stevanović (2013) considered the combination of vector autoregressive moving-average (VARMA) models and factor-augmented techniques. Moreover, Bai, Li and Lu (2016) derived the inferential theory that corresponds to a maximum likelihood estimation for FAVAR models. So far, the aforementioned papers are based on the assumption that the coefficients of the factor-augmented regression models are constant over time. However, the structural instability of factor-augmented models was also witnessed by numerous studies. For instance, Corradi and Swanson (2014) constructed a test for the joint hypothesis of structural stability of both factor loadings and coefficients in factor-augmented forecasting model.

To address these inherent issues in static models, recently, Li, Tosasukul and Zhang (2020) proposed a univariate factor-augmented predictive regression model with functional coefficients, which allows the coefficients to vary with a variable. On the other hand, Yan and Cheng (2020) studied a parametric factor-augmented forecasting model in the presence of threshold effects. Nevertheless, it still remains unclear to us how to apply factor-augmented methods to reducing the number of coefficients in the nonlinear VAR models under high dimensional settings. In addition, the impulse response function, one of the most important tools for analyzing the influences of monetary policy shocks, may not be derived trivially when the coefficients of FAVAR models are time-varying. Furthermore, since the performance of parametric FAVAR models may be undermined by model misspecification and parameter instability, it is reasonable to consider nonparametric approaches for estimating FAVAR models with time-varying coefficients.

In this article, we propose a functional coefficient FAVAR, termed as FC-FAVAR, model to fill these gaps in the literature and is presented in (2) later. In particular, we capture nonlinearities in data by using a functional coefficient setting, which allows coefficients of traditional FAVAR models to vary with a variable. Actually, as elaborated by Cai, Das, Xiong and Wu (2006) and Cai (2010), a functional-coefficient model can be a good approximation to a fully nonparametric model and has a great ability to capture heteroscedasticity; see Cai (2010) for more details. In the last two decades, the functional-coefficient modeling approach has received much attention on time series studies, to name just a few, Chen and Tsay (1993), Cai, Fan and Yao (2000), Cai (2007), Dahlhaus and Subba Rao (2006), Chen and Hong (2016). Technically, we adapt local linear regression method to estimate the coefficient functions, which has been throughout discussed in Cai et al. (2000). Moreover, to address the problem of high-dimensionality, we impose an approximate factor modeling structure commonly used in the analysis of economic and financial time series data. In recent years, there has been increasing interest on studying the approximate factor model, see, for example, Chamberlain and Rothschild (1983), Fama and French (1992), Bai and Ng (2002), Fan, Liao and Mincheva (2013) and the references therein. It is well known in the literature of dynamic factor models that the information from a large number of time series can be summarized by a relatively small set of estimated factors, see, e.g., Stock and Watson (2002) and Bernanke and Boivin (2003). Note that our FC-FAVAR model allows both observed and unobserved factors to jointly follow a VAR process, which is different from the predictive factor-augmented model with the structure that the lagged observed variables have no effect on unobserved factors as in Li et al. (2020). Under our model setting, additional restrictions of identification are required to obtain corresponding impulse response functions.

Indeed, the motivation of this study arises from a debate over the issue that was found by various studies that a contractionary monetary policy is often followed by an increase of the price level, which is contrary to the standard economic theory, the so-called "price puzzle", see, e.g., Sims (1992), and Christiano, Eichenbaum and Evans (1999). Sims (1992) suggested that this puzzle results from the VAR analysis not fully capturing the information. In order to reduce the price puzzle, Sims (1992) considered adding commodity prices as an "information variable" in monetary VAR models because it contains information that helps the Federal Reserve forecast inflation, while Hanson (2004) questioned this explanation about the role for commodity prices in VAR models, finding that the ability that commodity prices have to resolve the price puzzle varies over the sample periods. Meanwhile, Bernanke et al. (2005) followed the idea in Sims (1992) and reduced the huge dimension of information set by using a FAVAR model. Other researches of solving the price puzzle include, to name a few, attributing the omission of output gap (or potential output) to the occurrence of price puzzle in Giordani (2004), referring cost channel as an alternative explanation for the price puzzle in Henzel, Hülsewig, Mayer and Wollmershäuser (2009), considering a broad Divisia monetary aggregate as monetary police indicator rather than the fed funds rate (FFR) in a structural VAR model in Keating, Kelly and Valcarcel (2014), and among others. To the best of our knowledge, there is little literature to consider the relations between structural changes of economic variables and the existence of price puzzle. However, Hanson (2004) found that the price puzzle is more pronounced in specific sample periods. This observation indicates that the significance of the price puzzle may be related to the dynamic features of general economy. In addition, since the driving force for structural changes may be the institutional changes or the policy interventions, such as the changes of exchange rate systems and the U.S. quantitative easing policy, features about structural changes can apparently enrich the information set that the researchers and policy-makers care about and help correct abnormal results caused by the price puzzle. Thus, due to its ability of capturing features of structural changes, the proposed FC-FAVAR model may have the potential to reduce the price puzzle. In this study, we consider the estimation of FC-FAVAR models and apply these models to reducing the price puzzle. The detailed analysis results are reported in Section 3.

The rest of this paper is organized as follows. In Section 2, the model setup is presented for the FC-FAVAR model, and a two-stage procedures for estimating functional coefficients as well as corresponding impulse response functions are also discussed in this section, together with a simple inference via a Bootstrap approach. In Section 3, our models are applied to exploring the dynamic effects of innovations to monetary policy on large amounts of economic variables. Section 4 concludes the paper. Finally, a detailed description of our dataset and some assumptions for inducing probabilistic properties of FC-FAVAR model are gathered in Appendix.

2 Econometric Modeling

2.1 Functional Coefficient FAVAR Model

Let $X_t = (X_{1t}, \ldots, X_{Nt})^T$ be an $N \times 1$ vector of available predictive variables at time t for $1 \leq t \leq n$. For $1 \leq j \leq M$ with $M \geq 1$, consider following factor-augmented forecasting model with functional coefficients

$$y_{j,t} = \gamma_{j0}(Z_{jt}) + \sum_{s=1}^{q_y} \boldsymbol{\gamma}_{j,s,y}^T(Z_{jt})Y_{t-s} + \sum_{r=1}^{q_f} \boldsymbol{\gamma}_{j,r,F}^T(Z_{jt})F_{t-r} + \sum_{l=1}^{p} \boldsymbol{\beta}_{j,l}^T(Z_{jt})W_{t-l} + v_{j,t} \tag{1}$$

for some q_y, q_f and p, where $\gamma_{j0}(\cdot)$ is a scalar function, $Y_t = (y_{1,t}, \ldots, y_{M,t})^T$ is an $M \times 1$ vector of observable economic variables that are contained in X_t, $F_t = (F_{1t}, \ldots, F_{Kt})^T$ is a $K \times 1$ vector of unobservable factors, and W_t is a $\kappa \times 1$ vector of observable covariates, including possibly some or all of $\{y_{j,t}\}_{j=1}^M$ and/or some exogenous variables. In addition, both $\boldsymbol{\gamma}_{j,s,y}(\cdot) = (\gamma_{sj1,y}(\cdot), \ldots, \gamma_{sjM,y}(\cdot))^T$ and $\boldsymbol{\gamma}_{j,r,F}(\cdot) = (\gamma_{rj1,F}(\cdot), \ldots, \gamma_{rjK,F}(\cdot))^T$ are $M \times 1$ and $K \times 1$ vectors of functional coefficients, respectively. Finally, $\boldsymbol{\beta}_{j,l}(\cdot) = (\beta_{lj1}(\cdot), \ldots, \beta_{lj\kappa}(\cdot))^T$ is a $\kappa \times 1$ vector of functional coefficients and $v_{j,t}$ is an error term. Here, Z_{jt} is an observable scalar smoothing variable, which might be one part of W_{t-l} and/or time or other exogenous variables or their lagged variables. Of course, Z_{jt} can also be an economic index to characterize economic activities. It is worthwhile to note that Z_{jt} can be set as a multivariate variable. In such a case, the estimation procedures and the related theory for the univariate case still hold for multivariate case, but more complicated notations are involved and models with Z_{jt} in very high dimension are often not practically useful due to the curse of dimensionality; see Cai et al. (2000) for details.

Importantly, in the case of estimating high dimensional VAR models with functional coefficients in our empirical studies, we assume that Y_t and F_t jointly follow a VAR process. In addition, for easy exposition, we let $p = 0$ and $q_y = q_f \equiv q$, and the smoothing variable Z_{jt} is allowed to vary only across different time periods but keeps constant over individual units. Therefore, model (1) can be written as a VAR model with functional coefficients. In particular, by letting $Z_{jt} = Z_t$ for all $1 \leq j \leq M$, $1 \leq \iota \leq Q$ and $1 \leq \ell \leq Q$ with $Q = M + K$, our FC-FAVAR model is

$$\mathbb{P}_t = \boldsymbol{\gamma}_0(Z_t) + \boldsymbol{\Phi}(Z_t)\mathbb{P}_{t-1} + \mathbb{U}_t, \tag{2}$$

where $\mathbb{P}_t = (P_t^T, \ldots, P_{t-q+1}^T)^T$ with $P_t = (Y_t^T, F_t^T)^T$, $\boldsymbol{\gamma}_0(\cdot) = (\gamma_{10}(\cdot), \ldots, \gamma_{Q0}(\cdot), 0_{1\times(Qq-Q)})^T$ is a vector of scalar function $\gamma_{\iota 0}(\cdot)$, and $\mathbb{U}_t = (u_{1,t}, \ldots, u_{Q,t}, 0_{1\times(Qq-Q)})^T$ is a vector of error terms. In addition, $\mathbf{\Phi}(Z_t)$ is a functional coefficient matrix and is expressed as follows

$$\mathbf{\Phi}(Z_t) = \begin{pmatrix} \Gamma_1(Z_t) & \Gamma_2(Z_t) & \dots & \Gamma_{q-1}(Z_t) & \Gamma_q(Z_t) \\ I_Q & 0 & \dots & 0 & 0 \\ 0 & I_Q & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I_Q & 0 \end{pmatrix},$$

where I_Q is a $Q \times Q$ identity matrix and $\Gamma_k(Z_t) = (\gamma_{k\iota\ell,P}(Z_t))_{Q\times Q}$ is a $Q \times Q$ matrix with $\gamma_{k\iota\ell,P}(\cdot)$ being the functional coefficient for $1 \leq k \leq q$. Notice that process P_t in (2) is presented as

$$P_t = \gamma_0(Z_t) + \Gamma_1(Z_t)P_{t-1} + \Gamma_2(Z_t)P_{t-2} + \cdots + \Gamma_q(Z_t)P_{t-q} + \boldsymbol{u}_t, \tag{3}$$

where $\gamma_0(\cdot) = (\gamma_{10}(\cdot), \ldots, \gamma_{Q0}(\cdot))^T$ and $\boldsymbol{u}_t = (u_{1,t}, \ldots, u_{Q,t})^T$. With models (1) and (2) at hand, our FC-FAVAR model further assumes that X_t is affected by P_t through a factor model

$$X_t = \Lambda_F F_t + \Lambda_Y Y_t + e_t, \tag{4}$$

where F_t is a $K \times 1$ vector of common factors, Λ_F is an $N \times K$ matrix of factor loadings, Λ_Y is an $N \times M$ matrix of coefficients, and $e_t = (e_{1t}, \ldots, e_{Nt})^T$ is an $N \times 1$ vector of idiosyncratic errors. To demonstrate high-dimensional setting, the number N is large and it is commonly assumed to be much greater than the number of factors and observed variables $(K + M \ll N)$.

Clearly, the model in (3) covers many well known models in literature as a special case. In particular, when Y_{t-k} in P_{t-k} is assume to have no effect on F_t in P_t for $1 \leq k \leq q$, then the model in (3) includes the model in Li et al. (2020). In addition, when $M = 1$ (univariate case) and factor part is excluded, this model nests that in Chen and Tsay (1993), Cai et al. (2000) and Cai (2010), and the model in Cai (2007) for Z_{jt} being time. In addition, if Z_{jt} is time, then model (3) is called time-varying FAVAR model, which includes static FAVAR model in Bernanke et al. (2005), Bai et al. (2016) and Yamamoto (2019), as well as the threshold FAVAR model in Yan and Cheng (2020).

Remark 2.1. *(Strictly stationary and α-mixing). To apply our estimation procedures, one has to show that the model given in* (2) *can generate strictly stationary and α-mixing process. It is well-established that a geometrically ergodic Markov process initiated from its invariant distribution is (strictly) stationary and α-mixing (Pham, 1986). Notice that model (2) can* *also be expressed as a vector valued Markov model. Thus, it is common practice to prove ergodicity to establish the stationarity for FC-FAVAR models and we present an assumption that induces strictly stationary and α-mixing for process $\{(P_t, Z_t)\}$ in Appendix. Notice that the detailed proof of this stationarity is similar to that in Cai and Liu (2020) and omitted.*

Remark 2.2. *(Selection of Z_t). Of importance is to choose an appropriate smoothing variable Z_t in applying the functional-coefficient FAVAR model in* (2)*. Knowledge on physical background or economic theory of the data may be very helpful, as we have witnessed in modeling the real data in Section 3 by choosing Z_t to be the monthly series of the difference between Moody BAA-rated corporate bond and FFR. Without any prior information, it is pertinent to choose Z_t in terms of some data-driven methods such as the Akaike information criterion (AIC), cross-validation (CV), and other criteria. Ideally, one would choose Z_t as a linear function of given explanatory variables according to some optimal criterion or an economic index based on economic theory or background. Nevertheless, here we would recommend using a simple and practical approach proposed by Cai et al. (2000) in practice.*

2.2 Two-stage Estimation Procedures

Our estimation procedures consist of two steps similar to the estimation method in Cai et al. (2006) for functional coefficient instrumental variables model and in Li et al. (2020) for univariate case. The first is to estimate vector of latent factors F_t in (4), and then we perform locally weighted estimation for functional coefficients in (3) using the estimated $\hat{F}_t$ from the first step. The methods of obtaining $\hat{F}_t$ include the maximum likelihood estimation as discussed in Bai et al. (2016), a direct principle component estimation with orthogonality restriction between the observable and unobservable factors in Yamamoto (2019) and the construction of unobservable factors based on economic theory in Bernanke et al. (2005) and among others. In this paper, we prefer the skill proposed in Bernanke et al. (2005) by identifying unobservable factors as residuals of the regression of the principal components from the entire dataset on the principal components from "slow-moving variables" and Y_t. This method has the merit of preserving the information of interest rate and interest rate spread that is closely related to forecast of inflation rate (see Bernanke et al., 1990) without imposing the assumption that the unobserved factors do not respond to monetary policy innovations within the period (here, a month). The loss of information of interest rate and interest rate spread can cause the mis-measurement of inflation rate and amplify the price puzzle. The detail of this method is presented in Section 3.

After obtaining the estimated $\hat{F}_t$ and given $\hat{P}_t = (Y_t^T, \hat{F}_t^T)^T$, the second step follows from estimating (3) by a local linear approach, although a general local polynomial method is also applicable. The local (polynomial) linear method has been widely used in nonparametric regression during the past two decades due to its attractive mathematical efficiency, bias reduction, and adaptation of edge effects; see, for example, Cai et al. (2000). More specifically, let $\boldsymbol{\Gamma}(\cdot) = (\gamma_0(\cdot), \Gamma_1(\cdot), \ldots, \Gamma_q(\cdot))$ and by assuming that each entry $\gamma_{k\iota\ell,P}(\cdot)$ of matrix $\Gamma_k(\cdot)$ has a continuous second derivative, $\boldsymbol{\Gamma}(Z_t)$ can be approximated by a linear function at any given grid point $z_0 \in \mathbb{R}$ as follows

$$\mathrm{vec}[\boldsymbol{\Gamma}(Z_t)] \approx \mathrm{vec}[\boldsymbol{\Gamma}(z_0)] + \mathrm{vec}[\boldsymbol{\Gamma}^{(1)}(z_0)](Z_t - z_0),$$

where $\mathrm{vec}(\cdot)$ stacks the elements of a $m \times n$ matrix as a $mn \times 1$ vector, $\approx$ denotes the first-order Taylor approximation and $\boldsymbol{\Gamma}^{(1)}(\cdot)$ is the first-order derivative of each element of $\boldsymbol{\Gamma}(\cdot)$. Thus, (3) is approximated by

$$P_t \approx \hat{\boldsymbol{P}}_t^{*T} \boldsymbol{\theta} + \boldsymbol{u}_t,$$

where $\boldsymbol{\theta} = \begin{pmatrix} \mathrm{vec}[\boldsymbol{\Gamma}(z_0)] \\ \mathrm{vec}[\boldsymbol{\Gamma}^{(1)}(z_0)] \end{pmatrix}$ and $\hat{\boldsymbol{P}}_t^{*} = \begin{pmatrix} \hat{\boldsymbol{P}}_t \\ (Z_t - z_0)\hat{\boldsymbol{P}}_t \end{pmatrix}$ with $\hat{\boldsymbol{P}}_t = (1, \hat{P}_{t-1}^T, \ldots, \hat{P}_{t-q}^T)^T \otimes I_Q$, which becomes a local linear model. Therefore, the locally weighted sum of squares is

$$\sum_{t=1}^{n} [P_t - \hat{\boldsymbol{P}}_t^{*T} \boldsymbol{\theta}]^T [P_t - \hat{\boldsymbol{P}}_t^{*T} \boldsymbol{\theta}] K_h(Z_t - z_0), \tag{5}$$

where $K(\cdot)$ is a kernel function, $K_h(x) = K(x/h)/h$, and $h = h(n)$ is called bandwidth, which is a sequence of positive numbers tending to zero and controls the amount of smoothing used in estimation. By minimizing (5) with respect to $\boldsymbol{\theta}$, we obtain the local linear estimate of $\boldsymbol{\Gamma}(z_0)$, denoted by $\hat{\boldsymbol{\Gamma}}(z_0)$, consisting of the first $(qQQ + Q)$ elements of $\hat{\boldsymbol{\theta}}$, and the local linear estimator of the derivative of $\boldsymbol{\Gamma}(z_0)$, denoted by $\hat{\boldsymbol{\Gamma}}^{(1)}(z_0)$, consisting of the last $(qQQ + Q)$ elements of $\hat{\boldsymbol{\theta}}$.

In practical implementations of (5), there are some practical issues that need to be addressed. First, to obtain $\hat{\boldsymbol{\theta}}$, one indeed needs to run a weighted least squares regression. Second, the number of factors d and lags q are selected by minimizing some well known criteria such as the nonparametric Bayesian information criterion proposed in Li et al. (2020) and the nonparametric AIC in Cai and Tiwari (2000). Finally, given the selected $\hat{d}$ and $\hat{q}$, we choose the optimal bandwidth h based on some bandwidth selectors such as the modified multifold CV criterion developed in Cai et al. (2000) or the nonparametric AIC type criterion in Cai and Tiwari (2000), which are attentive to the structure of stationary time series data.

Remark 2.3. *(Asymptotics) Notice that the asymptotic theory for $\hat{\mathbf{\Gamma}}(z_0)$ can be obtained by following the ideas in Cai et al. (2006) and Li et al. (2020) and it may not be the exactly same as that in Cai et al. (2000) because P_t contains vector of latent factors F_t. It would be very interesting to investigate the asymptotic theory for $\hat{\mathbf{\Gamma}}(z_0)$ and sequentially for the impulse response functions, which is not a trivial task. It is conjectured that the asymptotic variance of $\hat{\mathbf{\Gamma}}(z_0)$ might have an additional term to account for variability of the estimated latent factors at the first step. We leave this theoretical justification as a future research topic.*

Remark 2.4. *(Identification restrictions on factors and policy shocks). It is well known that the model in (3)-(4) can only be estimated after imposing identification restrictions on factors and policy shocks. To this end, let $X = (X_1, \ldots, X_n)^T$, $F = (F_1, \ldots, F_n)^T$ and $\hat{F} = (\hat{F}_1, \ldots, \hat{F}_n)^T$, we use the standard normalization implicit in the principal components in the same way as in Bernanke et al. (2005). That is, we restrict $\hat{F}^T\hat{F}/n = I_K$. Moreover, define the rotation matrix for factors as*

$$H = V^{-1}(\hat{F}^T F/n)(\Lambda_F^T \Lambda_F/N),$$

where V is an $K \times K$ diagonal matrix with main diagonal elements as the K largest eigenvalues of $XX^T/(nN)$, in descending order. As discussed in Bernanke et al. (2005) and Yamamoto (2019), by assuming a recursive structure where all the factors in (3) respond with a lag to change in the monetary policy instrument, ordered last in Y_t (e.g., Assumption B4 holds), then, $\hat{\mathcal{P}} = Chol(\hat{\mathbf{u}}_t^T \hat{\mathbf{u}}_t/n)$ is a consistent estimate for $H\mathcal{P}$. Thus, no further restrictions are required on factors and on the equation (4), and the identification of the policy shock can be achieved in (3) as if it were a standard VAR.

2.3 Impulse Response Function

The focus in this subsection is on presenting impulse responses with functional coefficients below, which capture the dynamic interactions among the variables of interest in a wide range of practical cases. Under the assumption of stationarity of the process P_t, (3) has a vector

MA(∞) expression as

$$P_t = \boldsymbol{\mu}_t(Z_t) + \boldsymbol{u}_t + \Psi_1(Z_t)\boldsymbol{u}_{t-1} + \Psi_2(Z_t)\boldsymbol{u}_{t-2} + \ldots, \tag{6}$$

where $\boldsymbol{\mu}_t(Z_t) = \gamma_0(Z_t) + \sum_{k=1}^{\infty} \Psi_k(Z_t)\gamma_0(Z_t)$, $\Psi_k(Z_t) = J\boldsymbol{\Phi}^k(Z_t)J^T$ for $k \geq 1$, and $J = [I_Q, 0_{Q\times Q(q-1)}]$. In the analysis of structural models, the impulse response functions for orthogonal shocks are required. For this purpose, we consider the Cholesky decomposition of covariance matrix $\Omega = \text{var}(\boldsymbol{u}_t)$. Let $\mathcal{P}$ be the lower triangular matrix from the Cholesky decomposition such that $\Omega = \mathcal{P}\mathcal{P}^T$. In addition, let $\boldsymbol{\omega}_t$ be the corresponding structural shocks with the relation that $\boldsymbol{u}_t = \mathcal{P}\boldsymbol{\omega}_t$. Then, (6) can be written as

$$P_t = \boldsymbol{\mu}_t(Z_t) + \boldsymbol{B}_0\boldsymbol{\omega}_t + \boldsymbol{B}_1(Z_t)\boldsymbol{\omega}_{t-1} + \boldsymbol{B}_2(Z_t)\boldsymbol{\omega}_{t-2} + \ldots, \tag{7}$$

where $\boldsymbol{B}_0 = \mathcal{P}$ and $\boldsymbol{B}_k(Z_t) = \Psi_k(Z_t)\mathcal{P}$ for $k \geq 1$ is the impulse response function corresponding to the structural shocks $\boldsymbol{\omega}_t$. Therefore, impulse response functions of all variables in X_t can eventually be recovered through (4). Notice that the estimator of $\boldsymbol{\Phi}(\cdot)$ is defined as $\hat{\boldsymbol{\Phi}}(\cdot)$, in which we replace $\Gamma_k(\cdot)$ of (3) with the corresponding estimator in $\hat{\boldsymbol{\Gamma}}(\cdot)$ obtained from the previous subsection. Thus, corresponding impulse response functions can be derived at each grid point z_0 too, which is different from the impulse response function with constant coefficients. In Section 3, we present the estimated impulse response function $\hat{\boldsymbol{B}}_k(z_0)$ at given grid points z_0 and analyze the responses of key macroeconomic variables in X_t to monetary policy shocks.

2.4 Bootstrap Inference for Impulse Response Functions

To construct the confidence interval for structural impulse response functions in our FC-FAVAR model, we adopt a Bootstrap method, outlined below, which is similar to that in Gonçalves and Perron (2014) and Yamamoto (2019) for linear factor-augmented model, and yet the asymptotic validity of our procedure is significantly different from the existing literature due to our focus on nonparametric Bootstrapping under a functional coefficient setting. We leave the derivation of the asymptotic validity of our Bootstrap method as a future study. Under the assumptions in Appendix, our Bootstrap procedure is presented as follows.

1. Estimate the model in (3) using the two-step procedure outlined in Section 2.2, obtain $\hat{\Lambda}_F$, $\hat{\Lambda}_Y$ and matrices of functional coefficient estimates $\hat{\Gamma}_k(Z_t)$, $\hat{\mathcal{P}}$ and $\hat{\gamma}_0(Z_t)$ at each Z_t for

$1 \leq k \leq q$, as well as the residuals $\hat{\boldsymbol{u}}_t$ in (3) and $\hat{e}_t$ in (4), and construct the structural impulse response estimate $\hat{\boldsymbol{B}}_k(z_0)$ at given grid point z_0.

2. After demeaning the functional coefficient VAR residuals $\{\hat{\boldsymbol{u}}_t\}_{t=1}^n$ in the time direction, resample with replacements the residuals $\{\hat{\boldsymbol{u}}_t\}_{t=1}^n$ as $Q \times 1$ vectors in an i.i.d. fashion, denoted by $\{\boldsymbol{u}_t^*\}_{t=1}^n$, and generate the Bootstrapped sample P_t^* using

$$P_t^* = \hat{\gamma}_0(Z_t) + \hat{\Gamma}_1(Z_t)P_{t-1}^* + \hat{\Gamma}_2(Z_t)P_{t-2}^* + \cdots + \hat{\Gamma}_q(Z_t)P_{t-q}^* + \boldsymbol{u}_t^*$$

for $1 \leq t \leq n$, with bias correction that was discussed by Kilian (1998) and Yamamoto (2019).

3. Demean the the idiosyncratic residuals $\{\{\hat{e}_{it}\}_{t=1}^n\}_{i=1}^N$ in both time and cross-sectional directions. For each $i = 1, \ldots, N$, we propose the i.i.d. resampling of $\{\hat{e}_{it}\}_{t=1}^n$ to obtain $\{e_{it}^*\}_{t=1}^n$. Then, generate the Bootstrapped sample X_t^* from $X_t^* = \hat{\Lambda}_F F_t^* + \hat{\Lambda}_Y Y_t^* + e_t^*$, where $(Y_t^{*T}, F_t^{*T})^T = P_t^*$, for $1 \leq t \leq n$.

4. Use the Bootstrapped sample X_t^* to estimate $\hat{F}_t^*$, $\hat{\Lambda}_F^*$ and $\hat{\Lambda}_Y^*$ following the first step of the estimation procedure, and then, estimate the functional coefficient FAVAR in (3) using $\hat{F}_t^*$ and Y_t^* to obtain the Bootstrapped estimates $\hat{\Gamma}_k^*(z_0)$ and $\hat{\mathcal{P}}^*$ with the same identification restriction as the original estimate. This yields the Bootstrap estimate of the structural impulse response $\hat{\boldsymbol{B}}_k^*(z_0)$.

5. Repeat Steps 2–4 B times (say, $B = 500$ in Section 3) to obtain $\{\hat{\boldsymbol{B}}_k^*(z_0)\}_{k=1}^B$.

6. Store the re-centered statistic $c_k \equiv \hat{\boldsymbol{B}}_k^*(z_0) - \hat{\boldsymbol{B}}_k(z_0)$ and compute the $100\alpha/2$th and $100(1-\alpha/2)$th percentiles $c_k^{(\alpha/2)}$ and $c_k^{(1-\alpha/2)}$, respectively. The resulting $100(1-\alpha)\%$ pointwise confidence interval for $\boldsymbol{B}_k(z_0)$ is $[\hat{\boldsymbol{B}}_k(z_0) - c_k^{(\alpha/2)}, \hat{\boldsymbol{B}}_k(z_0) - c_k^{(1-\alpha/2)}]$ for the given grid point z_0.

Finally, for the sake of simplicity, we use the same bandwidth throughout the procedure and assume that the idiosyncratic errors in the factor model are serially and cross-sectionally independent, given in Assumption C in Appendix, which is consistent with settings of Gonçalves and Perron (2014). Note that the Bootstrap method presented in Li et al. (2020) can not be directly applied in this article, since our Bootstrap method requires generating factors F_t^* at Step 2 and renewing them by $\hat{F}_t^*$ at Step 4, which implies a different large sample theory compared to the procedure without estimating new factors based upon the Bootstrapped sample X_t^* in Li et al. (2020). For this regard, the read is referred to Yamamoto (2019) for a comparative study between these two schemes of Bootstrap.

3 Empirical Analysis

3.1 Literature Review on Price Puzzle

To clearly describe the common view of the cause of price puzzle, we first formalize the well-known reaction function of monetary policy that illustrates the relationship between the policy instrument variable and the data of economic activities. In particular, suppose that one element of Y_t defined in Section 2.1 is the policy instrument of the monetary authority, denoted as $r_{f,t}$, then, the monetary policy reaction function is written as follows

$$r_{f,t} = \beta(\pi_{e,t} - \tilde{\pi}) + D(Y_t, F_t) + \mu_t, \tag{8}$$

where $\pi_{e,t}$ is the expected future rate of inflation based on the information at time t and $\tilde{\pi}$ is the Fed's target inflation rate. In addition, μ_t is a exogenous policy shock which is an element of $\boldsymbol{u}_t$ in (3) and $D(Y_t, F_t)$ represents other observable or unobservable arguments of the reaction function (e.g., the output gap or lags of the policy instrument). Note that $r_{f,t}$ is selected to be the FFR in this paper. In the impulse response analysis, an impulse is imposed on μ_t and then all variables in X_t can be affected by this impulse through (7) and (4). As pointed out in Sim (1992) and Hanson (2004), if there is a measurement error on $\pi_{e,t}$ such that

$$\pi_{e,t} = \pi_{m,t} + \pi_{\xi,t},$$

where $\pi_{m,t}$ represents the "measured" inflation expectations based only upon the information contained in the estimated model by the researcher, while $\pi_{\xi,t}$ captures information excluded from the estimation of $\pi_{e,t}$, then, (8) becomes the following misspecified model

$$r_{f,t} = \beta(\pi_{m,t} - \tilde{\pi}) + D(Y_t, F_t) + \nu_t, \tag{9}$$

where $\nu_t = \beta\pi_{\xi,t} + \mu_t$.

Notice that the estimated policy shock ν_t is contaminated by a bias $\beta\pi_{\xi,t}$, where β is the degree to which the Fed reacts to inflationary pressures. Given the misspecified reaction function in (9), even the impact of a "true" policy shock μ_t upon price level is negative or zero followed by macroeconomic theory, the impulse response of price level to the estimated policy shock ν_t can be positive if $\beta\pi_{\xi,t}$ has positive impact on the price level. As a result, an empirical researcher would incorrectly infer that a contractionary policy shock had raised price level, which cause the price puzzle. For more discussion in detail, the reader is referred to the paper by Hanson (2004). Therefore, the attempt of reducing price puzzle faces two challenges. First, the feature of $\pi_{\xi,t}$, which is associated with some omitted variables for forecasting inflation rate $\pi_{e,t}$, needs to be captured. One possible way to resolve the first challenge is to introduce more variables that could improve the forecast power of inflation rate into the VAR model. However, Bernanke et al. (2005) argued that the forecast equation of inflation rate $\pi_{e,t}$ involves the measurement of potential output and cost-push shock, which can not be directly observed by both the central bank and the econometrician. Under this circumstance, a factor-augmented VAR may demonstrate a strength of investigating models with unobserved variables. Second, the β needs to be estimated with correct specification. As documented in Hanson (2004), the magnitude of β is different substantially across regimes, which is obviously referred to as a nonlinear feature. For this reason, it is unnecessarily feasible to apply linear FAVAR model to studying the effect of monetary policy shock to macroeconomic variables. Thus, our FC-FAVAR model is well-suitable to reducing price puzzle because it can not only capture nonlinearities in data, but also extract unobservable information from a huge dataset. It is worth mentioning that the aim of this empirical study is to demonstrate the usefulness of the proposed FC-FAVAR model in reducing the price puzzle compared to the classical FAVAR, rather than eradicating the existence of price puzzle. Further extensions may be realized through advocating alternative monetary instrument variables (e.g., Divisia index proposed in the seminal work of Barnett, 1980), which is out of the scope of this paper.

3.2 Data and Implementation

In this section, the proposed FC-FAVAR model is applied to exploring the effects of innovations to monetary policy on large amounts of economic variables. In particular, our dataset X_t in (4) consists of a balanced panel of 100 monthly macroeconomic time series, which are updates of series used in Bernanke et al. (2005) and Michael, McCracken and Ng (2016). The data span the period from February 1960 through July 2020. These series are initially transformed to induce stationarity. The description of the series in the dataset and their transformation are described in Appendix. In our empirical study, Y_t in (1)-(3) could include policy indicator and observable measures of real activity and prices such as the FFR, the industrial production index (IP) and the consumer price index (CPI), while $\hat{C}(X_t)$ is the vector of principal components estimated from the entire dataset X_t. Since both Y_t and

$\hat{C}(X_t)$ involve the series of FFR, denoted by FFR_t, it would be invalid to identify the effect of policy shocks when simply estimating a VAR in Y_t and $\hat{C}(X_t)$. Thus, the direct dependence of $\hat{C}(X_t)$ on FFR_t should be removed. By following the procedures in Bernanke et al. (2005), we first regress $\hat{C}(X_t)$ on FFR_t in the form of $\hat{C}(X_t) = b_C\hat{C}(\hat{F}_t) + b_{\text{FFR}}\text{FFR}_t + e_t$, where $\hat{C}(\hat{F}_t)$ is an estimate of all the common components other than FFR_t. One way to obtain $\hat{C}(\hat{F}_t)$ is to extract principal components from the subset of "slow-moving variables", which are not affected contemporaneously by FFR_t. The reader is referred to Bernanke et al. (2005) for more discussion about "slow-moving variables". Next, we construct $\hat{F}_t$ as $\hat{C}(X_t) - \hat{b}_{\text{FFR}}\text{FFR}_t$ and finally estimate the FC-FAVAR model (2) in Y_t and $\hat{F}_t$, with FFR ordered last. To fully demonstrate the usefulness of our FC-FAVAR models, we revisit one of issues that was discussed in Bernanke et al. (2005) and compare classical FAVAR models with FC-FAVAR models in the performance of reducing the price puzzle.

3.3 Empirical Results

The analysis in this section aims at comparing the results generated from a standard FAVAR model in Bernanke et al. (2005) to that from our proposed FC-FAVAR model in (2) based on the dataset X_t from a new time span. Different from the standard FAVAR, the proposed FC-FAVAR model allows coefficients to vary with an exogenous variable Z_t. Here, without adding new variable in the model, we choose one of variables from X_t to be Z_t. In this way, the information that contained in the original X_t could not only be preserved, but also be applied to capturing the structural changes that lie in the multivariate time series without introducing new variables into the model. Therefore, we choose Z_t as the sixth-lagged spread between Moody BAA-rated corporate bond and FFR, denoted by SFYBAAC in what follows and in Appendix. This choice of smoothing variable is reasonable, because the SFYBAAC contains information of both the spread between Moody AAA-rated corporate bond and FFR, denoted by SFYAAAC in what follows and in Appendix, and the spread between Moody BAA-rated corporate bond and Moody AAA-rated corporate, denoted by DEFAULT in what follows. Indeed, Bernanke (1990) suggested that DEFAULT should be used as a measure of the behavior of perceived default risk in the economy, which has an influence on the Federal Reserve Bank for making monetary policy decision. Furthermore, SFYAAAC can serve as a nice indicator of monetary policy changes. Thus, the temporal changes of SFYBAAC may indicate the shift of environment of decision making for monetary policy. For this regard, the reader is referred to the paper by Bernanke (1990) for more discussions. It is worth emphasizing that SFYBAAC is not the only choice for smoothing variable, of course, other variables of economic status may also be suitable to serve as the smoothing variable and this may be left in a future study.

It is crucial to first show that the coefficients in (3) change over SFYBAAC in the empirical example. To this end, 36 entries of functional coefficients matrix $\Gamma_1(\cdot)$ with respect to the changes of SFYBAAC are reported in Figure 1, in which the vertical axis of each panel measures the functional coefficient and the horizontal axis of each panel measures SFYBAAC. It is obvious that all functional coefficients in Figure 1 are not constant, but significantly vary with the changes of SFYBAAC. This observation indicates that the dataset possesses features of nonlinearity. Therefore, adopting classical FAVAR model in this dataset may result in severe problem of misspecification and subsequently, the price puzzle.

Based on the analysis in Section 3.1, the reduction of price puzzle follows from the decrease of the time that the response of price level to policy shocks spends to become negative. Thus, the faster the curve of impulse response of CPI to monetary policy shocks becomes negative, the better the model performs in reducing price puzzle. In the next group of figures, we compare the results of impulse response functions estimated by our FC-FAVAR model and by classical FAVAR in Bernanke et al. (2005). We choose four grid points as the data of SFYBAAC on four time points: 1966:09, 1980:01, 2006:09 and 2011:09, and then obtain impulse responses at these grid points. In October 1979, Fed Chairman Paul Volcker announced a shift from effectively targeting the federal funds rate to explicitly targeting non-borrowed reserves. A number of researchers observed that this change of policy instrument significantly enlarged the β in monetary policy reaction function in (8), causing a regime-specific phenomenon on structural parameters; see, for instance, Taylor (1999), Clarida, Galì and Gertler (2000) and Hanson (2004). Therefore, the first and second time points 1966:09 and 1980:01 can act as nice proxies of the"pre-Volcker" and "post-Volcker" periods, respectively, to demonstrate our model's ability of capturing structural changes in data. In addition, the third and fourth time points 2006:09 and 2011:09 represent periods of "pre-financial crisis" and "post-financial crisis", respectively. It is well-known that the FFR has been stuck at or near the zero lower bound (ZLB) since 2008, which poses a criticism about the effectiveness of FFR as monetary policy indicator. Thus, the studies based on the third and fourth time points are suitable for checking the reliability of FC-FAVAR model

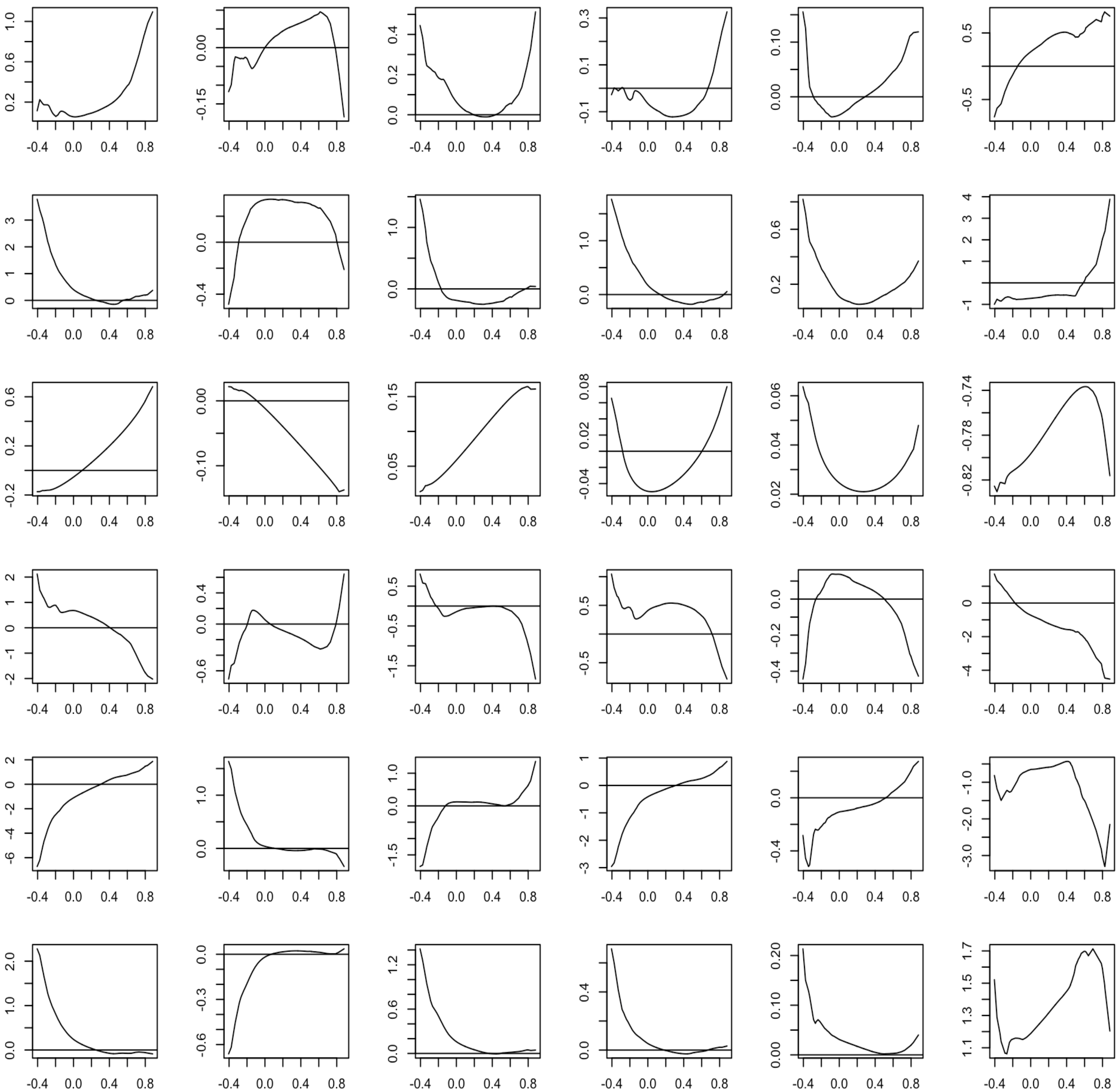

Figure 1: 36 entries of functional coefficients matrix $\Gamma_1(\cdot)$ with respect to the changes of SFYBAAC estimated by the FC-FAVAR model with 5 factors and 7 lags.

under extreme conditions of economy.

In Figure 2, the vertical axis measures impulse response functions estimated from classical FAVAR model, while in Figures 3 and 4, the vertical axis represents $\hat{\boldsymbol{B}}_k(z_0)$ estimated by

FC-FAVAR model at given grid point z_0, and the horizontal axis in Figures 2-4 represent the time lag k. Notice that the $\iota\ell$-th element of $\hat{\boldsymbol{B}}_k(z_0)$ is interpreted as the effect on variable ι of a unit innovation in the ℓ-th variable that has occurred k periods ago. In addition, we standardize the monetary shock to correspond to a 25-basis-point innovation in the FFR. Figure 2 presents the resulting impulse response functions of FFR, industrial production and consumer price index of all items for the classical FAVAR proposed in Bernanke et al. (2005) for two sample periods: 1960:02-2001:08 (the top panel) and 1960:02-2020:07 (the bottom panel), respectively. The first period ends in August 2001 following Bernanke et al. (2005), and the second period extends the sample to July 2020. In both two sample periods, we employ seventh lags and the number of factors is five. Observed that in the first time span, the response of all variables move in the same way as in Bernanke et al. (2005), while in the second time span, the response of CPI goes up to positive and fail to return to negative within 50 lags, which indicates that there is still a strong price puzzle in the classical FAVAR specification. These results are not surprising, because as the sample periods enlarged, the information about changes of general economy become significant and may eventually undermine the estimation results given by the linear FAVAR model.

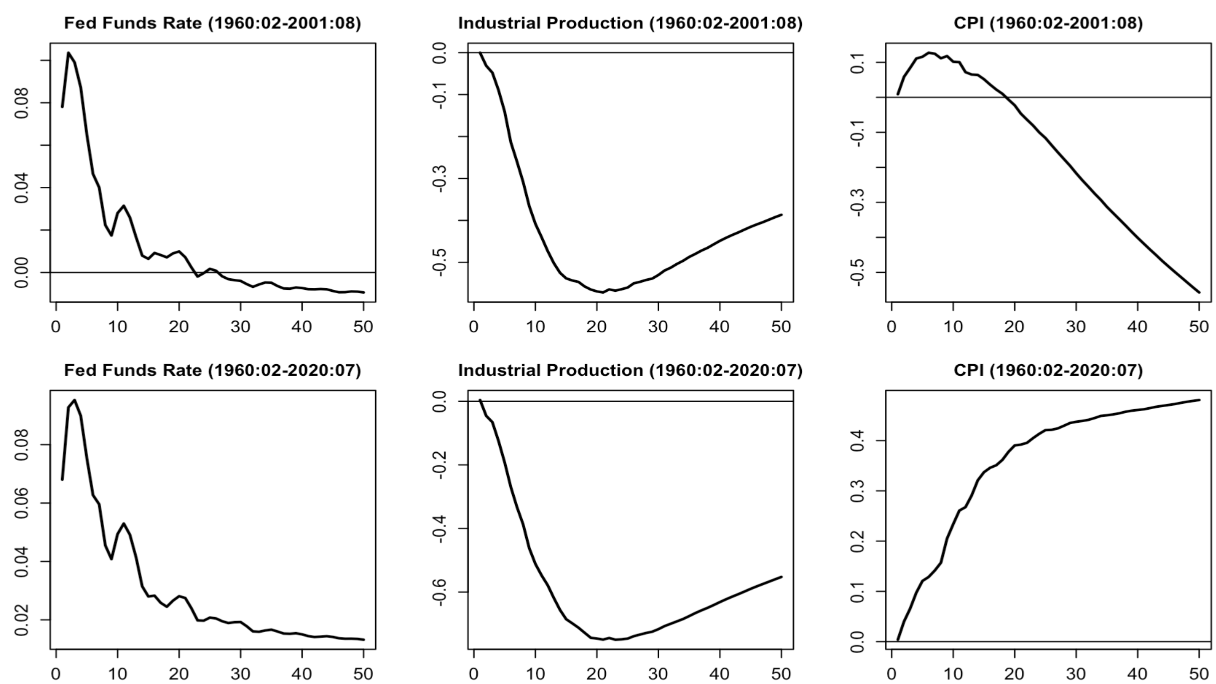

Figure 2: Impulse response functions of FFR, industrial production and consumer price index of all items for the classical FAVAR model with 5 factors and 7 lags for two sample periods: 1960:02-2001:08 (the top panel) and 1960:02-2020:07 (the bottom panel).

In contrast, Figure 3 displays impulse response functions on four time points obtained by the proposed FC-FAVAR model for the sample period ends in July 2020 and we use seven lags for $\hat{q}$, and five factors for $\hat{d}$. It is interesting that after considering the changes of economic environment, the responses of CPI go down to negative within 20 lags at all four grid points, suggesting that the price puzzle is considerably reduced compared to the results of classical FAVAR. More specifically, the responses of CPI at time points 1966:09, 1980:01 and 2006:09 drop to negative within 20 lags, which show that FC-FAVAR model can nicely reduce price puzzle by correcting the measurement error discussed in Section 3.1. For the result on time point 2011:09, even when FFR reaches to ZLB, the estimated response of CPI still returns to negative within 20 lags. In this case, although there exists macroeconomic models with alternative policy instruments that work fairly well in correcting the abnormal of price level, our FC-FAVAR model can reasonably reduce price puzzle without introducing new structures in the conventional macroeconomic model and replacing policy instruments. Of course, it is of great interest to use other variables as policy instruments instead of the FFR in FC-FAVAR model and we leave this as a future topic.

Finally, Figure 4 shows the impulse responses of selected macroeconomic variables to monetary policy shocks with 90% confidence intervals generated by the proposed FC-FAVAR on time point 1980:01, which are obtained by the Bootstrap procedure presented in Section 2.4. The responses are generally of the expected sign and magnitude: following a contractionary monetary policy shock, prices go down to negative rapidly, money aggregates decline, and the dollar appreciates. The dividend yields initially jump above the steady state and finally go down. To sum up, these results seem to demonstrate that measures of the effects of monetary policy are consistent and sensible. Notice that we only display 12 responses of all 100 that could also be investigated technically. The results for the rest responses are available upon request.

4 Conclusion

In this paper, we investigate a functional coefficient FAVAR model with an application to resolving the price puzzle and coefficients functionals are estimated by using a two-stage kernel smoothing method. In addition, there is little literatures regarding the relationship between the existence of price puzzle and the structural changes in the economic environment. After considering the changes of specific state of economy, the proposed framework mitigates

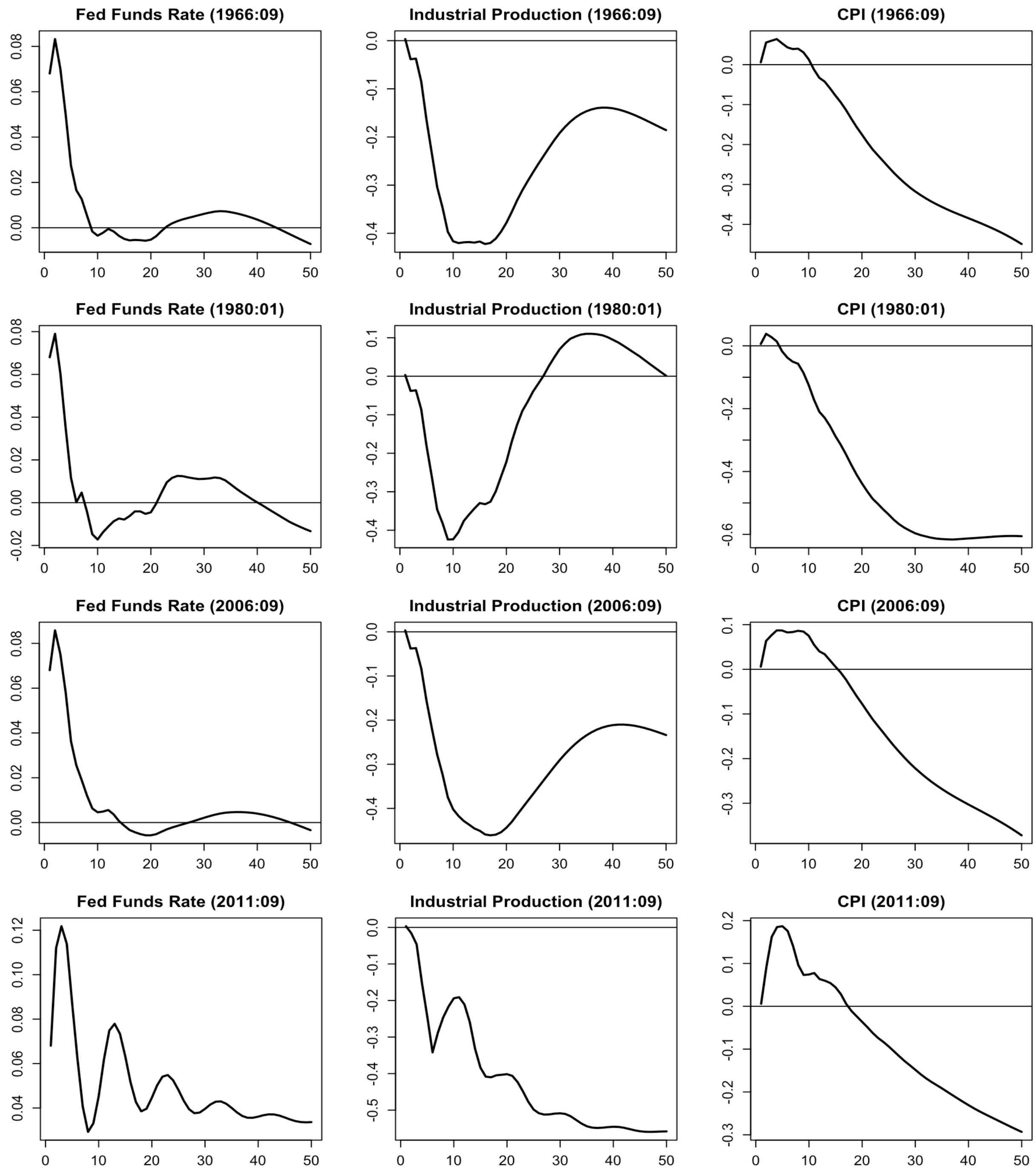

Figure 3: Impulse response functions of FFR, industrial production and consumer price index of all items for the FC-FAVAR model with 5 factors and 7 lags on 1966:09 (the first row of panel), 1980:01 (the second row of panel), 2006:09 (the third row of panel) and 2011:09 (the fourth row of panel).

the issue of price puzzle and still allows to estimate responses of large amounts of economic variables to monetary policy shocks.

There are several issues still worth of further studies. First, it is interesting to introduce

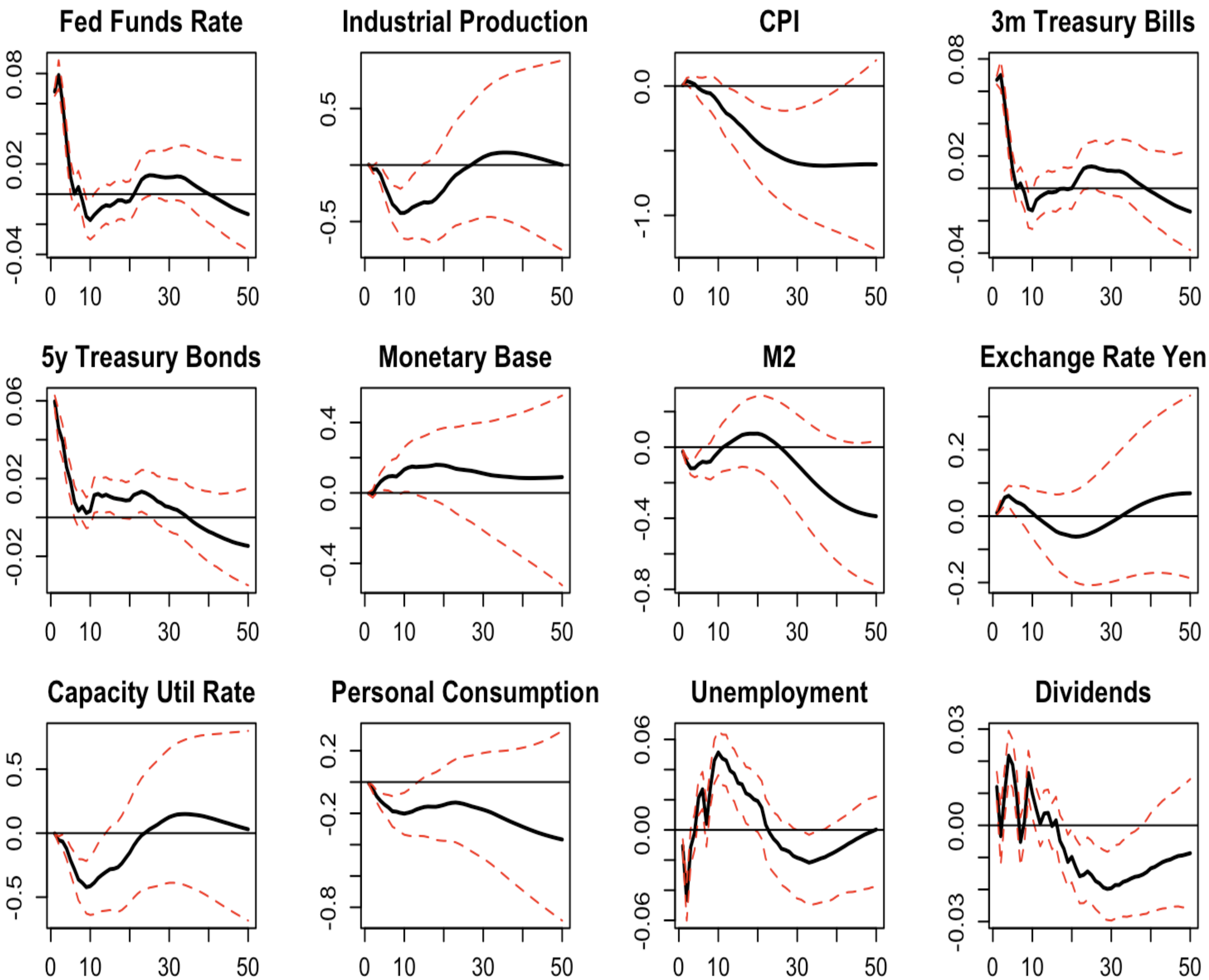

Figure 4: Impulse responses of 12 variables generated from FC-FAVAR with 5 factors and 7 lags on time point 1980:01 and corresponding 90% confidence intervals (the dashed lines) obtained from Bootstrapping.

heteroscedasticity into model (2), although a functional coefficient model has an ability to capture partial heteroscedasticity as argued by Cai (2010), so that the dynamics of monetary policy shocks can also be captured. Second, the asymptotic properties of functional coefficients and impulse response functions need to be derived and this should not be hard given the similar precedents of theoretical work in Cai et al. (2000), Cai et al. (2006) and Li et al. (2020). We leave these important issues as future research topics.

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Appendix

A.1 Descriptions of Dataset

All series are directly taken from the Federal Reserve Bank of St. Louis with a dataset proposed in Michael et al. (2016) and the format is as that in Bernanke et al. (2005): series number; series mnemonic; transformation code and series description as appearing in the database for the data span from 1960:02 to 2020:07. The transformation codes are 1-no transformation; 2-first difference; 4-logarithm; 5-first difference of logarithm. An asterisk $*$, next to the mnemonic, denotes a variable assumed to be slow-moving in the estimation.

Table A1: Description of data

	Real output and income	
1.	IPF*	5 INDUSTRIAL PRODUCTION: FINAL PRODUCTS (1992=100, SA)
2.	IPC*	5 INDUSTRIAL PRODUCTION: CONSUMER GOODS (1992=100, SA)
3.	IPCD*	5 INDUSTRIAL PRODUCTION: DURABLE CONS. GOODS (1992=100, SA)
4.	IPCN*	5 INDUSTRIAL PRODUCTION: NONDURABLE CONS. GOODS (1992=100, SA)
5.	IPE*	5 INDUSTRIAL PRODUCTION: BUSINESS EQUIPMENT (1992=100, SA)
6.	IPM*	5 INDUSTRIAL PRODUCTION: MATERIALS (1992=100, SA)
7.	IPMD*	5 INDUSTRIAL PRODUCTION: DURABLE GOODS MATERIALS (1992=100, SA)
8.	IPMND*	5 INDUSTRIAL PRODUCTION: NONDUR. GOODS MATERIALS (1992=100, SA)
9.	IPMFG*	5 INDUSTRIAL PRODUCTION: MANUFACTURING (1992=100, SA)
10.	IPMIN*	5 INDUSTRIAL PRODUCTION: MINING (1992=100, SA)
11.	IPUT*	5 INDUSTRIAL PRODUCTION: UTILITIES (1992=100, SA)
12.	IP*	5 INDUSTRIAL PRODUCTION: TOTAL INDEX (1992=100, SA)
13.	IPXMCA*	1 CAPACITY UTIL RATE: MANUFAC., TOTAL (% OF CAPACITY,SA) (FRB)
14.	GMPYQ*	5 PERSONAL INCOME (CHAINED) (SERIES #52) (BIL 92$,SAAR)
15.	GMYXPQ*	5 PERSONAL INC. LESS TRANS. PAYMENTS (CHAINED) (#51) (BIL 92$,SAAR)
	Employment and hours	
16.	LHEM*	5 CIVILIAN LABOR FORCE: EMPLOYED, TOTAL

		(THOUS., SA)
17.	LHUR*	1 UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%, SA)
18.	LHU680*	1 UNEMPLOY. BY DURATION: AVERAGE (MEAN) DURATION IN WEEKS (SA)
19.	LHU5*	1 UNEMPLOY. BY DURATION: PERS UNEMPL. LESS THAN 5 WKS (THOUS., SA)
20.	LHU14*	1 UNEMPLOY. BY DURATION: PERS UNEMPL. 5 TO 14 WKS (THOUS., SA)
21.	LHU15*	1 UNEMPLOY. BY DURATION: PERS UNEMPL. 15 WKS + (THOUS., SA)
22.	LHU26*	1 UNEMPLOY. BY DURATION: PERS UNEMPL. 15 TO 26 WKS (THOUS., SA)
23.	LPNAG*	5 EMPLOYEES ON NONAG. PAYROLLS: TOTAL (THOUS., SA)
24.	LPGD*	5 EMPLOYEES ON NONAG. PAYROLLS: GOODS-PRODUCING (THOUS., SA)
25.	LPMI*	5 EMPLOYEES ON NONAG. PAYROLLS: MINING (THOUS., SA)
26.	LPCC*	5 EMPLOYEES ON NONAG. PAYROLLS: CONTRACT CONSTRUC. (THOUS., SA)
27.	LPEM*	5 EMPLOYEES ON NONAG. PAYROLLS: MANUFACTURING (THOUS., SA)
28.	LPED*	5 EMPLOYEES ON NONAG. PAYROLLS: DURABLE GOODS (THOUS., SA)
29.	LPEN*	5 EMPLOYEES ON NONAG. PAYROLLS: NONDURABLE GOODS (THOUS., SA)
30.	LPSP*	5 EMPLOYEES ON NONAG. PAYROLLS: SERVICE-PRODUCING (THOUS., SA)
31.	LPTU*	5 EMPLOYEES ON NONAG. PAYROLLS: TRANS. & PUBLIC UTIL. (THOUS., SA)
32.	LPTW*	5 EMPLOYEES ON NONAG. PAYROLLS: WHOLESALE (THOUS., SA)
33.	LPTR*	5 EMPLOYEES ON NONAG. PAYROLLS: RETAIL (THOUS., SA)
34.	LPFR*	5 EMPLOYEES ON NONAG. PAYROLLS: FINANCE, INS. & REAL EST (THOUS., SA)
35.	LPGOV*	5 EMPLOYEES ON NONAG. PAYROLLS: GOVERNMENT (THOUS., SA)
36.	LPHRM*	1 AVG. WEEKLY HRS. OF PRODUCTION WKRS.: MANUFACTURING (SA)
37.	LPMOSA*	1 AVG. WEEKLY HRS. OF PROD. WKRS.: MFG., OVERTIME HRS. (SA)
38.	HWI*	2 HELP-WANTED INDEX FOR USA
39.	HWIURATIO*	2 RATIO OF HELP WANTED/NO. UNEMPLOYED
	Consumption	
40.	GMCQ*	5 PERSONAL CONSUMPTION EXPEND (CHAINED)-TOTAL (BIL 92$, SAAR)
41.	GMCDQ*	5 PERSONAL CONSUMPTION EXPEND (CHAINED)-TOT. DUR. (BIL 96$, SAAR)
42.	GMCNQ*	5 PERSONAL CONSUMPTION EXPEND

		(CHAINED)-NONDUR. (BIL 92$, SAAR)
43.	GMCSQ*	5 PERSONAL CONSUMPTION EXPEND (CHAINED)—SERVICES (BIL 92$, SAAR)
	Housing starts and sales	
44.	HOUST	4 HOUSING STARTS: TOTAL NEW PRIV
45.	HSNE	4 HOUSING STARTS: NORTHEAST (THOUS.U.) S.A.
46.	HSMW	4 HOUSING STARTS: MIDWEST (THOUS.U.) S.A.
47.	HSSOU	4 HOUSING STARTS: SOUTH (THOUS.U.) S.A.
48.	HSWST	4 HOUSING STARTS: WEST (THOUS.U.) S.A.
49.	HSBR	4 HOUSING AUTHORIZED: TOTAL NEW PRIV HOUSING (THOUS., SAAR)
	Consumption, orders and inventories	
50.	AMDMNOx	5 NEW ORDERS FOR DURABLE GOODS
51.	AMDMUOx	5 UNFILLED ORDERS FOR DURABLE GOODS
52.	BUSINVx	5 TOTAL BUSINESS INVENTORIES
53.	ISRATIOx	2 TOTAL BUSINESS: INVENTORIES TO SALES RATIO
	Stock prices	
54.	FSPCOM	5 S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-1943=10)
55.	FSPIN	5 S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-1943=10)
56.	FSDXP	1 S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM)
57.	FSDXE	1 S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%, NSA)
	Exchange rates	
58.	EXRSW	5 FOREIGN EXCHANGE RATE: SWITZERLAND (SWISS FRANC PER U. S.$)
59.	EXRJAN	5 FOREIGN EXCHANGE RATE: JAPAN (YEN PER U. S.$)
60.	EXRUK	5 FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)
61.	EXRCAN	5 FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U. S.$)

	Interest rates	
62.	FYFF	1 INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM, NSA)
63.	FYGM3	1 INTEREST RATE: U. S. TREASURY BILLS,SEC MKT,3-MO. (% PER ANN, NSA)
64.	FYGM6	1 INTEREST RATE: U. S. TREASURY BILLS,SEC MKT,6-MO. (% PER ANN, NSA)
65.	FYGT1	1 INTEREST RATE: U. S. TREASURY CONST MATUR., 1-YR. (% PER ANN, NSA)
66.	FYGT5	1 INTEREST RATE: U. S. TREASURY CONST MATUR., 5-YR. (% PER ANN, NSA)
67.	FYGT10	1 INTEREST RATE: U. S. TREASURY CONST MATUR., 10-YR. (% PER ANN, NSA)
68.	FYAAAC	1 BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM)
69.	FYBAAC	1 BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)
70.	SFYGM3	1 Spread FYGM3-FYFF
71.	SFYGM6	1 Spread FYGM6-FYFF
72.	SFYGT1	1 Spread FYGT1-FYFF
73.	SFYGT5	1 Spread FYGT5-FYFF
74.	SFYGT10	1 Spread FYGT10-FYFF
75.	SFYAAAC	1 Spread FYAAAC-FYFF
76.	SFYBAAC	1 Spread FYBAAC-FYFF
	Money and credit quantity aggregates	
77.	FM1	5 MONEY STOCK: M1 (BIL$,SA)
78.	FM2	5 MONEY STOCK: M2 (BIL$,SA)
79.	FM3	5 MONEY STOCK: M3 (BIL$,SA)
80.	FMFBA	5 MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES (MIL$,SA)
81.	FMRRA	5 DEPOSITORY INST RESERVES: TOTAL, ADJ FOR RES. REQ CHGS (MIL$, SA)
82.	FMRNBA	5 DEPOSITORY INST RESERVES: NONBOR., ADJ RES REQ CHGS (MIL$,SA)
83.	FCLNQ	5 COMMERCIAL & INDUST. LOANS OUTSTANDING IN 1992 DOLLARS (BCI)
84.	CCINRV	5 CONSUMER CREDIT OUTSTANDING NONREVOLVING G19
	Price indexes	

85.	PWFSA*	5 PRODUCER PRICE INDEX: FINISHED GOODS (82=100, SA)
86.	PWFCSA*	5 PRODUCER PRICE INDEX: FINISHED CONSUMER GOODS (82=100, SA)
87.	PWIMSA*	5 PRODUCER PRICE INDEX: INTERMED MAT. SUP & COMPONENTS (82=100, SA)
88.	PWCMSA*	5 PRODUCER PRICE INDEX: CRUDE MATERIALS (82=100, SA)
89.	PUNEW*	5 CPI-U: ALL ITEMS (82-84=100, SA)
90.	PU83*	5 CPI-U: APPAREL & UPKEEP (82-84=100, SA)
91.	PU84*	5 CPI-U: TRANSPORTATION (82-84=100, SA)
92.	PU85*	5 CPI-U: MEDICAL CARE (82-84=100, SA)
93.	PUC*	5 CPI-U: COMMODITIES (82-84=100, SA)
94.	PUCD*	5 CPI-U: DURABLES (82-84=100, SA)
95.	PUS*	5 CPI-U: SERVICES (82-84=100, SA)
96.	PUXF*	5 CPI-U: ALL ITEMS LESS FOOD (82-84=100, SA)
97.	PUXHS*	5 CPI-U: ALL ITEMS LESS SHELTER (82-84=100, SA)
98.	PUXM*	5 CPI-U: ALL ITEMS LESS MIDICAL CARE (82-84=100, SA)
	Average hourly earnings	
99.	LEHCC*	5 AVG HR EARNINGS OF CONSTR WKRS: CONSTRUCTION ($, SA)
100.	LEHM*	5 AVG HR EARNINGS OF PROD WKRS: MANUFACTURING ($, SA)

A.2 Probabilistic Property: Strictly Stationary and α-Mixing

Let $\mathcal{F}_a^b$ be the σ-algebra generated by $\{(P_t, Z_t)\}_{t=a}^b$. Then, a stationary process $\{(P_t, Z_t)\}_{t=-\infty}^{\infty}$ is said to be α-mixing (strongly mixing) if the mixing coefficient $\alpha(t)$ defined by

$$\alpha(t) = \sup\{|P(A \cap B) - P(A)P(B)| : A \in \mathcal{F}_{-\infty}^0, B \in \mathcal{F}_t^\infty\} \ \rightarrow \ 0$$

$t \to \infty$. Denote matrix $\mathbf{\Phi}$ as the same way as $\mathbf{\Phi}(Z_t)$ in (2). To show strictly stationary and α-mixing of process $\{\mathbb{P}_t\}$ in (2), the following assumptions are needed.

Assumption A.

A1: *Let $\{\mathbb{P}_t\}$ in (2) be a ϕ-irreducible and aperiodic Markov chain. For all $1 \le \iota \le Q$ and*

$1 \le \ell \le Q$, $\gamma_{k\iota\ell,P}(\cdot)$ *in (2) is bounded such that* $|\gamma_{k\iota\ell,P}(\cdot)| \le \gamma_{k\iota\ell,P}$ *and the density function of* $u_{\ell,t}$ *in (2) is positive every where on the real line* $\mathbb{R}$ *for all* $1 \le \ell \le Q$*. Furthermore, the roots of* $I_Q - \Gamma_1 L - \cdots - \Gamma_q L^q = 0_{Q\times Q}$ *all lie outside the unit circle.*

A2: *Let* $\boldsymbol{u}_t$ *in (3) be an i.i.d. process with* $\boldsymbol{u}_t = \Omega^{1/2}\boldsymbol{\eta}_t$*, where* $E(\boldsymbol{\eta}_t) = 0$, $\mathrm{var}(\boldsymbol{\eta}_t) = I_Q$*, and* $\Omega > 0$*,* $E(\|\boldsymbol{\eta}_t\|^4) < \infty$ *and the elements of* $\boldsymbol{\eta}_t$ *are mutually independent.*

Assumption B.

B1: *Let* $\lambda_{i,F}$ *be the* i*th column of* Λ_F *and let* $\lambda_{i,Y}$ *be the* i*th column of* Λ_Y*. Then, there exists a positive constant* C *large enough such that*$\|\lambda_{i,F}\| \le C < \infty$*,* $\|\lambda_{i,Y}\| \le C < \infty$ *for all* $1 \le i \le N$*.*

B2: $C^{-2} \le \sigma_i^2 \le C^2$ *for all* $1 \le i \le N$*, where* σ_i^2 *is defined in Assumption C.*

B3: *The kernel function* $K(\cdot)$ *is a bounded, symmetric density with a bounded support region.*

B4: $\lim_{N\to\infty} \frac{1}{N}\Lambda_F^T \Sigma_{ee}^{-1}\Lambda_F = \boldsymbol{Q}$ *exists and is a positive-definite matrix, where* Σ_{ee} *is defined in Assumption C. Furthermore,* $\lim_{n\to\infty} \frac{1}{n} F^T F = \Sigma_F$ *and* $\lim_{N\to\infty} \frac{1}{N}\Lambda_F^T \Lambda_F = \Sigma_{\Lambda_F}$ *exist and are diagonal matrices, and* $\mathcal{P}$ *in (7) is an upper or lower triangular matrix.*

Assumption C.

C1: $E(e_t) = 0$*;* $E(e_t e_t^T) = \Sigma_{ee} = \mathrm{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_N^2)$*;* $E(e_{it}^4) < \infty$ *for all* $1 \le i \le N$ *and* $1 \le t \le n$*. The errors* $\{e_{it}\}$ *are independent over* i *and* t*. The* $N \times 1$ *vector* e_t *is identically distributed over* t*. Furthermore,* e_{it} *is independent with* $\boldsymbol{u}_\tau$ *for all* i*,* t *and* $1 \le \tau \le n$*.*

C2: *Variances* σ_i^2 *are estimated in the compact set* $[C^{-2}, C^2]$*.*

Assumption A makes the regularity conditions on P_t. It guarantees that P_t is strictly stationary and α-mixing, which is similar to that in Chen and Tsay (1993) and Cai et al. (2000). Assumptions B and C are the same as that in Bai et al. (2016) and Yamamoto (2019). Assumption B is standard and is made on the factors and factor loadings. Notice that Assumption B4 requires the columns in Λ_F to be linearly independent. Otherwise, $\boldsymbol{Q}$ should be a singular matrix. In addition, Assumption B4 also guarantees that the rotation matrix H has no effect on the estimation of impulse response functions and the identification of the policy shock can be achieved in (3) as if it were a standard VAR. Assumption C centers on the idiosyncratic errors, allowing the correlations over time and cross-section and the heteroscedasticity over time to be ruled out. Finally, the following theorem is presented without proof, which might be derived in a similar way as in Cai and Liu (2020).

Theorem A.1. *Under Assumption A, if* $\mathbb{P}_0$ *is initialized from the invariant measure, then,* $\{\mathbb{P}_t\}$ *defined in* (2) *is a strictly stationary and* α*-mixing process.*