The Demand for Money for EMU: A Flexible Functional Form Approach

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Monetary aggregates have a special role under the "two pillar strategy" of the ECB. Hence, the need for a theoretically consistent measure of monetary aggregates for the European Monetary Union (EMU) is needed. This paper analyzes aggregation over monetary assets for the EMU. We aggregate over the monetary services for the EMU-11 countries, which include Estonia, Finland, France, Germany, Ireland, Italy, Luxembourg, Malta, Netherlands, Slovakia, and Slovenia. We adopt the Divisia monetary aggregation approach, which is consistent with index number theory and microeconomic aggregation theory. The result is a multilateral Divisia monetary aggregate in accordance with Barnett (2007). The multilateral Divisia monetary aggregate for the EMU-11 is found to be more informative and a better signal of economic trends than the corresponding simple sum aggregate. We then analyze substitutability among monetary assets for the EMU-11 within the framework of a representative consumer's utility function, using Barnett's (1983) locally flexible functional form, the minflex Laurent Indirect utility function. The analysis of elasticities with respect to the asset's user-cost prices shows that: (i) transaction balances (TB) and deposits redeemable at notice (DRN) are income elastic, (ii) the DRN display large variation in price elasticity, and (iii) the monetary assets are not good substitutes for each other within the EMU-11. Simple sum monetary aggregation assumes that component assets are perfect substitutes. Hence simple sum aggregation distorts measurement of the monetary aggregate. The ECB has Divisia monetary aggregates provided to the Governing Council at its meetings, but not to the public. Our European Divisia monetary aggregates will be expanded and refined, in collaboration with Wenjuan Chen at the Humboldt University of Berlin, to a complete EMU Divisia monetary aggregates database to be supplied to the public by the Center for Financial Stability in New York City.

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> “The simple arithmetic should not be used under any circumstances, being always biased and usually freakish as well. Nor should the simple aggregative ever be used; in fact this is even less reliable.” (cited in Barnett (2012, p. 37) from Fisher (1922, p. 361))

1. Introduction

The European Central Bank (ECB) is among a few central banks that attribute a special role to money under its two pillar strategy. Achieving and maintaining price level stability in the medium and long-term in the Euro area is the goal of ECB, which it achieves by economic analysis and monetary analysis. Monetary analysis, which is one of its pillars, includes analyzing monetary aggregates. Money is found to play a prominent role in the Euro area’s long run policy. The long run correlation between money growth and inflation appears to be robust to different policy regimes. See, e.g., Benati 2008,2009). Many studies with European data have confirmed the relationship between monetary growth and inflation.[1]

The need for a measure of monetary aggregates, which are theoretically consistent for economic unions, such as the EU and EMU, is highly relevant. The field of monetary aggregation and index number theory were first rigorously connected with the literature on microeconomic aggregation and index number theory by Barnett (1980). His initial paper is based on the assumption that the data were produced by a single closed economy. Subsequent studies with those data demonstrated that Divisia monetary aggregates are better measures than simple sum monetary aggregates in terms of policy criteria, such as causality and information content of the aggregate and stability of money demand equations. See, e.g., Barnett, Offenbacher and Spindt (1981, 1984), Belongia and Ireland (2006, 2014, 2015a,b,2016), Serletis and Rahman (2013), and Serletis and Gogas (2014).

In Barnett (2003, 2007), the theory for construction of Divisia monetary aggregates for the Euro area was developed. The theory for a single country was extended to the multi-country case, for an economic union, both prior to and after the introduction of common currency. A few studies have used Divisia monetary aggregates for the Euro area, such as Stracca (2004) and Darvas (2015), but under

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[1] Neumann and Greiber (2004); Bruggeman Camba-Mandez, Fischer and Sausa (2005); Gerlach and Assenmacher-Wesche (2005).

restrictive assumptions. In this paper, we develop multilateral Divisia monetary aggregates for the European Monetary Union (EMU) eleven countries (Estonia, Finland, France, Germany, Ireland, Italy, Luxembourg, Malta, Netherlands, Slovakia, and Slovenia), following Barnett (2003, 2007). We find that the resulting multilateral Divisia monetary aggregates are more informative than the corresponding simple sum aggregates. Following Barnett's work on multilateral Divisia monetary aggregates for the EMU, the ECB began producing and providing Divisia monetary aggregates to its Governing Council for use at its meetings, but the ECB does not provide those aggregates to the public. Our European Divisia monetary aggregates will be expanded and refined, in collaboration with Wenjuan Chen at the Humboldt University of Berlin, to a complete EMU Divisia monetary aggregates database to be supplied to the public by the Center for Financial Stability in New York City.

A basic question that this paper answers is: are the simple sum aggregates for the EMU-11 justified? A necessary condition would be that the monetary assets within the EMU-11 are perfectly substitutable. In this paper, the substitutability of the monetary assets within the EMU-11 is investigated using minflex Laurent consumer demand model, a flexible functional form. In the United States, the available results have shown that the monetary assets are not good substitutes. See, e.g., Serletis and Robb (1986). With US data, Serletis and Shahmoradi (2007) have used various functional forms for consumer demand modeling of money demand, including the generalized Leontief, the translog, and the almost Ideal demand system. For the Euro Area, extensive literature exists on the demand for money. Much of that work uses linear combinations of variables, including inflation, output gap, interest rate, and monetary aggregates (e.g., Stracca 2004). A few studies have also included wealth (e.g., Beyer (2008) and Boone et al (2004)).

The paper proceeds to discuss our multilateral Divisia monetary aggregates for the EMU-11. We then discuss the minflex Laurent model, our estimation procedure, our results, and conclusions.

2. EMU-11 Divisia Monetary Aggregation

We implement multilateral Divisia monetary aggregates for the eleven countries of European Monetary Union (EMU): Estonia, Finland, France, Germany, Ireland, Italy, Luxembourg, Malta, Netherlands, Slovakia, and Slovenia. The following are the theoretical results we use from Barnett (2003, 2007). The theory was developed for use both prior to and after the introduction of the common currency. The theory progresses from a general heterogeneous agents' approach, to a multilateral agent approach, to the most restrictive case of unilateral agent, representing the direction in which the EMU is planned to progress into the future.

2.1. **Aggregation Within Euro Area**

Let m_{kji} be the nominal per capita holdings of asset $i \in \{1, 2, ..., N\}$ located or purchased in country $j \in \{1, 2, ..., K+Z\}$, where Z is the number of relevant countries that are outside economic union, K be the number of countries in the economic union, and r_{kji} is the holding period after tax yield on asset i located or purchased in country j and owned by an economic agent in country k. In this study, *N* = 4 and *K* = 11. Let R_k be the benchmark rate of return in country k, where the benchmark rate of return is received on a pure investment providing no services other than its yield.[2] The real user-cost price of asset i, first derived by Barnett (1978), is

$$\pi^*_{kji}(t) = (R_k(t) - r_{kji}(t)) / (1 + R_k(t)). \tag{1}$$

In this application, the real user-cost price is of asset i located or purchased in country j and owned by residents of country k at time t, while $\pi_{kji} = p^*_k \pi^*_{kji}$ is the corresponding nominal user-cost. The user-cost of a monetary asset measures the foregone interest or opportunity cost of holding monetary asset i, when the higher yielding benchmark asset could have been held.

We define the set $S_k = \{(i, j); m_{kji} > 0 \ \textit{for all} \ \ i, j\}$. Then the real per-capita monetary services aggregate M^*_k and the nominal per-capita monetary services aggregate M_k for each country k are

$$d \log M^*_k = \sum_{(i,j) \in S_k} w_{kji} d \log m^*_{kji} \ , \tag{2}$$

$$d \log M_k = \sum_{(j,i) \in S_k} w_{kji} d \log m_{kji} \ . \tag{3}$$

[2] A detailed discussion about the benchmark rate is in Appendix B.

Similarly, the monetary real user-cost price aggregate, Π_k^*, and the monetary nominal user-cost price aggregate, Π_k, are

$$d\log \Pi_k^* = \sum_{(j,i)\in S_k} w_{kji} d\log \pi_{kji}^* \ , \tag{4}$$

$$d\log \Pi_k = \sum_{(j,i)\in S_k} w_{kji} d\log \pi_{kji} \ , \tag{5}$$

where the expenditure shares,

$$w_{kji} = \frac{\pi_{kji} m_{kji}^*}{\boldsymbol{\pi}_k \mathbf{m}_k^*} = \frac{\pi_{kji}^* m_{kji}^*}{\boldsymbol{\pi}_k^* \mathbf{m}_k^*} = \frac{(R_k - r_{kji}) m_{kji}^*}{\sum_{(j,i)\in S_k} (R_k - r_{kji}) m_{kji}^*} = \frac{(R_k - r_{kji}) m_{kji}}{\sum_{(j,i)\in S_k} (R_k - r_{kji}) m_{kji}}$$ and $0 \le w_{kji} \le 1$ for all $k \in \{1,...,K\}$, $j \in \{1,...,K+Z\}$, and $i \in \{1,...,N\}$. Also it follows that $\sum_{(j,i)\in S_k} w_{kji} = 1$ for all k.

Figure 1 displays the year-over-year percentage change of the Divisia M2 aggregate for the 11 EMU countries. The M2 aggregate has component assets of currency in circulation, overnight deposits, deposits with agreed maturity, and deposits redeemable at notice.[3]

2.2. **Aggregation Over Countries**

The euro area's nominal per-capita monetary service flow, M, and real per-capita monetary service flow, M^*, are given by

$$d\log M = \sum_{k=1}^{K} W_k d\log(s_k M_k e_k) \ , \tag{6}$$

$$d\log M^* = \sum_{k=1}^{K} W_k d\log(s_k M_k^*) \ . \tag{7}$$

Similarly the euro area's nominal monetary user-cost price, Π, and real monetary user-cost prices, Π^*, are

$$d\log \Pi = \sum_{k=1}^{K} W_k d\log(\Pi_k e_k) \ , \tag{8}$$

[3] Appendix A provides the definition of these monetary assets in accordance with the ECB glossary.

$$d\log\Pi^* = \sum_{k=1}^{K} W_k d\log(\Pi_k^*) \quad . \tag{9}$$

where $s_k = H_k / \sum_{k=1}^{K} H_k$ is country k's share of total economic union population, and H_k is the population of country k. The variable e_k is the exchange rate of country k's currency relative to a market basket of currencies. In this analysis, since all the countries of the EMU-11 now use the same currency, the euro, e_k becomes 1.0. Country k's expenditure share of the economic union's monetary services flow is given by $W_k = \frac{M_k^* \Pi_k e_k s_k}{\sum_{k=1}^{K} M_k^* \Pi_k e_k s_k}$.

The corresponding discrete time Divisia index replaces the differentials $d\log(z_t)$ by finite changes $\log(z_t) - \log(z_{t-1})$ and replaces W_{kt} by $(W_{kt} + W_{k,t-1})/2$. The resulting index is the Törnqvist approximation to the continuous time Divisia index.

These results are in per-capita form, in accordance with the representative agent theory from which the theory was derived and as is used in this paper for modeling the demand for money. Figure 2 shows our EMU-11 multilateral Divisia monetary aggregates, aggregating over currency in circulation, overnight deposits, deposits with agreed maturity, and deposits redeemable at notice. The figure also displays the corresponding simple sum aggregates.

3. The Data and the Variables

The Euro zone was formed in 1999, but the data for some of the monetary services and their corresponding rates of return are not available until January 2003. Hence, our data for the EMU countries begin in January 2003. Our data for the monetary services, their corresponding rates of return, and the populations and consumer prices of the eleven EMU countries are acquired from the Statistical Data Warehouse, which is the source provided on the ECB's website. Apart from the ECB, the central banks of the member countries are also sources of some of our data. Our household data on deposits and interest rates are from the ECB for deposits at Monetary and Financial Institutions (MFI). Our currency data are from the central banks of member countries.

Barnett and Chauvet (2011) observe that from the 1960s to 2005, the U.S. monetary aggregates and their Divisia counterparts diverge more during periods of high uncertainty than in times of stability. They suggest that this divergence can provide a signal for impending financial instability. For the U.K.

Rayton and Pavlyk (2010) demonstrate that the Divisia and simple sum monetary aggregates did not correlate at the start of the recent crisis. During the Great Recession in Germany, Chan and Nautz (2015) found that the information content of the two indices diverged for the recession period.

Our Divisia monetary aggregate for all the EMU-11 countries showed similar divergence, as seen in figure 1. This divergence is evident for all the eleven countries in the union. In addition, the Divisia monetary aggregates' growth rates were lower than the simple sum monetary aggregates' growth rates prior to the start of the Great Recession.

4. The Consumer's Maximization Problem

Our assumptions are sufficient for two stage budgeting, as introduced by Strotz (1957, 1959) and Gorman (1959). Hence consumers behave as if they were using sequential expenditure allocation. In the first stage, expenditure allocation is to broad categories. In the second stage, the expenditure allocation is within each broad category.

In the economy, individuals allocate over three types of goods and services: consumption goods, leisure, and the monetary asset services. The services from three enter the representative individual's utility function,

$$u = u(\mathbf{c}, l, \mathbf{m}) \tag{10}$$

where $\mathbf{c}$ is the vector of services of consumption goods, l is leisure, and $\mathbf{m}$ is the vector of services of monetary assets. The consumer maximizes utility subject to the budget constraint, $\mathbf{q}'\mathbf{c} + wl + \mathbf{p}'\mathbf{m} = z$ where $\mathbf{q}$ is the vector of prices of the consumption goods $\mathbf{c}$, w is the wage rate, $\mathbf{p}$ is a vector of user-cost of the monetary services $\mathbf{m}$, and z is the quantity of expenditure allocated to the current period in the prior stage intertemporal allocation.

The vector of monetary services is assumed to be weakly separable from consumption goods and leisure.[4] Hence equation (10) can be written as $u = u(\mathbf{c}, l, f(\mathbf{m}))$ where $f(\mathbf{m})$ is the aggregator function over monetary services. That aggregator function is assumed to be continuous and twice differentiable. Weak separability in $\mathbf{m}$ requires $\partial(\frac{\partial u / \partial m_i}{\partial u / \partial m_j}) / \partial\xi = 0$ for $\xi = c_i, l$. The consumer's

[4] A substantial literature exists on testing the hypothesis of blockwise weak separability. See, e.g., Hjertstrand, Swofford, and Whitney (2016) and Cherchye, Demuynck, Rock, and Hjerstrand (2015).

second stage utility maximization problem can be written as $\max_{\mathbf{m}} f(\mathbf{m})$ subject to $\mathbf{p}'\mathbf{m} = y$, where $\mathbf{m} = (m_1, m_2, m_3)$ is the vector of monetary assets, with m_1 = transaction balances, m_2 = deposits with agreed maturity (DAM), and m_3 = deposits redeemable at notice (DRN); $\mathbf{p} = (p_1, p_2, p_3)$ is the corresponding vector of user-costs, and y is the total expenditure on monetary assets allocated during the first stage allocation of z over the three categories of goods and services.

4.1. **Minflex Laurent Model**

The minflex Lauren model, originated by Barnett (1983), is a special case of the Full Laurent model. Barnett and Lee (1985) showed that among the three flexible functional forms, translog, generalized Leontief, and minflex Laurent, the minflex Laurent model has the largest regular region, and its regular region expands as real income grows. We use the minflex Laurent model to estimate the demand for money with our European data. The full Laurent reciprocal indirect utility function is given by

$$V(\mathbf{v}) = a_0 + 2\sum_{i=1}^{n} a_i v_i^{1/2} + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} v_i^{1/2} v_j^{1/2} - 2\sum_{i=1}^{n} b_i v_i^{-1/2} - \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} v_i^{-1/2} v_j^{-1/2}, \qquad (11)$$

where a_0, a_i, a_{ij}, b_i, b_{ij} are unknown parameters, and v_i and v_j denote the income normalized prices, p_i / y and p_j / y respectively.

By assuming that $b_i = 0$, $b_{ii} = 0$ for all i, $a_{ij} b_{ij} = 0$ for all i, j, and forcing the off diagonal elements of the symmetric matrices $A \equiv [a_{ij}]$ and $B \equiv [b_{ij}]$ to be nonnegative, equation (11) reduces to the minflex Laurent reciprocal indirect utility function

$$V(\mathbf{v}) = a_0 + 2\sum_{i=1}^{n} a_i v_i^{1/2} + \sum_{i=1}^{n} a_{ii} v_i + \sum_{i=1_{i \neq j}}^{n}\sum_{j=1}^{n} a_{ij}^2 v_i^{1/2} v_j^{1/2} - \sum_{i=1_{i \neq j}}^{n}\sum_{j=1}^{n} b_{ij}^2 v_i^{-1/2} v_j^{-1/2}. \qquad (12)$$

By applying Roy's identity to the equations of the indirect utility function of minflex Laurent, the share equations are

$$s_i = \frac{a_i v_i^{1/2} + a_{ii} v_i + \sum_{\substack{j=1 \\ i \neq j}}^{n} a_{ij}^2 v_i^{1/2} v_j^{1/2} + \sum_{\substack{j=1 \\ i \neq j}}^{n} b_{ij}^2 v_i^{-1/2} v_j^{-1/2}}{\sum_{i=1}^{n} a_i v_i^{1/2} + \sum_{i=1}^{n} a_{ii} v_i + \sum_{i=1_{i \neq j}}^{n} \sum_{j=1}^{n} a_{ij}^2 v_i^{1/2} v_j^{1/2} + \sum_{i=1_{i \neq j}}^{n} \sum_{j=1}^{n} b_{ij}^2 v_i^{-1/2} v_j^{-1/2}} \ . \tag{13}$$

Since the share equations are homogenous of degree zero in the parameters, a normalization is required. Following Barnett and Lee (1985), we impose the following normalization in the estimation of the share equations

$$\sum_{i=1}^{n} a_{ii} + 2\sum_{i=1}^{n} a_i + \sum_{\substack{j=1 \\ i \neq j}}^{n} a_{ij}^2 - \sum_{\substack{j=1 \\ i \neq j}}^{n} b_{ij}^2 = 1 \ . \tag{14}$$

5. Estimation Procedure

The three monetary assets in the consumer utility function are transaction balances (computed as a Divisia aggregate over currency in circulation and overnight deposits), deposits with agreed maturity, and deposits redeemable at notice. The user-costs for these monetary assets are computed using equation 9. To estimate the share equation system, (13), a stochastic version is specified. We assume that the observed share in the i^{th} equation deviates from the true share by an additive term, u_i .

We assume $\mathbf{u} \sim N(\mathbf{0}, \mathbf{\Omega})$, where $\mathbf{u} = (u_1, ..., u_n)'$, $\mathbf{0}$ is a null matrix, and $\mathbf{\Omega}$ is the $n \times n$ symmetric positive definite error covariance matrix. The share equations, (13), can be written as

$$\mathbf{s} = \mathbf{g}(\mathbf{v}; \boldsymbol{\theta}) + \mathbf{u} \ , \tag{15}$$

where $\boldsymbol{\theta}$ is the parameter vector to be estimated. The fact that the budget shares s_i sum to 1, implies that the disturbance covariance matrix is singular. Barten (1969) has shown that full information maximum likelihood estimates of the parameters can be obtained by arbitrarily deleting one equation from the system. The parameters in this paper are estimated following Barten (1969). Estimation is performed using nonlinear full-information maximum likelihood estimation with the TSP (version 5.1) program.

5.1. **Estimation Results**

Tables 1 to 5 show the estimated parameters and the elasticity estimates. Table 1 shows the parameter estimates of the minflex Laurent model. The positivity condition is checked by computing the indirect utility function, to confirm $V(\mathbf{v}) > 0$ for all t. The monotonicity is checked by computing the gradient vector, to confirm $V'(v) < 0$ for all t. Curvature is checked by examining negative semidefiniteness of the Allen elasticities of substitution matrix. The elasticities can be calculated from the estimated budget share equations, which can be written as $x_i = \frac{s_i y}{p_i}$.[5] The elasticity values are evaluated at the mean values of the variables. The values in the parenthesis are t-values (estimate divided by standard error) of the estimates.

The income elasticity is calculated by

$$\eta_{iy} = 1 + \frac{y}{s_i}\frac{\partial s_i}{\partial y} \qquad (16)$$

for $i = 1,2,3$. Table 2 shows that the income elasticities for the three monetary assets are positive, so that the monetary assets are normal goods. The transaction balances (TB) and deposits with agreed maturity (DAM) are income elastic, with income elasticities exceeding 1.0, while deposits redeemable at notice (DRN) are income inelastic, with income elasticities less than 1.0. Income elasticity paths over time are shown in Figure 3. The elasticity of TB with respect to income is high and does not display much variation, attaining its highest value of 1.182 and lowest value of 1.223. The elasticity of DAM with respect to income displayed its highest value of 1.77 for period April 2009 and lowest value of 1.179 for January 2004. The elasticity of DRN with respect to income is uniformly low, attaining its highest value of 0.64 for period January 2004 and lowest value, 0.285, for April 2009.

The Marshallian (uncompensated) price elasticities are calculated from

$$\eta_{ij} = \left(\frac{p_j}{s_i}\right)\left(\frac{\partial s_i}{\partial p_j}\right) - \delta_{ij} \;, \qquad (17)$$

[5] Appendix C shows the income and price elasticity derived from the estimated budget share equation.

where $i = 1, 2, 3$; and δ_{ij} is the Kronecker delta, so that $\delta_{ij} = 1 \text{ if } i = j$ and $\delta_{ij} = 0 \text{ if } i \neq j$. The own and cross price elasticities are shown in Table 3. All own price elasticities are negative, ruling out Giffen goods. All cross-price elasticities are negative, so that all of the assets are found to be gross complements. All price elasticities are less than 1.0 in absolute value, so all of the monetary assets are price inelastic. The income and price elasticities are consistent with the results in Serletis and Robb (1986) in case of US.

Elasticity of TB with respect to the user-costs over time is shown in Figure 4, top left graph. The own-price elasticity of TB is low, although it increased slightly in January 2009 to -0.48. The elasticity of TB with respect to the user-cost of DAM is comparatively high and does not show much variation, although it fell slightly in January 2009 to -0.18. The elasticity of TB with respect to the user-cost of DRN showed high variation, with highest value, -0.28, in January 2014 and lowest value, -0.57, in January 2009. Elasticity of DAM with respect to the user-costs over time is shown in Figure 4, top right graph. The own-price elasticity of DAM is low, with its highest value, -0.37 in January 2009 and lowest value, -0.58 in May 2009. The elasticity of DAM with respect to the user-costs of TB is comparatively high, with its lowest value, -0.34, in July 2003 and its highest value, -0.095, in April 2009. The elasticity of DAM with respect to the user-costs of DRN displays high variation, increasing to -0.07 in May 2009 and decreasing to -0.46 in July 2003. Price Elasticity of DRN with respect to the user-costs over time is shown in Figure 4, bottom left. The own-price elasticity of DRN is low but very volatile, with a sharp increase to -0.26 in January 2009 and a decrease to -0.55 in October 2013. The elasticity of DRN with respect to the user-cost of TB is less volatile, with its highest value, -0.33 in October 2003 and its lowest value, -0.48 in April 2009. The elasticity of DRN with respect to the user-cost of the DAM is comparatively high and less volatile, with its highest value, -0.105 in May 2009 and its lowest value, -0.25 in November 2008.

The own-Allen elasticities of substitution are negative, as is consistent with theory. The Allen cross elasticities are positive and less than 1.0 indicating that monetary assets are weak substitutes, but far from perfect substitutes. Allen elasticities of substitution over time are shown in Figure 5. The cross elasticities of substitution are positive and less than 1.0, indicating that the monetary assets are weak substitutes but far from perfect substitutes. Substitutability shows sharp decrease in 2009 and 2013.

Blackorby and Russell (1989) have shown that cross Allen elasticity of substitution may provide ambiguous information and suggest that Morishima elasticity of substitution may be a better measure of substitutability. Blackorby and Russell (1989) and Serletis and Shahmoradi (2005) advocate computing

Morishima elasticities using the equation $\sigma_{ij}^{m}(p,y) = s_i(p,y)((\sigma_{ji}^{a}(p,y) - \sigma_{ii}^{a}(p,y))$, where $\sigma_{ji}^{a}(p,y)$ and $\sigma_{ii}^{a}(p,y)$ are Allen elasticity of substitution. The Morishima elasticity measures the net change in the compensated demand for good j, when the price of good i changes. Goods will be Morishima complements (substitutes) if an increase in the price of i causes x_i/x_j to decrease (increase). By this measure, all three monetary services are found to be weak substitutes, but far from perfect substitutes, with the Morishima elasticity of substitution being less than 1.0.

6. Conclusion

This paper examines the monetary services in the European Monetary Union (EMU) of 11 countries. We aggregated over the monetary services of currency in circulation, overnight deposits, and deposits with agreed maturity and deposits redeemable at notice. We produce the Divisia monetary aggregates, which are consistent with index number theory and economic aggregation theory. The multilateral Divisia monetary aggregate for EMU-11 is more informative than the simple sum aggregate and is a good signal of economic trends. When the country Divisia monetary aggregates and the EMU-11 multilateral Divisia monetary aggregate diverge from the simple sum aggregates, the results signaled the recent economic crisis, as observed with US data by Barnett and Chauvet (2011).

The monetary assets are further analyzed within the framework of a representative consumer's utility function, specified by the minflex Laurent Indirect utility function. We find that the monetary assets are weak substitutes, far from perfect substitutes. As a result, the theoretically correct monetary aggregate cannot be linear, and certainly cannot be simple sum.

In the two-pillar strategy of ECB, monetary aggregates are important in maintaining price stability in the long run. Although the ECB has Divisia monetary aggregates, provided to its Governing Council in its meetings, ECB makes available to the public only simple sum monetary aggregates, which are inappropriate measures of the monetary service flow. Public availability of the Divisia monetary aggregates for Euro area would be valuable in research on monetary policy and in inflation forecasts. Divisia monetary aggregates for the United States are currently provided to the public by the Center for Financial Stability (CFS) in New York City through regular monthly releases. See Barnett, Liu, Mattson, and van den Noort (2013). Our European Divisia monetary aggregates will be expanded and refined, in collaboration with Wenjuan Chen at the Humboldt University of Berlin, to a complete EMU Divisia monetary aggregates database to be supplied to the public by the Center for Financial Stability in New York City.

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Table 1

Minflex Laurent model parameter estimates.

Variable	Estimate
a1	0.002406
	(0.172985)
a2	-0.003857
	(-1.03508)
a3	-0.006280
	(-0.88275)
a11	0.28184
	(2.42884)
a22	0.259754
	(3.96397)
a33	0.473870
	(3.72716)
a31	0.00028649
	(0.00000159944)
b21	0.00081911
	(5.26623)
Loglikelihood	802.568
Positivity violations	0
Monotonocity violations	0
Curvature violations	59

Numbers in parentheses are t-values. Sample period: 2003-01 to 2014-01

Table 2

Estimated Income elasticities

Monetary Asset	Income Elasticity
Transaction balances	1.21652
	(26.8955)
Deposits with agreed maturity	1.34478
	(9.08806)
Deposits redeemable at notice	0.536689
	(11.0111)

Numbers in parentheses are t-values. Sample period: 2003-01 to 2014-01

Table 3

Estimated price elasticities for the monetary assets

Monetary Assets *i*	η_{i1}	η_{i2}	η_{i3}
Transaction balances	-0.511041	-0.142338	-0.346621
	(-24.0263)	(-3.83083)	(-15.5284)
Deposits with agreed maturity	-0.245840	-0.461104	-0.293057
	(-3.94550)	(-4.20578)	(-5.75193)
Deposits redeemable at notice	-0.376363	-0.196048	-0.427589
	(-16.4524)	(-6.21254)	(-22.7411)

Numbers in parentheses are t-values. Sample period: 2003-01 to 2014-01

Table 4

Estimated Allen elasticity of substitution for the monetary assets

Monetary Assets i	σ^a_{i1}	σ^a_{i2}	σ^a_{i3}
Transaction balances	-0.063959	0.617414	0.262455
	(-1.10478)	(3.24950)	(7.48267)
Deposits with agreed maturity		-0.596014	0.538153
		(-1.02958)	(6.54606)
Deposits redeemable at notice			-0.640232
			(-12.9860)

Numbers in parentheses are t-values. Sample period: 2003-01 to 2014-01

Table 5

Estimated Morishima elasticity of substitution for the monetary assets

Monetary Assets i	σ^m_{i1}	σ^m_{i2}	σ^m_{i3}
Transaction balances		0.271938	0.130272
		(3.34982)	(5.40248)
Deposits with agreed maturity	0.288293		0.269461
	(1.58051)		(1.89831)
Deposits redeemable at notice	0.327957	0.428121	
	(13.3180)	(9.60369)	

Numbers in parentheses are t-values. Sample period: 2003-01 to 2014-01

Figure 1

Year-over-year percentage change of the Divisia and simple sum monetary aggregates for EMU-11 countries.

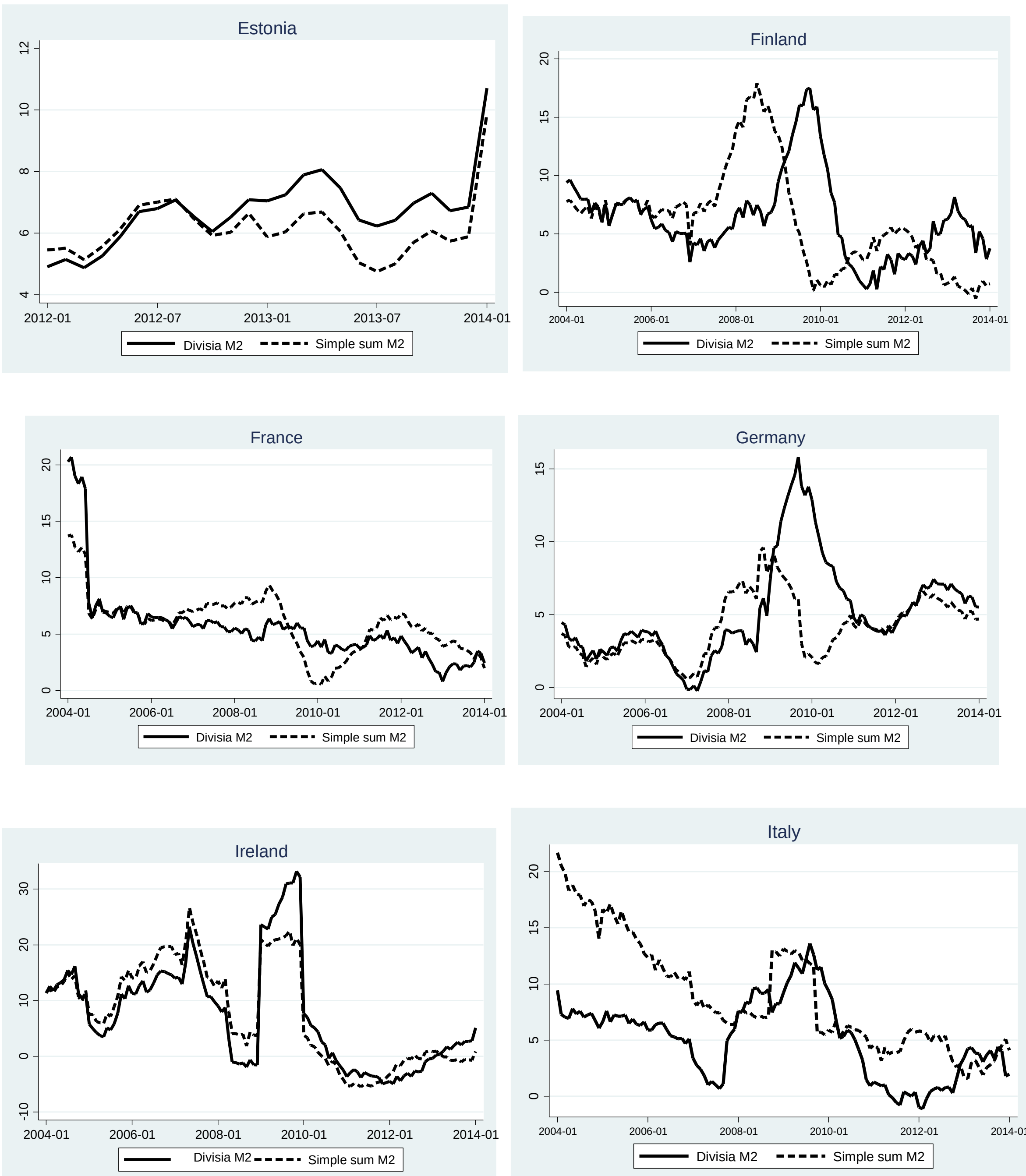


Figure 1 Continued

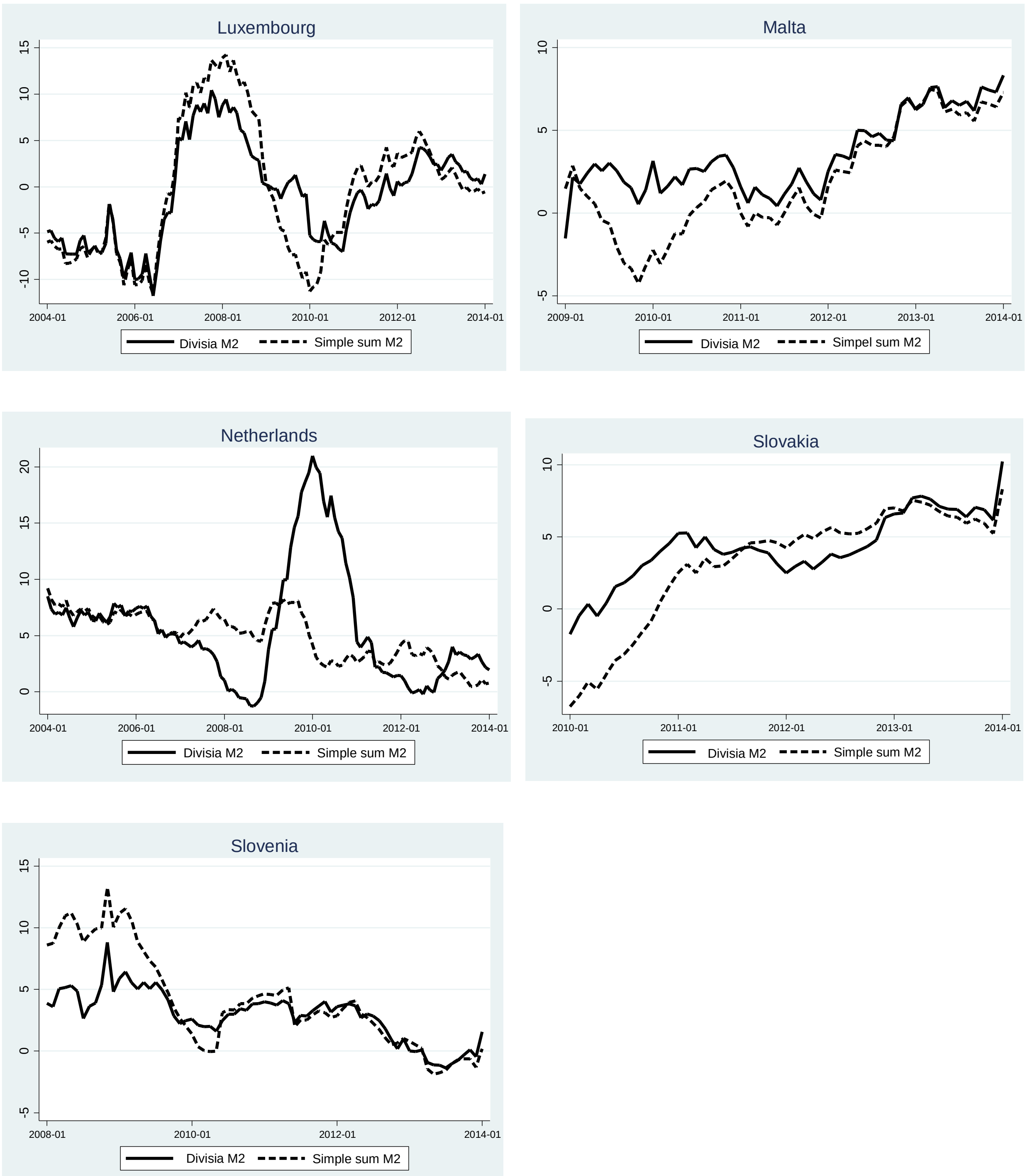


Figure 2

Year-over-year percentage change in Divisia and simple sum monetary aggregate for EMU-11

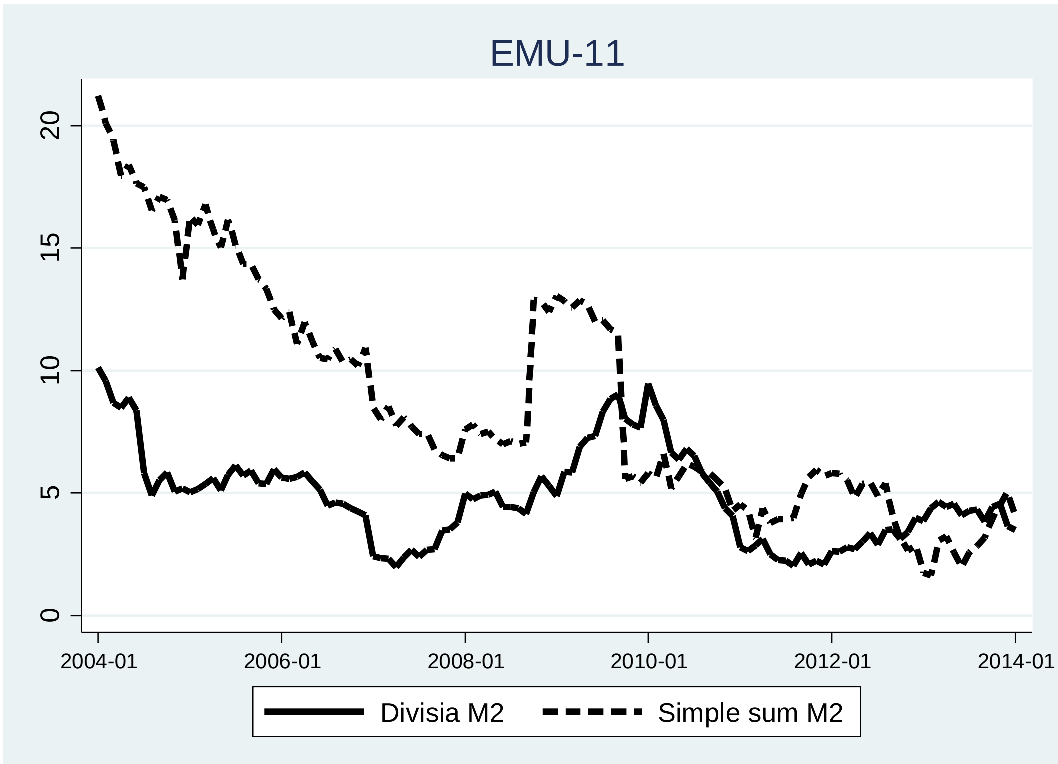


Figure 3

Income elasticity

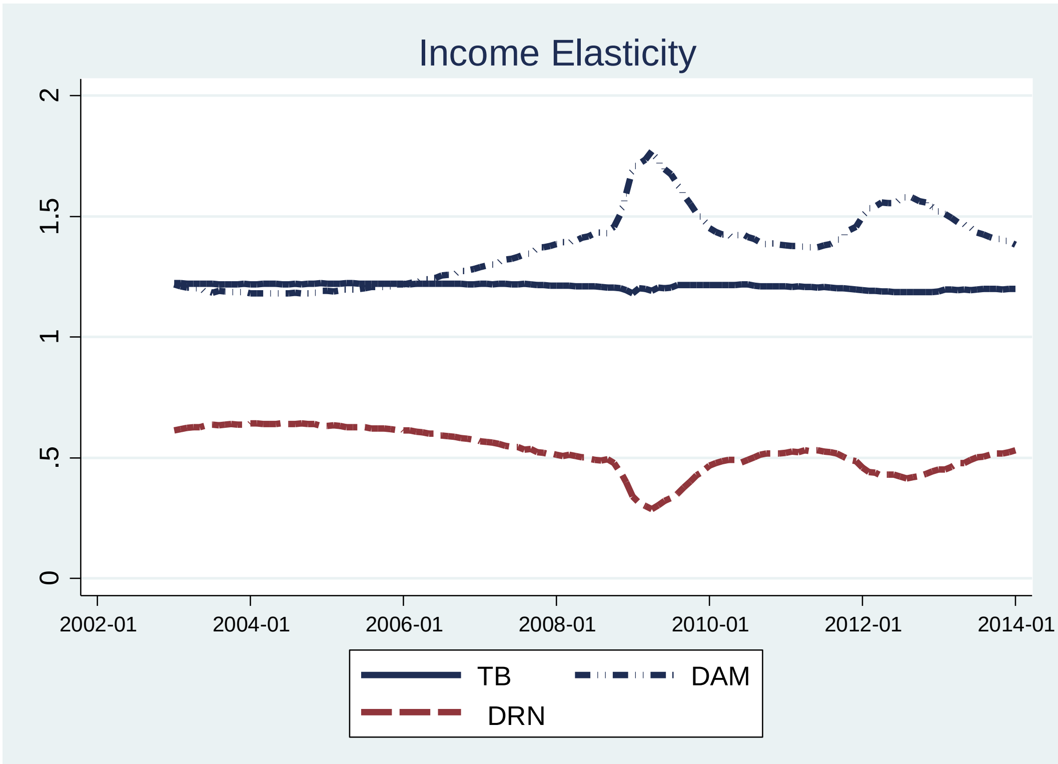
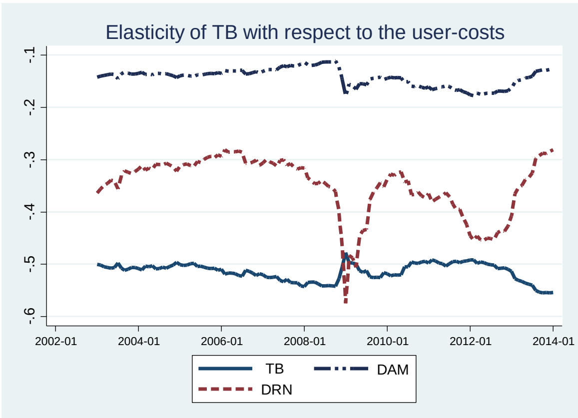
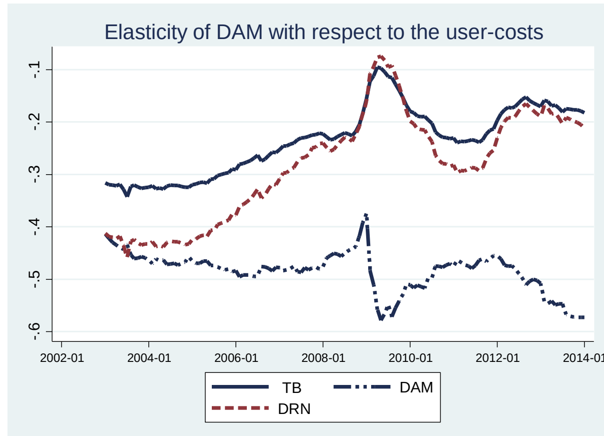


Figure 4

Elasticity of monetary assets with respect to the user-costs





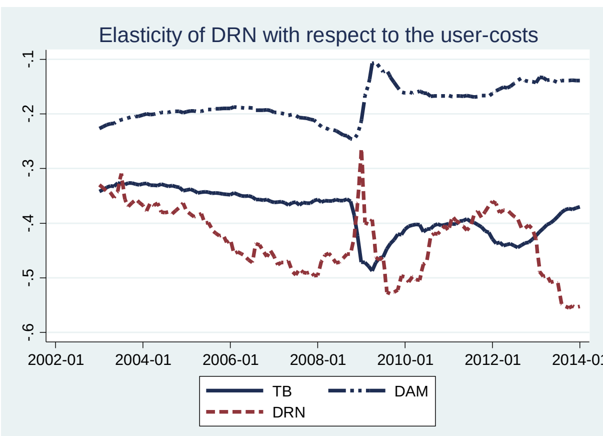
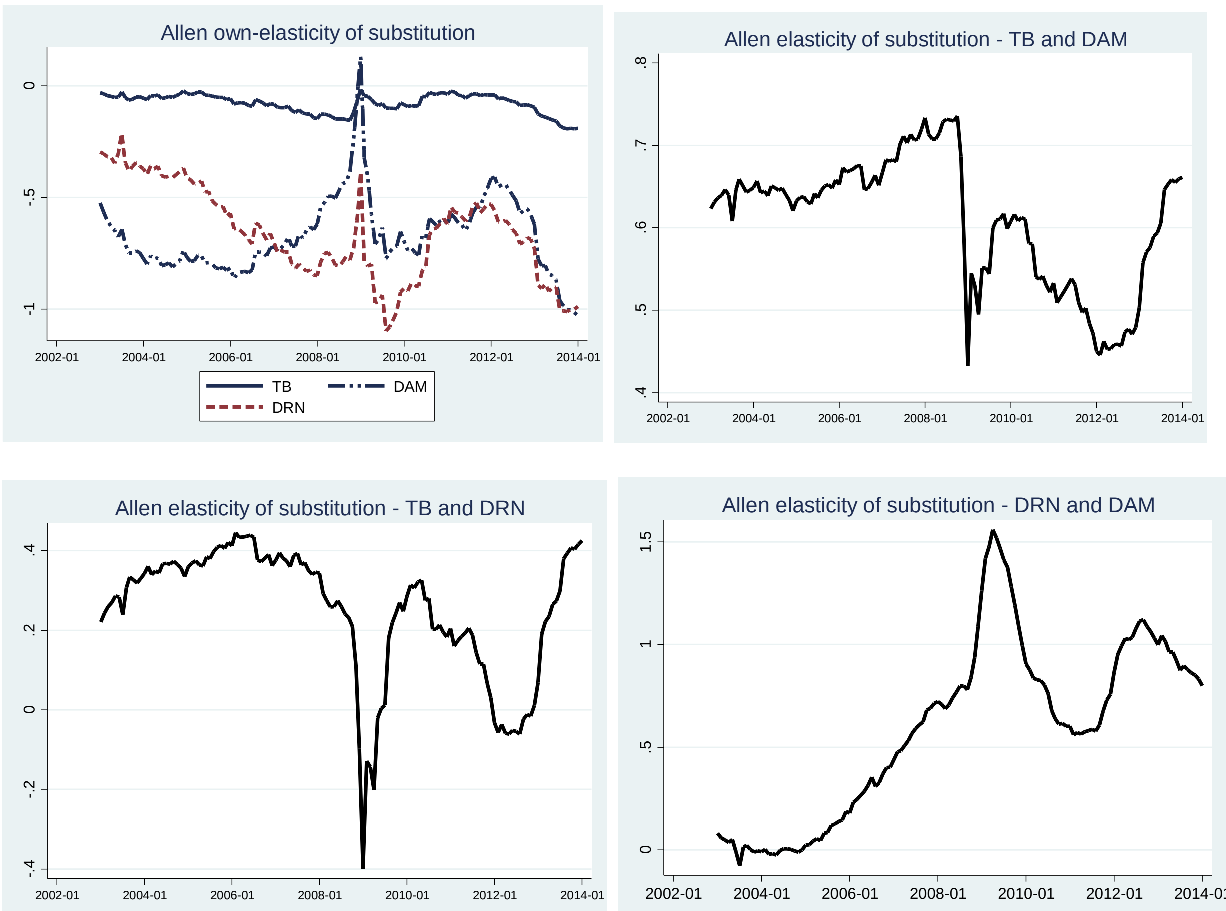


Figure 5

Allen elasticity of substitution



Appendices

Appendix A

Definitions:

Monetary and financial institutions (MFI) from the ECB Glossary: MFIs are Central Bank, resident credit institutions as defined by community law, and other resident financial institutions whose business is to receive deposits and /or close substitutes for deposits from entities other than MFIs and for their own account to grant credits and/or make investments in securities.

Overnight deposits from the ECB Glossary, deposits with next-day maturity: This instrument category comprises mainly those sight/demand deposits that are fully transferable by check or similar instrument. It also includes non-transferable deposits that are convertible on demand or by close of business the following day. Overnight deposits are included in M1 and hence in M2 and M3.

Deposits redeemable at notice (DRN) from the ECB Glossary: These deposits are savings deposits for which the holder must respect a fixed period of notice before withdrawing the funds. In some cases there is the possibility of withdrawing on demand a certain fixed amount in a specified period or of early withdrawal subject to the payment of a penalty. Deposits redeemable at a period of notice up to three months are included in M2 and hence in M3, while those with a longer period of notice are part of the non-monetary longer term financial liabilities of the MFI sector.

Deposits with an agreed maturity (DAM) from the ECB Glossary: These deposits are mainly time deposits with a given maturity that, depending on national practices, may be subject to the payment of a penalty in the event of early withdrawal. Some non-marketable debt instruments, such as non-transferable retail certificates of deposit, are also included. Deposits with an agreed maturity of up to two years are included in M2 and hence in M3, while those with an agreed maturity of over two years are included in the non-monetary long term financial liabilities of the MFI sector.

Non-profit institutions serving households (NPISH) from the Eurostat Glossary: These institutions make up an institutional sector in the context of national accounts consisting of non-profit institutions which are not mainly financed and controlled by government and which provide goods or services to households for free or at prices that are not economically significant. Examples include churches and religious societies, sports and other clubs, trade unions, and political parties. NPISH are private, non-market producers which are separate legal entities. Their main resources, apart from those derived from occasional sales, are derived from voluntary contributions in cash or in kind from households in their capacity as consumers, from payments made by general governments, and from property income.

Non-financial corporation (NFC) from the ECB Glossary: These firms are corporation or quasi-corporation that is not engaged in financial intermediation but is active primarily in the production of market goods and non-financial services.

Appendix B

The M1 monetary aggregate contains the most liquid monetary asset components. The ECB has defined the M1 monetary aggregate to include currency in circulation and overnight deposits. Overnight deposits are deposits with next-day maturity and comprises mainly of sight deposits or demand deposits which are fully transferable by check or similar instruments. The ECB definition of the M2 aggregate includes currency in circulation, overnight deposits, deposits with agreed maturity (DAM) up to 2 years and deposits redeemable at notice (DRN) up to 3 months . The data on currency in circulation is taken from the individual countries' central banks, as currency held by banks. The ECB website does not provide the data on currency in circulation. The data for the outstanding amount of OD, DAM, and DRN are for the households and non-profit institutions serving households. The data for the interest rate for OD, DAM and DRN are the Monetary Financial Institutions (MFI) interest rates for the households and non-profit institutions serving households. Currency is assumed is to have a zero own rate of return. The outstanding amount and interest rate data for OD, DAM and DRN are from the ECB Data Warehouse.

The benchmark rate is the expected rate of return received on a pure investment providing no services other that its yield. In short, the benchmark rate is the rate of return on pure capital. Since it provides no services other than its yield, the benchmark rate must be at least as high as the upper envelope over all the monetary aggregate's component yield-curve-adjusted rates of return. In that upper envelope, we also include the interest rate on loans of maturity of up to one year.

In case of a few countries like Finland, France, and the Netherlands, the interest rate on deposits with agreed maturity of 2 years was greater than the loan rate for a few months. For those periods, 100 basis points were added to the upper envelope to keep the user costs from becoming zero. This procedure is in accordance with Anderson and Jones (2011). In the case of Finland, the interest rate on DAM was higher than the loan rate for two periods of up to one year. For DAM and DRN, those periods were January 2009 to September 2009 and March 2012 to October 2012. For those periods, 0.01 point is added to the loan rate, so that the benchmark rate is highest of all the rates of return on monetary assets. The corresponding periods for France are March 2009 to January 2011 and December 2011 to January 2011. For the Netherlands, the periods are January 2009 to June 2010 and January 2012 to October 2013.

Appendix C

Income elasticity and price elasticity are computed from the budget share equation, which can be written as $x_i = \frac{s_i y}{p_i}$, where $i = 1,2,3$ or logarithmically $\log x_i = \log s_i + \log y - \log p_i$. The income elasticity can be calculated as

$$\eta_{iy} = \frac{\partial \log x_i}{\partial \log y} = \frac{\partial \log s_i}{\partial \log y} + 1, \text{ where } i = 1,2,3 .$$

The own price elasticity can be computed by

$$\eta_{ii} = \frac{\partial \log x_i}{\partial \log p_i} = \frac{\partial \log s_i}{\partial \log p_i} - 1, \text{ where } i = 1,2,3 ,$$

and the cross price elasticities as

$$\eta_{ij} = \frac{\partial \log x_i}{\partial \log p_j} = \frac{\partial \log s_i}{\partial \log p_j}, \text{ where } i, j = 1,2,3 \text{ and } i \neq j .$$