A Model of Monetary Policy Shocks for Financial Crises and Normal Conditions

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Abstract

In their classic 1999 paper, *Monetary policy shocks: What have we learned and to what end?*, Christiano, Eichenbaum, and Evans (CEE) investigate one of the most widely used methods for identifying monetary policy shocks of its time. Unfortunately, their approach is no longer viable, at least not in its original form. A major problem stems from the recent behavior of two key variables in their model, the Fed Funds rate and non-borrowed reserves. We develop a new identification scheme that remedies these difficulties but maintains the basic CEE framework. Our empirical specification is motivated by a standard New Keynesian DSGE model augmented by a simple financial structure. The model provides theoretical support for variables we use in place of certain variables that were used in the classic VAR approach outlined in CEE. One significant innovation is our use of Divisia M4, the broadest monetary aggregate currently available for the United States, as the policy indicator variable. We obtain four major empirical results that support the use of a properly measured broad monetary aggregate as the policy variable. First, policy shocks have significant effects on output and on the price level, even when an interest rate is included in our model – contradicting the New-Keynesian argument that monetary aggregates are redundant. Second, we develop a model that is not subject to the output, price or liquidity puzzles common to this literature – contradicting the view that using the interest rate as the policy indicator generally yields more reasonable responses than a monetary aggregate. Third, during normal conditions policy shocks from our Divisia-based model have similar effects on variables to those found in the Fed Funds model of monetary policy, and where there are differences our model with Divisia M4 obtains results that are more consistent with standard economic theory. Fourth, our preferred specification produces plausible responses to a monetary policy shock in samples that include or exclude the recent financial crisis.

Key words: Monetary Policy Rules, Output Puzzle, Price Puzzle, Liquidity Puzzle, Financial Crisis, Divisia Index Number, Dynamic Stochastic General Equilibrium (DSGE) Model

JEL classification codes: E3, E4, E5

1 Introduction

Modern models that analyze how monetary policy affects economic activity typically do so without reference to monetary aggregates. Ireland (2004) points out that virtually all standard New Keynesian presentations abstract from monetary aggregates and typically focus only on the behavior of output, inflation, and interest rates. However, the financial crisis, has raised significant doubts about the sufficiency of interest rate only models. Clarida, Gali, and Gertler (1999), for example, argue that monetary aggregates do not suffer from the same informational delays as inflation and output. This may validate money targeting as a viable monetary policy alternative given that monetary aggregates are immediately observable and correlated with inflation. McCallum (2001) argues that a common assumption in the New Keynesian framework, that the representative agent's utility function is strongly separable in money, is unlikely to be correct. While strong separability allows money to "aggregate away" from the equilibrium conditions, it implies that an agent may increase his/her consumption without changing the amount of money withdrawn from the bank or the frequency of withdrawals. Moreover, our theoretical model assumes strong separability in money and yet we find that an appropriate aggregate of money may still play an important role in policy.

Reasons for the absence of money in monetary models vary. From the demand side, the declining role of money in monetary models can be attributed to the difficulty of establishing a stable empirical money demand function with plausible structural parameter estimates. A common argument on the supply side is that the seemingly ubiquitous adoption of interest rate rules by central banks allows the modeler to drop the stock of money altogether from econometric specifications. This is because under certain conditions one can show that an interest rate rule makes the stock of money redundant (see e.g. Woodford, 2003, 2006). In other words, interest rate rules mean that the stock of money contains no information over and above that encapsulated in the interest rate. Though, our empirical model demonstrates statistically significant information content of properly measured monetary aggregates in a model with interest rates.

Another compelling reason to advocate monetary models devoid of money is the Barnett Critique (Chrystal and MacDonald, 1994). This refers to an internal inconsistency between standard assumptions in economic theory and the simple sum monetary aggregates supplied by central banks. Simple sum aggregates agglomerate nominal values of monetary assets while ignoring the fact that these assets provide different liquidity service flows and have different opportunity costs, also known

as user costs, (Belongia and Ireland, 2012). A consequence of informational “deficiencies” for money is that a policy of responding to monetary aggregates might destabilize the economy. For example, an analysis of the Volcker disinflation period by Friedman and Kuttner (1996) finds that a policy-maker’s use of broad money targets exposes the economy to money market shocks which may be more volatile than aggregate demand shocks. This empirical finding harkens back to Poole’s (1970) famous theoretical result. Belongia and Ireland (2012, p. 1) state “...if pressed on this issue, virtually all monetary economists today would no doubt concede that the Divisia aggregates proposed by Barnett are both theoretically and empirically superior to their simple-sum counterparts.” For example, Barnett et al. (2005) and Barnett et al. (2008) find that simple sum aggregates significantly overstate the money stock, and Kelly (2009) and Kelly et al. (2011) find that simple sum aggregates obfuscate the liquidity effect (an inverse relationship between money and interest rates following a change in monetary policy).

Despite the theoretical and empirical superiority of Divisia aggregates, there has been some reticence on the part of policymakers to adopt Divisia.¹In addition to theoretical arguments, empirical evidence has also been used to justify omission monetary aggregates. For example, (Leeper and Roush, 2003) point to a lack of clear evidence on the role of money in reduced-form models of monetary policy; Svensson (2000) argues that the European Central Bank should eliminate its money-growth pillar; and many in-sample and out-of-sample studies find that money growth does not help to predict output or inflation (see, e.g., Estrella and Mishkin, 1997, and Stock and Watson, 1999). Of course, most of the empirical studies, including the citations referenced here, base their conclusions on evidence derived using simple sum aggregates which are generally flawed.

Some have argued that monetary aggregates should be dismissed in favor of short-term interest rates because empirical models with money aggregates often have yielded puzzling results. For example Eichenbaum (1992), Christiano, Eichenbaum, and Evans (1999) (henceforth CEE) and Gordon and Leeper (1994), in VAR models that include the simple sum M1 monetary aggregate, each find a contrary response to M1 shocks. Leeper and Gordon, 1992, Strongin, 1995, and Bernanke et al. (2005), among others, find a positive correlation between the nominal interest rate and the stock of money. And Sims (1992), Barth and Ramey (2002) and others, have noted that an unexpected

¹For example, in a written statement to Businessweek, John Driscoll, a senior economist in the Fed’s Division of Monetary Affairs, said, “While Divisia indexes may be a theoretically useful way to think about money demand, they don’t solve certain practical issues that arise in using money as an indicator for such policy.”

n.d. <http://www.businessweek.com/articles/2013-03-28/the-fed-may-be-miscounting-the-money-supply>

increase in the Fed Funds rate was typically associated with a delayed increase in the price level. Dubbed the “output puzzle,” “liquidity puzzle,” and “price puzzle,”² respectively, these and other puzzling results have become a standard criticisms of using monetary aggregates to measure monetary policy shocks. We contend, contrarily, that the failure of monetary aggregates to properly capture sensible dynamic responses does not constitute a general indictment against money but instead that the problem may stem from a poor informational content of the monetary aggregates that have been chosen (see, e.g., Kelly, 2009, Kelly et al., 2011, and Keating et al., 2013 who each provide empirical evidence that the these puzzles vanish when a properly measured monetary aggregate is used).

Instead of relying on estimation of debatable money demand functions, or inclusion of extraneous data to get at sensible responses to monetary shocks, our approach is to rely on measurement and accounting improvements by using a superior measure of the stock of money. The standard response puzzles vanish as a result of this new policy variable. In Section 2 we present a relatively standard New-Keynesian model that is augmented by a simple financial structure and allows for two different types of liquid assets: deposits which pay interest and currency which does not. We use that model in Section 3 to theoretically study how the economy behaves under a Taylor Rule. We then compare that rule with a rule that essentially replaces the interest rate in Taylor’s Rule with a Divisia monetary aggregate. This substitution is motivated by one of the three key theoretical propositions we derive. These propositions guide variable choices in our empirical investigation. Section 4 builds a block-recursive structural VAR model following the classic approach that is surveyed and extended by CEE. Changes to the original formulation of CEE are motivated by our theoretical analysis. Our new model allows us to estimate policy shocks even when an economy experiences a financial crisis and some of the variables in the classic model of policy shocks misbehave. Our model obtains surprisingly robust impulse responses and variance decompositions over different sample periods. Section 5 concludes by highlighting the main results and suggesting potentially important avenues for future work.

²One notable exception is Barth and Ramey (2002) who argue that the price puzzle is no puzzle at all. They maintain that contractionary monetary policy affects supply as well as demand. When the central bank raises short-term rates, it makes it more expensive to replace inventory. The ensuing reductions in inventory investment act as a cost push shock (a negative supply shock) which serve to increase the price level. This is referred to as the cost channel of monetary policy.

2 A Minimal DSGE Model for Analyzing Monetary Shocks

This section develops a standard DSGE model that includes a simple banking system and two liquid assets. One of these assets pays no interest (currency) while the other asset may pay interest (deposits). The model largely follows from Belongia and Ireland (2012), however, we make a few significant adjustments. First, consumption and the monetary aggregate are assumed separable, implying a monetary aggregate does not appear in the log-linearized IS equation and the New-Keynesian Phillips Curve. Each of these restrictions is consistent with estimates based on U.S. data (see for example Ireland (2004)). We also alter the timing of when the bond market and goods market open and close to bring about a typical IS curve. This section sets the stage for section 3, which compares and contrasts policy rules based on a Divisia monetary aggregate with a benchmark Taylor rule in the context of a relatively standard New-Keynesian framework.

Below, all uppercase variables are real and all lowercase variables are nominal, including interest rates. Also, variables with a tilde over them denote log-deviations from the non-stochastic steady-state. A more detailed description of the equilibrium model is in the appendix.

2.1 The Household

The representative household enters any period $t = 0, 1, 2, \dots$ with a portfolio consisting of 3 assets. The household holds maturing bonds B_{t-1} , shares of monopolistically competitive firm $i \in [0, 1]$ $s_{t-1}(i)$, and currency totaling M_{t-1} (In equilibrium, this will be equal to the monetary base). The household faces a sequence of budget constraints in any given period. This budgeting can be described by dividing period t into 2 separate periods: first a securities trading session and then bank settlement period.

In the securities trading session the household can buy and sell stocks, bonds, receive wages W_t for hours worked h_t during the period, purchase consumption goods C_t and obtain any loans L_t needed to facilitate these transactions. Any government transfers are also made at this time, denoted by T_t . Any remaining funds can be allocated between currency N_t and deposits D_t . This is summarized in the constraint below.

$$N_t + D_t = \frac{M_{t-1}}{\pi_t} + \frac{B_{t-1}}{\pi_t} - \frac{B_t}{r_t} - \int_0^1 Q_t (s_t(i) - s_{t-1}(i)) di + W_t h_t + L_t - C_t + T_t \quad (1)$$

At the end of the period, the household receives dividends $F_t(i)$ on shares of stock owned in period t , $s_t(i)$, and settles all interest payments with the bank. In particular, the household is owed interest on deposits made at the beginning of the period, $r_t^D D_t$ and owes the bank interest on loans taken out, $r_t^L L_t$. Any remaining funds can be carried over in the form of currency into period $t + 1$, M_{t+1} .

$$M_t = N_t + \int_0^1 F_t(i) s_t(i) di + r_t^D D_t - r_t^L L_t \quad (2)$$

The household seeks to maximize their lifetime utility, discounted at rate β : $\sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \{u_{t+j}\}$ subject to (1) and (2). The period flow utility of the household takes the following form:

$$u_t = a_t \left[\frac{C_t^{(1-\theta_c)}}{1-\theta_c} + v_t \frac{(M_t^A)^{(1-\theta_m)}}{1-\theta_m} + \eta(1-h_t) \right].$$

The utility function contains two time-varying preference parameters that will serve as structural shocks in the linearized model. In particular, a_t will enter the linearized Euler equation as an IS shock while v_t will enter the linearized money demand equation as a money demand shock. Each of these structural shock processes is assumed to follow an AR(1) in logs.

The true monetary aggregate, M_t^A , enters the period utility function and takes on a general CES form:

$$M_t^A = \left[\nu^{\frac{1}{\omega}} (N_t)^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} (D_t)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}. \quad (3)$$

In the calibrated model, ν is derived from the relative expenditure shares on currency and deposits and ω represents the elasticity of substitution between the two monetary assets. When $\omega \rightarrow \infty$, $M_t^A = N_t + D_t$, or in other words, if deposits and currency are perfect substitutes, the simple-sum monetary aggregate is the correct aggregator function for the quantity of money. However, when $0 < \omega < \infty$, currency and deposits are imperfect substitutes and the simple-sum aggregate will not equal the true aggregate. In general, if assets pay different yields they can not be perfect substitutes and so a finite value for ω is generally true in a modern economy.

2.2 The Goods Producing Sector

The goods producing sector features a final goods firm and an intermediate goods firm. There are a unit measure of intermediate goods producing firms indexed by $i \in [0, 1]$ who produce a differentiated

product. The final goods firm produces Y_t combining inputs $Y_t(i)$ using the constant returns to scale technology,

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$$

in which $\theta > 1$ governs the elasticity of substitution between inputs, $Y_t(i)$. The final goods producing firm sells its product in a perfectly competitive market, hence solving the profit maximization problem,

$$\max_{Y_t(i)_{i \in [0,1]}} P_t \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(i) Y_t(i) di.$$

The resulting first order condition defines the demand curve for each intermediate goods producing firm's product:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t. \quad (4)$$

Intermediate Goods Producing Firm

The representative intermediate goods producing firm i has access to the production technology,

$$Y_t(i) = Z_t h_t(i), \quad (5)$$

where Z_t is an aggregate technology shock that follows an AR(1) in logs. Given the downward sloping demand for its product in (4), the intermediate goods producing firm has the ability to set the price of its product above marginal cost. However, we assume the firm faces “menu cost” associated with changing prices (Rotemberg, 1982):

$$\Phi(P_t(i), P_{t-1}(i), Y_t) = \frac{\phi}{2} \left[\frac{P_t(i)}{P_{t-1}(i)} - 1 \right]^2 Y_t.$$

The intermediate goods producing firm is assumed to maximize its period t real stock price:

$$Q_t(i) = \mathbb{E}_t \left[\sum_{j=0}^{\infty} \mathcal{M}_{t|t+j} F_{t+j}(i) \right], \quad (6)$$

where $\mathcal{M}_{t|t+j} = \beta^j / r_{t+j}^L (C_{t+j}/C_t)^{-\theta_c}$ is the rate at which the household discounts period $t+j$ payoffs

in period t .³ Substituting the definition of dividends for F_{t+j} in (6) the firm's problem can be stated as:

$$\max_{\{Y_t(i), h_t(i), P_t(i)\}_{i=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \mathcal{M}_{t|t+j} \left[\frac{P_t(i)}{P_t} Y_t(i) - W_t h_t(i) - \frac{\phi}{2} \left[\frac{P_t(i)}{P_{t-1}(i)} - 1 \right]^2 Y_t \right]$$

subject to (4) and (5).

2.3 The Financial Firm

The financial firm produces deposits D_t and loans L_t for its client, the household. Following Belongia and Ireland (2012), we assume that producing D_t deposits requires $x_t D_t$ units of the final good. In this case, x_t is the marginal cost of producing deposits and varies according to an AR(1) process in logs. Therefore an increase in x_t can be interpreted as an adverse financial productivity shock.

In addition to this resource costs, the financial firm must satisfy the accounting identity which specifies assets (loans plus reserves) equal liabilities (deposits),

$$L_t + \tau_t D_t = D_t. \quad (7)$$

Although changes in banking regulation have effectively eliminated reserve requirements, banks may choose to hold excess reserves in lieu of making loans. In fact, excess reserves holdings have risen to an enormous level following the recent financial crisis. We assume τ_t varies exogenously according to an AR(1) process in logs. An increase in τ_t can therefore be interpreted as a reserves demand shock, as opposed to a change in policy.

The financial firm chooses L_t and D_t in order to maximize period profits:

$$\max_{L_t, D_t} r_t^L L_t - r_t^D D_t - L_t + D_t - x_t D_t,$$

subject to the balance-sheet constraint (7). The first order conditions from this problem define the interest rate spread between loans and deposits:

$$r_t^L - r_t^D = \tau_t(r_t^L - 1) + x_t. \quad (8)$$

³The $1/r_{t+j}^L$ term appears in the stochastic discount factor because dividends are paid at the end of the period while the shares of the firm are bought and sold in the beginning of the period. Therefore, dividends are discounted both between periods and within periods.

If banks choose to hold more reserves or become relatively less productive, consumers will have to pay higher interest rates on loans relative to the rate they receive on their deposits.

2.4 Market Clearing

It is now possible to define the equilibrium conditions which close the model. Equilibrium in the money market requires that at all times:

$$M_t = \frac{M_{t-1}}{\pi_t} + W_t h_t - C_t + T_t. \quad (9)$$

This particular equilibrium in the money market is appropriate as it implies the monetary base will consist of currency and bank-reserves, or $M_t = N_t + \tau_t D_t$. The equity and bond markets clear when $s_t(i) = s_{t-1}(i) = 1$ and $B_t = B_{t-1} = 0$, respectively. Finally, imposing symmetry among the intermediate goods producing firms requires that in equilibrium $Y_t(i) = Y_t$, $h_t(i) = h_t$, $P_t(i) = P_t$, $F_t(i) = F_t$ and $Q_t(i) = Q_t$. The above equilibrium conditions along with the household's budget constraints imply the goods market must clear:

$$Y_t = C_t + x_t \frac{D_t}{P_t} + \frac{\phi}{2} [\pi_t - 1]^2 Y_t. \quad (10)$$

2.5 The Log-Linearized Model

The above model economy has the following log-linear structure⁴:

$$\tilde{C}_t = \mathbb{E}_t[\tilde{C}_{t+1}] - \frac{1}{\theta_c} (\tilde{r}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]) + \left(\frac{1 - \rho_a}{\theta_c} \right) \tilde{a}_t \quad (2.5.1)$$

$$\tilde{\pi}_t = \kappa [\tilde{C}_t - \tilde{Z}_t] + \beta \mathbb{E}_t[\tilde{\pi}_{t+1}] \quad (2.5.2)$$

$$\tilde{M}_t^A = \eta_c \tilde{C}_t - \eta_r \tilde{r}_t - \eta_\tau \tilde{\tau}_t - \eta_x \tilde{x}_t + \eta_v \tilde{v}_t \quad (2.5.3)$$

⁴See the appendix for details of the linearization. The semi-structural parameters κ , η_c , η_r , η_τ , η_v are defined as functions of the deep-parameters as follows:

$$\begin{aligned} \kappa &= \frac{(\theta - 1)\theta_c}{\phi} & \eta_c &= \frac{\theta_c}{\theta_m} & \eta_r &= \frac{1}{\theta_m} \left[\bar{s}^N \frac{1}{\bar{r}^L - 1} + \bar{s}^D \frac{\bar{\tau} - \bar{x}}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \right] \\ \eta_\tau &= \frac{1}{\theta_m} \left[\bar{s}^D \frac{\bar{\tau}(\bar{r}^L - 1)}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \right] & \eta_x &= \frac{1}{\theta_m} \left[\bar{s}^D \frac{\bar{x}}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \right] & \eta_v &= \frac{1}{\theta_m} \\ \text{where} & & \bar{s}^N &= \frac{\nu \bar{u}^{N(1-\omega)}}{\nu \bar{u}^{N(1-\omega)} + (1-\nu) \bar{u}^{D(1-\omega)}} & \bar{s}^D &= 1 - \bar{s}^N. \end{aligned}$$

Equations (2.5.1), (2.5.2) and (2.5.3) are the Euler equation, New-Keynesian Phillips Curve and money demand equation, respectively. In addition to these standard equations in New-Keynesian analysis, our model also contains:

$$\tilde{Y}_t = \frac{\bar{C}}{\bar{Y}} \tilde{C}_t + \left(1 - \frac{\bar{C}}{\bar{Y}}\right) (\tilde{x}_t + \tilde{D}_t); \quad (2.5.4)$$

$$\tilde{u}_t^A = \bar{s}^N \tilde{u}_t^N + \bar{s}^D \tilde{u}_t^D \quad (2.5.5)$$

$$\tilde{u}_t^D = \left[\frac{\bar{\tau} - \bar{x}}{\bar{\tau}(\bar{r} - 1) + \bar{x}} \right] \tilde{r}_t + \left[\frac{\bar{\tau}(\bar{r} - 1)}{\bar{\tau}(\bar{r} - 1) + \bar{x}} \right] \tilde{\tau}_t + \left[\frac{\bar{x}}{\bar{\tau}(\bar{r} - 1) + \bar{x}} \right] \tilde{x}_t; \quad (2.5.6)$$

$$\tilde{D}_t = \tilde{M}_t^A - \omega (\tilde{u}_t^D - \tilde{u}_t^A); \quad (2.5.7)$$

$$\tilde{u}_t^N = \left[\frac{1}{(\bar{r} - 1)} \right] \tilde{r}_t; \quad (2.5.8)$$

$$\tilde{N}_t = \tilde{M}_t^A - \omega (\tilde{u}_t^N - \tilde{u}_t^A). \quad (2.5.9)$$

Equation (2.5.4) relates output to consumption and financial resources using the income accounting identity. Equation (2.5.5) defines the user-cost of the monetary aggregate and (2.5.6) defines the user-cost of deposits. Equation (2.5.7) is the demand for deposits from the household. Equations (2.5.8) and (2.5.9) are the respective user-cost and demand equations for currency.

3 DSGE Model Analysis of Alternative Policy Rules

This section analyzes the DSGE model's responses to a contractionary monetary policy disturbance. We show that if the central bank follows a Taylor rule, the responses to all key variables in the model to a monetary policy shock can be perfectly replicated by a particular parametrization of the Divisia growth rate rule. This result suggests that from the perspective of the Lucas Critique (1973) a Divisia growth rule may not be all that different from the sort of Taylor rule that some central banks, including the Federal Reserve, are thought to have pursued at times. We also show that Divisia and Simple-Sum monetary aggregates exhibit divergent responses to a monetary contraction. This result suggests that the unusual responses that economists have found in empirical models with money may be largely the consequence of using simple sum money which is in general an improper aggregate. We also find that the Fed Funds rate and Divisia user cost are highly correlated and

share important similarities in the model. This result suggests useful information in the Fed Funds rate is also contained in the user cost, and so using that information may well be helpful in modeling the effects of monetary policy on the economy. The implication of all these results is that Divisia data may prove useful in empirically identifying monetary policy shocks that are not plagued by the “price,” “output” and “liquidity” puzzles that are frequently found in empirical analysis.

3.1 Central Bank Policy

As is standard in the literature, the system of equations defined in Section (2.5) is under determined without a specification of monetary policy. Interest-rate rules are typically used to describe central bank policy since Taylor’s (1993) seminal paper. As is common, we augment Taylor’s rule slightly to include a smoothing parameter (for which there is much empirical evidence) and a white noise policy disturbance, ε^r :

$$\frac{r_t}{\bar{r}} = \left(\frac{r_{t-1}}{\bar{r}} \right)^{\rho_r} \left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi^r} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y^r} e^{\varepsilon_t^r}. \quad (3.1)$$

However, we will also consider a money growth rule. In contrast to much of the research, Belongia and Ireland’s (forthcoming) model contains multiple monetary assets, real currency (N_t) and real deposits (D_t). We eschew the assumption a central bank observes the true monetary aggregate (M^A) as that requires knowledge of the structural parameters in the monetary aggregator (ω and ν from equation (3)). If not the true aggregate, what monetary aggregate might a central bank control? One popular index produced by most central banks is the simple-sum aggregate. This aggregate implicitly assumes N_t and D_t are perfect, one for one, substitutes (for example, see our discussion of equation (3)).

Definition 1. *The growth rate of the nominal Simple-Sum monetary aggregate is defined by*

$$\ln(\mu^{SS}) = \ln \left(\frac{N_t + D_t}{N_{t-1} + D_{t-1}} \right) + \ln(\pi_t). \quad (3.2)$$

But, in general, since monetary assets have different nominal yields they are not perfect substitutes. An alternative to the simple sum index which allows for possible imperfect substitutability is the Divisia Monetary Aggregate proposed by Barnett (1980).

Definition 2. *The growth rate of the nominal Divisia monetary aggregate is defined as*

$$\ln(\mu_t^{Divisia}) = \left(\frac{s_t^N + s_{t-1}^N}{2} \right) \ln \left(\frac{N_t}{N_{t-1}} \right) + \left(\frac{s_t^D + s_{t-1}^D}{2} \right) \ln \left(\frac{D_t}{D_{t-1}} \right) + \ln(\pi_t) \quad (3.3)$$

where s_t^N and s_t^D are the expenditure shares of currency and interest bearing deposits respectively defined by

$$s_t^N = \frac{u_t^N N_t}{u_t^N N_t + u_t^D D_t} \quad (3.4)$$

$$s_t^D = \frac{u_t^D D_t}{u_t^N N_t + u_t^D D_t} \quad (3.5)$$

where $u_t^N = (r_t^L - 1)/r_t^L$ and $u_t^D = (r_t^L - r_t^D)/r_t^L$ are user costs for these two assets. This index has been extensively studied over the last 30 years and has been shown to outperform the simple-sum alternative in many empirical and theoretical applications.⁵

By weighting the growth rate of the individual assets (with time-varying weights) the Divisia monetary aggregate is able to successfully account for changes in the composition of the aggregate which may have no impact on the overall aggregate. This superior accuracy places the Divisia index amongst Diewart's (1976) class of *superlative* index numbers, meaning the Divisia index has the ability track any linearly homogenous function (with continuous second-derivatives) up to second-order accuracy.

In the appendix, we show an even stronger result in the context of this model. The log-linearized Divisia monetary aggregate can track the true monetary aggregate to first-order:

$$\tilde{\mu}_t^{Div} = \tilde{M}_t^A - \tilde{M}_{t-1}^A + \tilde{\pi}_t. \quad (3.6)$$

The more popular simple-sum aggregate fails to attain this same level of accuracy. In particular, we show in the appendix the log-linearized simple-sum monetary aggregate differs from the true monetary aggregate at first-order:

$$\tilde{\mu}_t^{SS} = \tilde{M}_t^A - \tilde{M}_{t-1}^A + \tilde{\pi}_t + \psi_r^{ss} \Delta \tilde{r}_t - \psi_\tau^{ss} \Delta \tilde{\tau}_t - \psi_x^{ss} \Delta \tilde{x}_t \quad (3.7)$$

⁵For more in depth discussion of the research examining the Divisia monetary aggregate's properties relative to alternative simple-sum measures see Barnett and Chauvet (2011b); Belongia (1996); Barnett and Singleton (1987); Belongia and Binner (2000); Barnett and Serletis (2000); Barnett and Chauvet (2011a); Barnett (2012).

where each ψ_j^{ss} for $j = r, \tau, x$ is equal to zero when currency and deposits are perfect substitutes and otherwise each of these is greater than zero. Due to its superior accuracy, we will define the money growth rule in terms of the Divisia monetary aggregate (augmented to include a white noise monetary policy shock):

$$\left(\frac{\mu_t^{Div}}{\bar{\mu}^{Div}}\right) = \left(\frac{\mu_t^{Div}}{\bar{\mu}^{Div}}\right)^{\rho_\mu} \left(\frac{\pi_t}{\bar{\pi}}\right)^{-\phi_\pi^\mu} \left(\frac{Y_t}{\bar{Y}}\right)^{-\phi_y^\mu} e^{-\varepsilon_t^\mu}, \quad (3.8)$$

or in log-linear form,

$$\tilde{\mu}_t^{Div} = \rho_\mu \tilde{\mu}_{t-1}^{Div} - \phi_\pi^\mu \tilde{\pi}_t - \phi_y^\mu \tilde{Y}_t - \varepsilon_t^\mu. \quad (3.9)$$

3.2 Observational Equivalence between Money Growth and a Taylor Rules

Suppose the central bank follows a Taylor rule:

$$\tilde{r}_t = .5\tilde{r}_{t-1} + 1.5\tilde{\pi}_t + .125\tilde{Y}_t + \varepsilon_t^r. \quad (3.1)$$

We generate impulse responses to an increase in ε_t^r and then search for coefficients of Divisia instrument rules to try to match these dynamics. The results are presented in Table (1) below.

Table 1: Monetary Policy Regimes

Taylor Rule	Divisia Growth ¹
$\sigma_r = 0.0025$	$\sigma_\mu = 0.0107$
$\rho_r = 0.500$	$\rho_\mu = 0$
$\phi_\pi^r = 1.500$	$\phi_\pi^\mu = 0$
$\phi_y^r = 0.125$	$\phi_y^\mu = 2.180$

¹ The coefficients on the Divisia instrument rules are found by minimizing the distance between IRFs to a policy shock in the Taylor rule and the Divisia rule.

A Divisia rule with no persistence ($\rho_\mu = 0$), no reaction to inflation and a strong reaction to output is able to match perfectly the dynamics of a monetary policy contraction under a Taylor rule regime.⁶ The resulting dynamics are featured below in Figure 1.

We characterize this result in the following:

⁶We are also able to match IRFs in the Taylor Rule (reacting to inflation or the price level) with a Divisia level rule. Furthermore, when we set the coefficients in the Taylor rule equal to the estimates of Clarida et al. (2000), $\tilde{r}_t = 0.76\tilde{r} + 0.47\tilde{\pi}_t + 0.13\tilde{Y}_t + \varepsilon_t^r$, and/or alter the slope of the Phillips Curve to $\kappa = 0.25$ we are still able to perfectly replicate the dynamics of a monetary policy contraction with an appropriately calibrated Divisia growth rule.

Figure 1: Contractionary Monetary Policy Shocks under alternative monetary policy regimes. Taylor Rule denotes the dynamics to a contractionary monetary policy shock under the policy rule in (3.1) and Divisia Growth denotes the rule defined in (3.9).

Proposition 1: A monetary policy shock to an interest rate rule is observationally equivalent to a monetary policy shock to an appropriately parametrized Divisia rule.

We examine the result using VARs.

3.3 Measurement Matters: Divisia Quantity vs. Simple-Sum

Regardless of the policy instrument, the simple-sum monetary aggregate fails to get the sign ‘right’ in response to a monetary contraction. Given this implication of the equilibrium model, it is not surprising that using the Simple-Sum aggregate in the policy-maker’s information set in an estimated VAR may be a source of the *output*, *price* or *liquidity puzzles* frequently observed in VAR studies.

Eichenbaum (1992) finds that increases in the simple-sum aggregate produce incredulous output responses - the *output puzzle*. Eichenbaum (1992) concludes that changes in the money supply fail to properly identify monetary policy disturbances in a VAR. Using various monetary aggregates as the policy instrument in their benchmark specification, Christiano, Eichenbaum and Evans (1999) also find a variety of puzzling responses depending on the choice of simple sum aggregate.⁷

In contrast, the DSGE model implies that the Divisia monetary aggregate has the theoretically correct correlations associated with a monetary contraction. This point is made numerically in Table 2. When monetary policy shocks are the only driving force of this sticky price economy, interest rates are perfectly negatively correlated with inflation and output and negatively correlated with Divisia.

Table 2: Model Correlations Under a Taylor Rule¹

	$\rho_{\tilde{u}_t^A, \tilde{r}_t}$	$\rho_{\Delta \tilde{u}_t^A, \Delta \tilde{r}_t}$
Monetary Policy Shocks	1.00	1.00
Full Model	0.5014	0.4725

¹ The full model correlations increase significantly under Divisia instrument rules which implicitly accommodate the financial disturbances. Therefore these should be considered a lower bound on the correlations between user costs and policy rates.

From the impulse responses observe that the monetary policy contraction increases the user cost of the true monetary aggregate, $\tilde{u}_t^A$. Naturally this results in a fall in the monetary aggregate. The relative costs of both currency and deposits also rise; However, the cost of currency increases by more than the cost of deposits.⁸ This results in a substitution into deposits, out of currency. Since the Divisia monetary aggregate properly weights the component assets (by expenditure share), it is able to internalize this substitution effect, hence getting the sign ‘right’ and falling in response to a monetary contraction.

However, the simple-sum aggregate equally weights currency and deposits, even though currency makes up a substantially larger expenditure share than do deposits. This results in simple-sum rising

⁷They find that a contractionary policy shock to money yields an output puzzle when monetary base or M1 are used as the policy instrument and they also find a price puzzle when M1 is the instrument. Interestingly, the common puzzles do not emerge when using M2 in the benchmark model. However, the positive and unusually large response of nonborrowed reserves after a few quarters is puzzling.

⁸This can be seen by inserting (8) into the user cost of deposits expression and comparing that with the user cost of currency. Given that τ is small, that proves that the user cost of currency is more sensitive to market rates than the user cost of deposits.

in response to the monetary contraction. This lack of a liquidity effect leads to simple-sum growth being positively correlated with interest rates, negatively correlated with output and inflation. These reverse correlations from our equilibrium model suggest a second empirical implication.

Proposition 2: The use of simple-sum monetary aggregates in estimated VARs may be a source of the various empirical puzzles.

3.4 The Relationship between Aggregate User Cost and the Policy Interest Rate

Theoretical motivation for using superlative index numbers in empirical applications extends beyond quantity aggregates. Price aggregates, such as the user cost index dual to the Divisia quantity index, also have appeal. This provides some promise that the Divisia user cost dual may prove useful in empirical applications.⁹ Belongia and Ireland (2006) for example show the user-cost captures the monetary transmission mechanism in a flexible price equilibrium model and a small VAR. Table (3) shows that their result carries over to this sticky-price model.

Table 3: Model Correlations Under a Taylor Rule¹

		$\tilde{u}_t^A$	$\tilde{Y}_t$	$\tilde{\pi}_t$	$\tilde{\mu}_t^{Div}$	$\tilde{\mu}_t^{SS}$
Monetary Policy Shocks	$\tilde{r}_t$	1.00	1.00	-1.00	-0.5729	0.5386
	$\tilde{u}_t^A$		-1.00	-1.00	-0.5729	0.5386
Full Model Shocks	$\tilde{r}_t$	0.9303	-0.8896	0.9259	0.2741	0.1440
	$\tilde{u}_t^A$		-0.8265	0.8639	0.2036	0.0789

The policy rate and user cost are perfectly correlated in the model when conditioning only on policy shocks and therefore the correlation between each of these variables and the other key variables are precisely the same. And even when we allow all shocks in the model to occur, each of the other variables has a similar correlation to the policy rate and the user cost, and those two

⁹Although simulations for the model have only included the true price aggregates (see Eq. (2.5.5)), two applications of Fisher's factor reversal test verify that in this linearized model, the Divisia price dual will track the true price dual (in growth rates) without error. Fisher's factor reversal test implies that for the Divisia price dual. (The price dual is defined by Fisher's factor reversal test. If we define the nominal Divisia level by $\frac{div_t}{div_{t-1}} = \mu_t^{Div}$, the Divisia price dual u_t^{Div} satisfies the equation $div_t u_t^{Div} = (p_t D_t u_t^D + p_t N_t u_t^N) u_t^{Div}$,

$$\tilde{\mu}_t^{Div} + \Delta \tilde{u}_t^{Div} = \tilde{M}_t^A - \tilde{M}_{t-1}^A + \tilde{\pi}_t + \Delta \tilde{u}_t^A.$$

Since we have previously shown the growth rate of the Divisia monetary aggregate tracks the growth rate of the true monetary aggregate to first order without error (See Eq. (3.6)), we have that $\Delta \tilde{u}_t^{Div} = \Delta \tilde{u}_t^A$.

variables remain highly correlated with one another. This suggests the user-cost contains valuable information for empirically identifying monetary policy disturbances and their transmission to the rest of the economy. This is the third implication we will examine in the data.

Proposition 3: The Divisia user-cost moves closely with the policy rate because they each capture similar information about the macroeconomy.

The remainder of the paper develops an empirical model that identifies the effects of monetary policy shocks under normal and crisis conditions. Each of the three theoretical propositions is used to help formulate our new empirical model.

4 A Robust Model of Monetary Policy Shocks

Christiano, Eichenbaum and Evans (1999) provide a detailed survey on the use of their particular recursive framework for identifying monetary policy shocks along with important empirical results and theoretical justification for their approach. Their recursive framework was at one time a very popular method for identifying monetary policy shocks, perhaps even the most popular. Today, however, it is no longer viable, at least not in its original form. There are major problems with three of the variables used in their original specification: (1) The Fed Funds rate has essentially been stuck at its lower bound since December of 2008; (2) A monetary aggregate that CEE makes extensive use of, namely non-borrowed reserves, was negative for nearly all of 2008; and (3) The commodity price index initially made popular by this line of work is no longer available. Before discussing the alternative variables used in place of these problematic series, we review the CEE methodology for identifying monetary shocks and then outline our modifications to the classic specification.

Their approach is based on a VAR that can be written as:

$$Z_t = B_1 Z_{t-1} + \dots + B_q Z_{t-q} + u_t \quad (4.1)$$

where $E u_t u_t' = V$ represents the covariance matrix for residuals.

A linear structural model may be written as:

$$A_0 Z_t = A_1 Z_{t-1} + \dots + A_q Z_{t-q} + \varepsilon_t \quad (4.2)$$

where $E\varepsilon_t\varepsilon_t' = D$ is the diagonal covariance matrix for structural shocks.

The variables in the model are sub-divided into three groups:

$$Z_t = \begin{pmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{pmatrix} \quad (4.3)$$

Each group might consist of multiple variables, although we follow CEE in specifying X_{2t} as a single variable and assume it serves as the policy indicator variable.

The structure is assumed to take on the following form:

$$A_0 = \begin{bmatrix} A_{11} & 0_{12} & 0_{13} \\ A_{21} & A_{22} & 0_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \quad (4.4)$$

where A_{ij} is an $n_i \times n_j$ matrix of parameters and 0_{ij} is an $n_i \times n_j$ zero matrix. The vector of structural shocks is given by: $\varepsilon_t = (\varepsilon_{1t}', \varepsilon_{2t}', \varepsilon_{3t}')'$. CEE prove that under these assumptions and with X_{2t} a single variable (i.e. $n_2 = 1$), the Cholesky factor of V will identify the effects of shocks to the structural equation for X_{2t} .¹⁰

This identification assumes the policy variable, X_{2t} , responds contemporaneously to X_{1t} a set of important macroeconomic variables that slowly adjust to policy and a variety of other nominal shocks. In the CEE benchmark specifications, as well as the vast majority of papers that identify monetary policy shocks using this block-recursive formulation, X_{1t} consists of real GDP, the GDP deflator, and a commodity price index. The first two variables are included because of a monetary policymaker's obvious concern for how these macroeconomic aggregates behave. If the policy variable is the Fed Funds rate, a response to these two variables is consistent with a Taylor Rule formulation which is often assumed to describe the central bank reaction function. The commodity price index is included to remedy the pervasive price puzzle, perhaps because it encapsulates current information about future price movements that a forward-looking central bank is concerned

¹⁰Keating (1996) generalizes CEE's identification result. That paper shows that if $n_2 > 1$ and A_{22} is a lower triangular matrix, the Cholesky factor of V will identify the effects of structural shocks for all n_2 shocks in ε_{2t} . Structures that take on this form are defined as partially recursive.

with.¹¹ A key identifying assumption is that monetary policy has no contemporaneous effect on X_{1t} . The third set of variables, X_{3t} , consists of variables that respond immediately to X_{1t} or X_{2t} . In the benchmark specifications of CEE, X_{3t} includes various reserves measures along with the Simple Sum M2 measure of money. It will also include the Fed Funds rate whenever Non borrowed Reserves is used as the policy variable. These money market variables are allowed to respond immediately to macroeconomic conditions and monetary policy. The key assumption is that the macro and monetary policy variables do not respond contemporaneously to money market variables. Ultimately, we make a number changes to each group of variables as we develop a model that is capable of identifying monetary policy shocks under a variety of different macroeconomic conditions.

4.1 Benchmark Specifications of CEE

One purpose of this research is to compare our specifications with the classic models from CEE. But , we also want to estimate models of monetary policy shocks using more recent data. A problem with that intention is the commodity price measure that CEE used has been discontinued. Thus our first task is to find a substitute commodity price index that is currently being produced and that is available over a reasonably long period of time. We employ the Commodity Research Bureau (CRB) spot price index provided in the Thomson-Reuters/Jefferies CRB index report. This index comprises 19 commodities falling under four different groups (Petroleum-based products, liquid assets, highly liquid assets, and diverse commodities – each group with different weightings).

First, we will estimate the classic Benchmark specifications of CEE to determine whether our data yields roughly the same results as they obtained. Obviously, one difference between our data and that of CEE is the commodity price index. A second difference is that we employ revised data of a more recent vintage. And a third difference stems from the fact we will use specifications that include a Divisia index of money. Given the starting date for Divisia series, we must begin our sample at a slightly later date than CEE did. Nevertheless, in spite of these data modifications when we replicate the Benchmark specifications of CEE we obtain estimates that are remarkably similar.

The Fed Funds and Non-borrowed Reserves Benchmark models of CEE (1999) are estimated using

¹¹If a change in monetary policy causes a change in future prices, the commodity price index might react immediately to anticipated future price level movements. In that case, placing the commodity price index in the first block of variables would be incorrect because it would not permit a policy shocks to affect commodity prices contemporaneously.

quarterly data from 1967:1 to 1995:2.¹² The starting point is determined by Divisia availability and the end of this sample is the same as CEE.¹³ Figure 2 shows our estimates of these two benchmark models. The first column considers the Federal Funds rate as the policy instrument while the second column assumes NBR is the policy variable. Whenever one of these variables is taken to be the policy variable, the other is placed in the third block of variables. In both of these models, output falls with some delay and has a U-shaped response to a monetary contraction. In contrast to CEE, we find that the response of output is not statistically significant in the NBR model, though it is almost significant at the 4th and 5th quarters. The negative output response is significant from three to eleven quarters in the Fed Funds model. The price level response is negative in both cases, and like CEE, the decline is more delayed in the Fed Funds model. In contrast to CEE, the price level response in the Fed Funds Model does not rise initially (in CEE's model it rose a small amount before it began to decline). Instead, the estimate is essentially zero for the first six quarters. Similar to CEE we find these price level responses are never significant. The commodity price response is initially zero, by construction, but afterwards it is consistently negative. This seems more natural than the responses in CEE's estimates where the response in the Fed Funds model is sometimes positive and where the response in the NBR model goes to zero rather quickly. Initially, the Fed Funds rate rises while NBR falls. Both effects are significant for a few quarters. Total reserves fall and with some delay in the Fed Funds model.¹⁴ M2 falls immediately and that response is always negative.

Overall, our estimates for both Benchmark Models are qualitatively and quantitatively quite similar to those reported in CEE. The primary exceptions are the responses of the commodity price index. Our responses are never positive, in contrast to the Fed Funds model of CEE. And following a delay, the decline in the commodity price index in our model is persistent, in contrast to CEE's NBR model. The typically modest quantitative differences in responses of other variables can be attributed to our using a different commodity price index, estimating with more recent vintage data and selecting VAR lags based on the Hannan-Quin criterion.¹⁵ We conclude that our Benchmark model estimates are very similar to – and in some cases marginally better than – CEE's results. These

¹²CEE also use another formulation of the policy variable, based on Strongin (1995), that we estimate but did not report. That model has the same problems we document for the NBR Benchmark.

¹³CEE model estimates begin in 1965:3 so given that they use a 4 lag VAR their data sample began in 1964:3 (two and a half years earlier than ours did). We end our initial sample period at 1995:2 as do CEE with the benchmark specifications reported in Figure 2 on that paper.

¹⁴A delayed response of total reserves is consistent with the identification assumptions in Strongin (1995).

¹⁵Our models were estimated with 2 lags whereas CEE used 4.

findings imply we have found a suitable substitute for the commodity price index used originally by CEE.

4.2 Using the Divisia Monetary Aggregate as the Policy Variable

As the recent financial crisis worsened, central banks began to rapidly reduce short-term nominal interest rates. The Fed was particularly aggressive in responding to the crisis and was the first of the world's major central banks to bring its policy rate down to essentially zero. When stuck at zero, downward movements in the Fed Funds rate can no longer occur and so the interest rate is unable to measure expansionary policy from Quantitative Easing and the various lending facilities created by the Fed. The lack of movement, and thus the lack of information, is particularly problematic when we are trying to gauge the effects of monetary policy.

To address the Fed Funds zero lower bound issue, we propose using Divisia M4 as the policy variable, in place of the Fed Funds rate. Our replacement is justified by Proposition 1 which shows theoretically that an interest rate rule is observationally equivalent to a Divisia quantity rule for a given choice in parameters. Divisia M4 is the broadest of Barnett's Divisia measures available for the United States, and is published by the Center for Financial Stability (for more information see http://www.centerforfinancialstability.org/amfm_data.php). A broad measure has a number of advantages. Theoretical advantages are: (i) a broad aggregate includes monetary services contained in assets that are weak substitutes for assets that serve primarily as a medium of exchange such as currency, (ii) Kelly (2013) (citation????) proved that the error resulting from incorrectly omitting an asset is much larger than the error from incorrectly including an asset. Empirically, a broad measure also has advantages. Barnett et al. (2008), Kelly (2009), and Barnett and Chauvet (2011c), for example, all find that broad measures outperform narrow monetary aggregates in useful ways (Say more ??).

The third column in Figure 2 shows results for this Divisia Benchmark Model, allowing us to easily compare Divisia model results with the other two Benchmark models. Following a negative shock to the Divisia quantity, we see that each variable in the model has a response that is quite similar to that found in the Fed Funds model, except for policy variables which are different. The output response is significant for a similar length of time and the shape of the response is very similar

Figure 2: **Benchmark Specifications (1967q1 - 1995q2)**

The Fed. Funds model corresponds to the first benchmark specification of CEE where the Effective Federal Funds Rate is used to indicate monetary policy. The Non-borrowed Reserves model corresponds to their second benchmark specification where the Non-borrowed Reserves is used to indicate monetary policy. The Divisia model corresponds to our first specification with Divisia M4 as the monetary policy indicator. This model replaces the Effective Federal Funds Rate in the first CEE benchmark model with Divisia M4.

to the other benchmark specifications. Qualitatively, the price response is identical but, in contrast to the two benchmark models, this price level response in the Divisia model is significant after about 10 quarters. Declines in NBR and M2 are similar to what we found for the Fed Funds model, and they have similar periods of statistical significance. The commodity price response resembles that of the Fed Funds model, except that in the Divisia model the response rises a small amount before turning negative. Also, the Divisia model yields a much longer period of significance for this response than either of CEE's Benchmark Models.¹⁶ Using roughly the same sample as CEE we find that Divisia works very well as the policy variable. Responses to a policy shock in the Divisia model are quite similar to responses in the Fed Funds Model, aside from the different policy variables. This finding provides empirical support for theoretical Proposition 1.

The overwhelmingly strong similarity of results from the Divisia and Fed Funds Benchmark models suggests that these variables serve the same purpose in the CEE framework, at least when estimated over the sample period that roughly coincides with the CEE's estimates. The interchangeability of the two policy indicators suggests that when the Fed Funds hits its lower bound Divisia may continue to perform well since it has not been subject to a binding constraint. Thus for much of this paper we will use the Divisia quantity index as the policy variable. This might have raised Lucas-critique concerns since the Fed has historically conducted policy with little interest in Divisia, however, Proposition 1 provides theoretical support for this substitution. And, our empirical results imply that if the Fed focused mostly on Fed Funds as the policy variable and the CEE framework is an appropriate way of estimating the effects of monetary policy, whether we use Divisia or Fed Funds is largely a matter of choice. These two variables apparently provide essentially the same information about monetary policy activities in the sample period originally used by CEE. And, when they yield different results, the Divisia model's estimates are more consistent with theoretical predictions.

¹⁶The short-run increase in the commodity price index is not surprising and it can be explained by a "working-capital hypothesis." [Cite Christiano and ???] Firms often incur short-term debt to produce the output that will be sold at some later date. This requires working capital to help the firm pay raw material and labor costs incurred prior to the sale of the final output. Firms borrow to finance working capital. And as interest rates go up so does the the cost of working capital. This rise in the cost of working capital may induce some firms to raise prices. The more competitive a market is, the more likely its price will initially rise when the Fed raises rates. Given that commodity markets appear to be highly competitive compared to most other markets, a Fed tightening could lead to the short-run rise in commodity prices that was found in the Divisia Model.

4.3 Benchmark Models in the Pre-Crisis Period

It is important to estimate the three benchmark models with more recent data to determine how well each specification performs and to ascertain how robust results are to an extended sample period. However, the extent to which the sample may be extended is limited by accounting irregularities in the Federal Reserve's measure of NBR and by the protracted stance of monetary policy at the ZLB. We estimate the three quarterly benchmark models up to 2007:4. Immediately after that, NBR becomes negative and remains so until almost the end of 2008, at which time the Fed Funds rate reaches its lower bound where it remains to this date. At the time of this writing, the Fed has committed to keeping the Fed Funds rate at roughly zero for an extended period according to the new "forward-guidance" mechanism.

Figure 3 reports the Fed Funds model, NBR model and the Divisia model for the period 1967:1 to 2007:4. We compare these models with one another as well with the results in Figure 2 to get a better picture for how these models perform as the sample is extended to just before the crisis.

There are many similarities between results from the shorter sample period (1967:1-1995:2) period and the longer sample (1967:1-2007:4) estimates. For example, there results for the Divisia and Fed Funds rate models share many similarities. However, there are also some noteworthy differences. This longer sample now obtains an output puzzle in the NBR Model. While output falls a small amount after a delay of two quarters, this response turns positive after 9 quarters and becomes larger in absolute value compared to the initial negative response. This is a very peculiar output response to a contractionary monetary policy shock. While it is never statistically significant, it distinguishes this model from the other two which find substantial portions of the negative output responses to be significant. All of these models show something of a price puzzle. But this puzzle is larger and statistically significant in the Fed Funds model. The Divisia model is the only one to have a correctly signed (negative) price level response that is nearly statistically significance after 16 quarters. The delayed negative price response is consistent with models of sticky nominal adjustment. NBR, total reserves and M2 each behave similarly in the Fed Funds and Divisia Models. For example, NBR and total reserves both fall initially, although these responses eventually turn positive. In the NBR model these responses are always negative. However, the NBR model shows an M2 puzzle. While initially negative, the response of money is positive after about 8 quarters, although that positive response is never significant. Overall, the Divisia model's responses of output and the price level

are as good as the other two models and sometimes better than both. However, the significant positive responses of NBR and total reserves at longer horizons following a monetary contraction in the Divisia and Fed Funds models are puzzling. So once again the Fed Funds and Divisia models yield very similar responses to a policy shock, consistent with Proposition 1.

4.4 Modifications to Benchmark Divisia Model: Pre-Crisis Estimates

We find that Divisia serves as a reasonable substitute for the Fed Funds interest rate as the policy variable in the CEE framework. A substitute for non-borrowed reserves in the information (third) block is still necessary, in order to develop a model that may be useful during the recent financial crisis. The problem is that it makes no sense for non-borrowed reserves to take on negative values. By construction, non-borrowed reserves equal total reserves less borrowed reserves. As the Fed has noted (<http://www.federalreserve.gov/feeds/H3.html>): “The negative level of non-borrowed reserves was an arithmetic result of the fact that borrowings from the Federal Reserve liquidity facilities were larger than total reserves.” In other words, certain borrowings were counted as borrowed reserves but not counted as total reserves. This accounting error was so large that it turned negative a number that should never be negative. That peculiarity suggests non-borrowed reserves may corrupt a model that attempts to measure the effects of monetary policy, particularly in the years since the crisis began.

It would be helpful if a replacement variable could be found that contained essentially all the information in non-borrowed reserves but was not subject to similar accounting problems. A natural candidate is the Monetary Base.¹⁷ The base has the advantage of never having been negative. And since it equals the sum of currency and total reserves, the monetary base internalizes the separation of reserves into borrowed and non-borrowed components.

The first column of Figure 4 labeled Divisia Model-A reports results when the monetary base replaces NBR in our Divisia Benchmark model (results for that were shown on the third column of Figure 3). The sample period from 1967:1 to 2007:4 is used to maintain comparability with our estimates in Figure 3.

¹⁷Some economists in the past discussed using the non-borrowed monetary base instead of non-borrowed reserves as a better policy measure. Cite them ??

Fed. Funds Model

Non-borrowed Reserves

Divisia Model

Figure 3: **Benchmark Specifications (1967q1-2007)**

Same models as in Figure 2 except that we extend the sample to just before the recent financial crisis.

Divisia Model-A

Divisia Model-B

Divisia Model-C

Figure 4: **Modified Divisia Models (1967q1 - 2007q4)**

Divisia Model-A replaces non borrowed reserves in the baseline Divisia model with the monetary base. Divisia Model-B adds to Model A the 10-Year Treasury Constant Maturity Rate. Divisia Model-C removes from Model-B the simple sum M2 monetary aggregate.

Except for the monetary base's response the results are largely the same as when we used NBR in the Divisia model. Interestingly, the base response is always negative, and this response is usually significant. Thus the peculiar positive NBR response found in the original Divisia model is not replicated by the monetary base. The total reserves response still starts out negative and eventually turns significantly positive, and so remains a puzzle that this Divisia model continues to share with the Fed Funds Benchmark model.

While the Divisia quantity aggregate is a non-linear function of the interest rates, our first two Divisia models do not explicitly allow for a separate interest rate reaction to policy shocks. This seems a glaring weakness of the previous Divisia models since macroeconomic theories and empirical models typically contain at least one interest rate. Therefore, we add the 10-year Treasury to the third block of variables in Model-A and call this Divisia Model B. Short term-to-maturity Treasury bills are not useful since these rates, like the Fed Funds rate, have been fairly close to lower bounds since the Great Recession began. Moreover, those rate have changed very little over the last few years. Basic term structure theory explains low rates on short-term securities as a consequence of the Fed's commitment to keeping the Fed Funds rate near zero well into the future. Results for Model B are also reported in Figure 4. Note that the interest rate rises initially but eventually it falls, and that decline eventually becomes statistically significant. This response pattern has an intuitive interpretation; A liquidity effect is initially of importance to the response of the 10-year Treasury, but eventually the Fisher effect becomes dominant. The results for the other variables in the model are almost the same as Model A, except that in Model-B a much greater portion of the total reserves response is now positive and it still becomes significant after 4 years. So once again the total reserves response is puzzling.

Our model contains Divisia which is a theoretically correct measure of the money supply. It also contains the simple sum which is known to be an inferior measure of the money stock. According to empirical results in Barnett et al., 2005, Barnett et al., 2008 and others (see, e.g., Kelly, 2009, Rotemberg, Driscoll, and Poterba, 1995, and Barnett, 1991), the simple sum is substantially upward biased compared with the theoretically correct monetary aggregate. Those findings suggest that simple sum may be a source of noise or bias in our estimates. Furthermore, Proposition 2 suggests that simple sum money has the types of peculiar correlations with output, the price level and the nominal interest rate which are consistent with the standard empirical puzzles found in the VAR literature. Therefore, we remove the simple sum from Model-B, labeling it Divisia Model-C. For

comparison purposes, results for these models are reported in Figure 4. The results for Model-C are essentially the same as Model-B for variables that are common to both models, although now the response of reserves is virtually nil and the response of monetary base is a bit smaller. Removing simple sum apparently eliminates the strong total reserves puzzle.

4.5 Modifications to Benchmark Divisia Model: Full Sample Estimates

We now extend the samples for Models A, B and C to 2013:2 to include the recent financial crisis period and report these results in Figure 5. The goal of this paper is to develop a model that works well under all conditions. Compared to the sample ending in 2007:4, we now have robust decreases in total reserves in all three models. In fact, using the longer sample we find that total reserves, the monetary base and the Divisia quantity of money all exhibit stronger negative responses than in the pre-crisis sample. The remaining responses look qualitatively similar to those earlier estimates, except for the interest rate. In Divisia Model-B, the 10 year Treasury continues to have the initial liquidity effect but, in this specification there is no Fisher effect at longer horizons. And in Divisia Model-C, the interest rate response is flat so in addition to no Fisher effect there is also no short-run liquidity effect.

4.6 Varying Term-to-Maturity on Treasury Rates

The peculiar rate response noted for Divisia Model-C suggests we consider alternative interest rates. Figure 6 reports responses when the model uses the 3-year, the 5-year and the 10-year Treasury rate, respectively, to determine if the unusual results are robust to different options for the term-to-maturity. Interest rate responses are mildly different as we vary the term, however, the peculiar response of the interest rate to a contractionary policy shock seems robust. No other variable's response is appreciably affected by the choice of Treasury rate. Initially, the 3-year Treasury falls more than the 5-year rate following a monetary contraction. The response of the 10-year rate to a monetary contraction is virtually zero. We expected a monetary contraction to have a liquidity effect, that would raise short-term interest rates and thus cause long rates to rise initially via the term structure of rates. The response for each of these long-term Treasury rates to a policy shock is disappointing.

Divisia Model-A

Divisia Model-B

Divisia Model-C

Figure 5: **Modified Divisia Models (1967q1 - 2013q2)**
The same models as in Figure 4 using the full sample period.

3-Year Treasury Rate 5-Year Treasury Rate 10-Year Treasury Rate Nominal User Cost

Figure 6: Divisia Model-C, Various Treasury Rates, (1967:01-2013:2)

4.7 Using the Divisia User Cost as the Interest Rate

Monetary aggregation theory suggests that rather than arbitrarily choosing a single interest rate, a properly weighted aggregate of all interest rates that affect the choice of a monetary portfolio should contain significantly greater information for monetary policy. The Divisia user cost provides a potentially useful interest rate aggregate. As shown in equation (??), the aggregate user cost provides a weighted average of interest rates spreads. In spite of that, Proposition 3 suggests that the user cost will contain much the same information as the Fed Funds rate.

Consistent with that perspective is time series evidence that compares the Fed Funds rate to the nominal User Cost for the M4 Divisia. Figure 2 shows that these two variables exhibit strikingly similar behavior, even though one is a nominal yield and the other an aggregate of nominal interest rate spreads. We are the first to recognize this interesting and unexpected empirical relationship.¹⁸ This combination of theory and evidence suggests we may be able to use these variables interchangeably in the model. An important advantage to the user cost, in contrast to Fed Funds rate, is that it has not bumped up against a lower bound.

We replace the Treasury rate in Divisia Model-C with the Divisia user cost, estimating the model over the three sample periods used earlier: 1967:1-1995:2; 1967:1-2007:4 and 1967:1-2013:2. Figure 8 reports the impulse responses. In all three periods, we find a significant U-shaped output response, a delayed drop in the price level that does not begin for at least a year and a decline in the commodity price that occurs much earlier than the drop in the price level. The decline in commodity prices preceding the decline in the aggregate price level supports the view that commodity prices react faster to news about monetary policy. The Divisia monetary aggregate, the monetary base and total reserves all fall while the user cost initially spikes upward. In the full sample the User Cost gradually shrinks to zero. In the other two sample periods the User Cost response eventually becomes negative. Note that this long run negative value was also observed in CEE for the Fed Funds Benchmark Model and was replicated by us (Column 1, Figure 2) Thus we obtain a short-term liquidity effect in all cases. And both pre-crisis samples find a significant Fisher effect after

¹⁸We also experimented with using the User Cost as the policy variable, and not surprisingly, results for User Cost are frequently much the same as with the Fed Funds rate. Consequently, the Divisia quantity also outperforms the User Cost as policy indicator in our VAR estimates.

Figure 7: Nominal Divisia M4 User Cost and the Fed Funds Rate

somewhere between two and three years. All of these effects are statistically significant, except for the negative price level response in the full sample estimate. Each effect has the correct sign and it almost always is statistically significant. Qualitatively, all of the impulse response results are robust to alternative sample periods, except for the eventual Fisher Effect which is not observed in the full sample estimate.

The responses of output, the price level, the commodity price index and the user cost all become somewhat more attenuated as we move from the shortest to the longest sample period, although these differences are almost never statistically significant. These cross-sample differences may well be a consequence of the variation in persistence of Divisia to a policy shock which decreases as the sample period is extended. It is possible that when a money supply shock has a less persistent effect on the quantity of money it may have relatively smaller effects on output and prices. This would likely be the case if the effects on interest rate spreads were also less persistent, since these spreads also factor into the calculation of a Divisia quantity aggregate.

Responses of monetary base and total reserves to a policy shock are the only responses which

are dramatically different depending on sample period. Both of these negative responses increase substantially in magnitude when we include the financial crisis period. These responses also have huge confidence intervals in the longer sample period. These quantitatively different results fit a standard view about the financial crisis. The Fed’s new tools such as Quantitative Easing injected massive amounts of reserves into the banking system and had a similar effect on the base. However, as a result of the financial crisis, banks are making far fewer loans and holding tremendous amounts of excess reserves. Holding a substantially larger share of excess reserves means it now takes a much larger increase in total reserves to generate a given level of monetary stimulus as measured by Divisia. And while the monetary stimulus seems to have had some beneficial effects on the economy, the economy’s responsiveness to policy has apparently become attenuated as a result of the crisis period. Whether this is due primarily to unwillingness of banks to lend or unwillingness of borrowers to take out loans remains a topic for another day. An interesting question is how much of the recent abatement in monetary policy’s effectiveness is attributable to persistent negative effects from the financial crisis, uncertainty resulting from the strident political disagreements that have taken place in Washington and elsewhere, an inappropriate fiscal response to the crisis, and other potential factors.

4.8 A Quantitative Assessment of Policy Shocks

Finally, we examine variance decompositions to determine the relative importance of policy shocks in the Divisia model. The variance decompositions are taken from Divisia Model-D as this model omits the series typical found in the traditional/Benchmark models of monetary policy shock and which exhibits impulse responses that are consistent with economic theory . Tables 4, 5, and 6 report the percentage variance due to a monetary policy shock for the 1967q1 - 1995q2, 1967q1 - 2007q4, and 1967q1 - 2013q2 sample periods, respectively. While a sizable share of output variance is attributed to policy shocks, our largest point estimate is roughly 28% and it is obtained in the sample that roughly corresponds to CEE’s original estimates. This suggests that monetary shocks have some effect though other shocks cumulatively are much more important for explaining output fluctuations. Notice that output variance falls when the estimation includes the recent financial crisis, and for that sample it is always less than 10 percent. The price variance attributable to policy

1967q1 - 1995q2

1967q1 - 2007q4

1967q1 - 2013q2

Figure 8: Divisia Model-C, Nominal User Cost of Divisia M4 as the Interest Rate, Various Sample Periods

shocks is very small even after 8 quarters; It rises substantially at longer horizons, except when the financial crisis period is added to the sample. In that case, very little price level variance is explained by policy shocks even after 16 quarters. Why are the variance results for price and output so different when the crisis period is included? One explanation may be there recently were large shocks arising from sources outside of monetary policy that affected the economy. Another reason may be that monetary policy loses strength when an economy goes into a severe financial crisis. Differentiating between these two possibilities and other explanations is an open question for future research.

The base variance rises as the horizon increases, but this variance is primarily explained by non-policy shocks. A rather small amount of total reserves variance, never more than 10%, is explained by the policy shock. The fact that disturbances other than monetary policy shocks explain the vast majority of variance for the base and total reserves can be attributed to the Fed's tendency to follow an interest rate rule. If the central bank smoothed interest rate movements, then fluctuations in the demand for highly liquid assets will tend to be the dominant source of variability for the monetary base and total reserves.

When we include the crisis period the User Cost variance explained by policy shocks shrinks and these shocks also explain less of the Divisia's variance particularly at longer horizons. Again this fits with the view that other shocks became more important for the economy, reducing the contributions made by policy disturbances.

5 Conclusion

This paper resuscitates a classic method of identifying monetary policy shocks that had fallen out of favor. Certain key variables in that model are replaced by arguably preferable time series drawing from the implications of a relatively standard New-Keynesian theoretical framework. We develop a VAR model that is well-grounded in economic theory. Our most important innovation is to use a broad Divisia measure of money as the policy indicator variable in place of the Fed Funds interest

Table 4: Percentage Variance Due to Monetary Policy Shocks: Divisia Model-D (1967q1-1995q2)

	4 Quarters Ahead	8 Quarters Ahead	20 Quarters Ahead
Real GDP	16.66 (6.94, 31.58)	26.36 (10.36, 41.48)	27.55 (9.02, 41.14)
GDP Deflator	0.22 (0.06, 5.17)	1.54 (0.36, 16.40)	38.29 (16.13, 62.88)
Divisia M4	84.66 (64.92, 91.34)	73.93 (49.08, 86.48)	61.54 (31.79, 80.53)
Monetary Base	11.92 (2.68, 28.58)	13.40 (2.17, 32.43)	18.87 (1.97, 42.68)
Total Reserves	9.11 (2.27, 22.95)	6.37 (1.56, 20.25)	5.16 (2.54, 21.31)
Nominal User Cost	19.08 (6.51, 35.64)	15.46 (6.82, 33.13)	19.23 (9.70, 37.11)

Note: numbers in parentheses are the boundaries of the associated 90 percent confidence interval.

Table 5: Percentage Variance Due to Monetary Policy Shocks: Divisia Model-D (1967q1-2007q4)

	4 Quarters Ahead	8 Quarters Ahead	20 Quarters Ahead
Real GDP	14.96 (6.91, 25.55)	19.68 (8.32, 31.89)	18.75 (6.04, 30.21)
GDP Deflator	0.02 (0.02, 2.24)	0.80 (0.29, 9.23)	19.34 (7.47, 44.45)
Divisia M4	89.71 (75.06, 93.65)	82.33 (61.19, 89.21)	69.75 (38.45, 79.37)
Monetary Base	5.43 (0.64, 17.13)	7.08 (0.93, 22.56)	9.26 (1.06, 31.23)
Total Reserves	1.12 (0.24, 8.86)	0.87 (0.24, 10.00)	0.50 (0.41, 13.11)
Nominal User Cost	14.75 (6.53, 26.40)	11.66 (5.99, 23.83)	11.78 (6.43, 25.02)

Note: numbers in parentheses are the boundaries of the associated 90 percent confidence interval.

Table 6: Percentage Variance Due to Monetary Policy Shocks: Divisia Model-D (1967q1-2013q2)

	4 Quarters Ahead	8 Quarters Ahead	20 Quarters Ahead
Real GDP	6.93 (1.37, 16.81)	7.92 (1.35, 19.28)	6.10 (1.54, 17.87)
GDP Deflator	0.08 (0.02, 2.29)	0.43 (0.05, 5.37)	1.22 (0.15, 13.54)
Divisia M4	86.82 (70.65, 93.55)	70.64 (43.85, 87.96)	33.79 (13.42, 62.15)
Monetary Base	6.00 (0.32, 17.98)	7.43 (1.04, 20.70)	15.05 (5.50, 31.15)
Total Reserves	3.97 (0.40, 15.03)	4.46 (0.84, 16.35)	9.04 (2.52, 22.20)
Nominal User Cost	9.14 (2.56, 21.05)	6.83 (2.14, 18.25)	5.96 (2.33, 17.46)

Note: numbers in parentheses are the boundaries of the associated 90 percent confidence interval.

rate. This is motivated by theoretical analysis as well as empirical evidence that the two may serve the same purpose in the VAR model, at least while the Fed Funds rate is well above its lower bound. Of course, a quantity of money is not likely to ever hit a lower bound, while the Fed Funds rate has been at that point for years. We also have replaced non-borrowed reserves with the monetary base. Non-borrowed reserves behaved strangely during the the recent financial crisis, taking on theoretically impossible negative values. This odd result was a consequence of peculiar accounting, and it calls into question the usefulness of non-borrowed reserves in empirical studies the deal with data from the recent financial crisis. In contrast, the monetary base is not subject to that problem. And since the base encompasses non-borrowed reserves it likely contains much the same information. We also removed the simple sum measure of the quantity of money from the empirical model. We show the simple sum exhibits peculiar correlations in the theoretical model, results which are consistent with various puzzles economists have often observed when using simple sum measures of money in VARs. Economic theory and empirical analysis typically attribute an important role to interest rates. Thus we include the User Cost of money which is an aggregate of the rates of return relevant to the Divisia quantity of money used in the empirical model. Based

on theoretical analysis and empirical evidence we show that the User Cost contains much the same information as the Fed Funds rate; A major advantage of User Cost is that it also has not bumped up against a zero lower bound. Furthermore, we show that the User Cost performs substantially better than long-term Treasury rates in our empirical models.

We find that the responses of all variables to shocks are theoretically plausible and broadly similar across various sample periods which exclude or include the recent financial crisis. Including the crisis period in the sample causes responses of the base and reserves to a policy shock to be significantly larger. We interpret that result to mean that during the crisis the Fed had to inject a much larger amount of money into the banking system to achieve a given amount of liquidity as measured by the Divisia quantity index. We also find that when the crisis period is added to the sample, the responses of price and output to a monetary shock are somewhat attenuated. These findings are consistent with the view that the effects of monetary policy were weakened during the crisis and its aftermath.

This paper finds the informational content of Divisia money is important for characterizing monetary shocks--and importantly, it does so under the financial crisis as well as normal conditions. Given that responses of base and reserves, in particular, are rather sensitive to whether or not the crisis is included in the sample, future work might allow for more general time variation in model parameters. One possibility might be to extend the work of Keating and Valcarcel (2013, 2014) using a similar VAR to the preferred specification in this paper. But given that our results are qualitatively robust to sample period, the only major benefit to time variation may be to more accurately measure how the quantitative effects of monetary policy shocks vary across alternative policy regimes or different market conditions. Another potentially productive area of future work would be use the identification strategy that we have developed to re-investigate the many interesting questions economists have addressed using our version of the classic model of monetary policy shocks.¹⁹

This paper's results call for a reconsideration of the role monetary aggregates may play in policy decisions. The field has largely moved away from considering monetary aggregates, in part because it has focused attention on the empirical and theoretical weaknesses of simple sum measures. Money was also expunged from most policy discussions based on the belief that the short-term interest rate captures all relevant information. But this is clearly not the case when that rate is stuck at its lower

¹⁹Many applications of the classic model are surveyed in Christiano, Eichenbaum and Evans (1999)

bound. Our findings suggest central banks should seriously consider allowing Divisia measures of money to play some role in monetary policy. We find that a rule based on a theoretically coherent measure of money seems to work just as well as one based on Fed Funds under normal conditions. And a Divisia rule still appears viable even when a financial crisis forced the central bank to push short-term nominal rates to zero. ²⁰Designing a monetary rule that performs well under alternative conditions, particularly financial crises that can emerge rapidly and with little warning, may have important social benefits. And more effort should focus on determining how best to use appropriately measured monetary aggregates in policy decisions.

²⁰Keating and Smith (2013) provide a formal theoretical investigation which finds welfare gains from using the Divisia quantity of money in policy rules even when the lower bound on rates has not been reached.

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A DSGE Model

This appendix describes the DSGE model used in the paper in detail. The model largely follows from Belongia and Ireland (2012), however, there are a few adjustments. For this reason, we describe the model in detail. Below, all uppercase variables are real and all lowercase variables are nominal, including interest rates. Also, Δ denotes the change over two consecutive time periods (in lag operator notation $\Delta = (1 - L)$). Finally, variables with a tilde over them denote log-deviations from steady-state.

A.1 The Household

The representative household enters any period $t = 0, 1, 2, \dots$ with a portfolio consisting of 3 assets. The household holds maturing bonds B_{t-1} , shares of monopolistically competitive firm $i \in [0, 1]$ $s_{t-1}(i)$, and currency totaling M_{t-1} . The household faces a sequence of budget constraints in any given period. This budgeting can be described by dividing period t into 2 separate periods: first a securities trading session and then bank settlement period.

In the securities trading session the household can buy and sell stocks, bonds, receives wages W_t for hours worked h_t during the period, purchases consumption goods C_t and obtains any loans L_t needed to facilitate these transactions. Any government transfers are also made at this time, denoted by T_t . Any remaining funds can be allocated between currency N_t and deposits D_t . This is summarized in the constraint below.

$$N_t + D_t = \frac{M_{t-1}}{\pi_t} + \frac{B_{t-1}}{\pi_t} - \frac{B_t}{r_t} - \int_0^1 Q_t (s_t(i) - s_{t-1}(i)) di + W_t h_t + L_t - C_t + T_t \quad (\text{A.1})$$

At the end of the period, the household receives dividends $F_t(i)$ on shares of stock owned in period t , $s_t(i)$, and settles all interest payments with the bank. In particular, the household is owed interest on deposits made at the beginning of the period, $r_t^D D_t$ and owes the bank interest on loans taken out, $r_t^L L_t$. Any remaining funds can be carried over in the form of currency into period $t + 1$, M_{t+1} .

$$M_t = N_t + \int_0^1 F_t(i) s_t(i) di + r_t^D D_t - r_t^L L_t \quad (\text{A.2})$$

The household seeks to maximize their lifetime utility, discounted at rate β , subject to these con-

straints. The period flow utility of the household takes the following form.

$$u_t = a_t \left[\frac{C_t^{(1-\theta_c)}}{1-\theta_c} + v_t \frac{(M_t^A)^{(1-\theta_m)}}{1-\theta_m} + \eta(1-h_t) \right]$$

The time-varying preference parameter a_t enters the linearized Euler equation as an IS shock and similarly, v_t enters the linearized money demand equation as a money demand shock. Both of these processes are assumed to follow an AR(1) (in logs).

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_t^a \quad \varepsilon_t^a \sim \mathcal{N}(0, \sigma_a) \quad (\text{A.3})$$

$$\ln(v_t) = (1 - \rho_v) \ln(\bar{v}) + \rho_v \ln(v_{t-1}) + \varepsilon_t^v \quad \varepsilon_t^v \sim \mathcal{N}(0, \sigma_v) \quad (\text{A.4})$$

The true monetary aggregate, M_t^A , which enters the period utility function takes a rather general CES form,

$$M_t^A = \left[\nu^{\frac{1}{\omega}} (N_t)^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} (D_t)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad (\text{A.5})$$

where ν calibrates the relative expenditure shares on currency and deposits and ω calibrates the elasticity of substitution between the two monetary assets.

The representative household faces the problem of maximizing its lifetime utility subject to its budget constraints. Letting $\zeta_t = [C_t, h_t, M_t^A, N_t, D_t, L_t, B_t, M_t, s_t(i)]$ denote the vector of choice variables, the household's optimization problem can be recursively defined using Bellman's method.

$$\begin{aligned} V_t(B_{t-1}, M_{t-1}, s_{t-1}(i)) = \max_{\zeta_t} & \left\{ a_t \left[\frac{C_t^{(1-\theta_c)}}{1-\theta_c} + v_t \frac{(M_t^A)^{(1-\theta_m)}}{1-\theta_m} + \eta(1-h_t) \right] \right. \\ & - \Lambda_t^1 \left(D_t + N_t + C_t - W_t h_t - L_t - M_{t-1} - T_t - B_{t-1} + \int_0^1 Q_t(i)(s_t(i) - s_{t-1}(i)) di + B_t/r_t \right) \\ & - \Lambda_t^2 \left(M_t^A - \left[\nu^{\frac{1}{\omega}} (N_t)^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} (D_t)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \right) \\ & \left. - \Lambda_t^3 \left(M_t - N_t - \int_0^1 F_t(i) s_t(i) di - r_t^D D_t + r_t^L L_t \right) + \beta \mathbb{E}_t [V_{t+1}(B_t, M_t, s_t(i))] \right\} \end{aligned}$$

The first order necessary conditions are given by the following equations. The system of equations (A.6)-(A.14) is under-determined in the sense that we have introduced various derivatives of the value function. However, we can complement these first order necessary conditions with the Bienveniste-

Scheinkman Envelope Conditions to eliminate the value function from the system above. These envelope conditions are given in equations (A.15)-(A.17) below.

$$a_t C_t^{-\theta_c} - \Lambda_t^1 = 0 \quad (\text{A.6})$$

$$-a_t \eta + \Lambda_t^1 W_t = 0 \quad (\text{A.7})$$

$$a_t v_t M_t^{A^{-\theta_m}} - \Lambda_t^2 = 0 \quad (\text{A.8})$$

$$N_t - \nu M_t^A \left[\frac{\Lambda_t^2}{\Lambda_t^1 - \Lambda_t^3} \right]^\omega = 0 \quad (\text{A.9})$$

$$D_t - (1 - \nu) M_t^A \left[\frac{\Lambda_t^2}{\Lambda_t^1 - \Lambda_t^3 r_t^D} \right]^\omega = 0 \quad (\text{A.10})$$

$$\Lambda_t^1 - \Lambda_t^3 r_t^L = 0 \quad (\text{A.11})$$

$$\frac{-\Lambda_t^1}{r_t} + \beta \mathbb{E}_t[V'_{t+1, B_t}(B_t, M_t, s_t(i))] = 0 \quad (\text{A.12})$$

$$-\Lambda_t^3 + \beta \mathbb{E}_t[V'_{t+1, M_t}(B_t, M_t, s_t(i))] = 0 \quad (\text{A.13})$$

$$-\Lambda_t^1 Q_t(i) + \Lambda_t^3 F_t(i) + \beta \mathbb{E}_t[V'_{t+1, s_t(i)}(B_t, M_t, s_t(i))] = 0 \quad (\text{A.14})$$

Envelope Conditions:

$$V'_{t, B_{t-1}}(B_{t-1}, M_{t-1}, s_t(i)) = \frac{\Lambda_t^1}{\pi_t} \quad (\text{A.15})$$

$$V'_{t, M_{t-1}}(B_{t-1}, M_{t-1}, s_t(i)) = \frac{\Lambda_t^1}{\pi_t} \quad (\text{A.16})$$

$$V'_{t, s_{t-1}(i)}(B_{t-1}, M_{t-1}, s_t(i)) = \Lambda_t^1 Q_t(i) \quad (\text{A.17})$$

Now update (A.15)-(A.17) and substitute the resulting equations into (A.12)-(A.14) yielding:

$$\frac{-\Lambda_t^1}{r_t} + \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}^1}{\pi_{t+1}} \right] = 0 \quad (\text{A.18})$$

$$-\Lambda_t^3 + \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}^1}{\pi_{t+1}} \right] = 0 \quad (\text{A.19})$$

$$-\Lambda_t^1 Q_t(i) + \Lambda_t^3 F_t(i) + \beta \mathbb{E}_t [\Lambda_{t+1}^1 Q_{t+1}(i)] = 0 \quad (\text{A.20})$$

The conditions (A.6)-(A.11) and (A.18)-(A.20) define the consumers optimal behavior.

A.2 The Goods Producing Sector

The goods producing sector features a final goods firm and an intermediate goods firm. There are a unit measure of intermediate goods producing firms indexed by $i \in [0, 1]$ who produce a differentiated product. The final goods firm produces Y_t combining inputs $Y_t(i)$ using the constant returns to scale technology,

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$$

in which $\theta > 1$ governs the elasticity of substitution between inputs, $Y_t(i)$. The final goods producing firm sells its product in a perfectly competitive market, hence solving the profit maximization problem,

$$\max_{Y_t(i)_{i \in [0, 1]}} P_t \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(i) Y_t(i) di.$$

The resulting first order condition defines the demand curve for each intermediate goods producing firm's product.

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t \quad (\text{A.1})$$

Intermediate Goods Producing Firm

Given the downward sloping demand for its product in (A.1), the intermediate goods producing firm has the ability to set the price of its product above marginal cost. Unlike the final goods market, the intermediate goods market is not purely competitive as evident by the downward sloping demand for its product in equation (A.1). To permit aggregation and allow for the consideration of a representative intermediate goods producing firm i , we assume all such firms have the same constant returns to scale technology which implies linearity in the single input labor $h_t(i)$,

$$Y_t(i) = Z_t h_t(i). \quad (\text{A.2})$$

The Z_t term in (A.2) is an aggregate technology shock that follows an AR(1) (in logs),

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + \varepsilon_t^Z \quad \varepsilon_t^Z \sim \mathcal{N}(0, \sigma_Z). \quad (\text{A.3})$$

The price setting ability of each firm is constrained in two ways. First, each intermediate goods producing firm faces a demand for its product from the representative final goods producing firm defined in (A.1). Second, each intermediate goods producing firm faces a convex cost of price adjustment proportional one unit of the final good defined by Rotemberg (1982) to take the form,

$$\Phi(P_t(i), P_{t-1}(i), P_t, Y_t) = \frac{\phi}{2} \left[\frac{P_t(i)}{P_{t-1}(i)} - 1 \right]^2 Y_t.$$

The intermediate goods producing firm maximizes its period t real stock price, $Q_t(i)$. Using the representative household's demand for firm i 's stock (A.20) and iterating forward defines the real (no-bubbles) share price as the discounted sum of future dividend payments.

$$Q_t(i) = \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \frac{\Lambda_{t+j}^3}{\Lambda_t^1} F_{t+j}(i) \right]. \quad (\text{A.4})$$

Though the firm maximizes period t share price, the costly price adjustment constraint makes the intermediate goods producing firm's problem dynamic. Mathematically summarizing, each intermediate goods producing firm solves to following dynamic problem.

$$\max_{\{Y_t(i), h_t(i), P_t(i)\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{\Lambda_{t+j}^3}{\Lambda_t^1} \left[\frac{P_t(i)}{P_t} Y_t(i) - W_t h_t(i) - \frac{\phi}{2} \left[\frac{P_t(i)}{P_{t-1}(i)} - 1 \right]^2 Y_t \right]$$

subject to

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta} Y_t$$

$$Y_t(i) = Z_t h_t(i)$$

The problem can be simplified by substituting the inverse of the technology constraint for $h_t(i)$ and then substituting the factor demand into the resulting expression for $Y_t(i)$ so that now the representative intermediate goods producing firm solves the following recursive problem defined by

Bellman's equation.

$$V_t(P_{t-1}(i)) = \max_{P_t(i)} \left\{ \frac{\Lambda_t^3 Y_t}{\Lambda_t^1} \left[\left[\frac{P_t(i)}{P_t} \right]^{1-\theta} - \frac{W_t}{Z_t} \left[\frac{P_t(i)}{P_t} \right]^{-\theta} - \frac{\phi}{2} \left[\frac{P_t(i)}{P_{t-1}(i)} - 1 \right]^2 \right] \right. \\ \left. + \beta \mathbb{E}_t [V_{t+1}(P_t(i))] \right\}$$

The first order necessary condition for the problem is given by

$$(1-\theta) \frac{\Lambda_t^3 Y_t}{\Lambda_t^1} \left[\frac{P_t(i)}{P_t} \right]^{-\theta} \frac{1}{P_t} + \theta \frac{\Lambda_t^3 Y_t}{\Lambda_t^1} \frac{W_t}{P_t Z_t} \left[\frac{P_t(i)}{P_t} \right]^{-1-\theta} \\ - \phi \frac{\Lambda_t^3 Y_t}{\Lambda_t^1} \left[\frac{P_t(i)}{P_{t-1}(i)} - 1 \right] \frac{1}{P_{t-1}(i)\pi} + \beta \mathbb{E}_t [V'_{t+1}(P_t(i))] = 0 \quad (\text{A.5})$$

Once again invoking the Bienveniste-Scheinkman Envelope Condition we have

$$V'_t(P_{t-1}(i)) = \frac{\Lambda_t^3 Y_t}{\Lambda_t^1} \phi \left[\frac{P_t(i)}{P_{t-1}(i)} - 1 \right] \frac{P_t(i)}{(P_{t-1}(i))^2}. \quad (\text{A.6})$$

Updating (A.6) one period and substituting into (A.5) and then multiplying the resulting equation by $\Lambda_t^1 P_t$ yields

$$(1-\theta) \left[\frac{P_t(i)}{P_t} \right]^{-\theta} + \theta \frac{W_t}{Z_t} \left[\frac{P_t(i)}{P_t} \right]^{-1-\theta} - \phi \left[\frac{P_t(i)}{P_{t-1}(i)} - 1 \right] \frac{P_t}{P_{t-1}(i)} \\ + \beta \phi E_t \left[\frac{\Lambda_{t+1}^3 Y_{t+1}}{\Lambda_t^3 Y_t} \left[\frac{P_{t+1}(i)}{P_t(i)} - 1 \right] \frac{P_t P_{t+1}(i)}{(P_t(i))^2} \right] = 0 \quad (\text{A.7})$$

In a symmetric equilibrium where $P_t(i) = P_t \forall i \in [0, 1]$, (A.7) can be log-linearized in which case it takes the form of a New-Keynesian Phillips Curve relating current inflation to the average real marginal cost and expected future inflation.

A.3 The Financial Firm

The financial firm produces deposits D_t and loans L_t for its client, the household. Following Belongia and Ireland (2012), assume that producing D_t deposits requires $x_t D_t$ units of the final good. In this case, x_t is the marginal cost of producing deposits and varies according to the AR(1) process (in logs),

$$\ln(x_t) = (1 - \rho_x) \ln(\bar{x}) + \rho_x \ln(x_{t-1}) + \varepsilon_t^x \quad \varepsilon_t^x \sim \mathcal{N}(0, \sigma_x). \quad (\text{A.1})$$

Therefore an increase in x_t can be interpreted as an adverse financial productivity shock.

In addition to this resource costs, the financial firm must satisfy the accounting identity which specifies assets (loans plus reserves) equal liabilities (deposits),

$$L_t + \tau_t D_t = D_t. \quad (\text{A.2})$$

Although changes in banking regulation have effectively eliminated reserve requirements, banks may often choose to hold reserves in lieu of making loans - a flight to quality of sorts. Therefore, instead of assuming the central bank controls the reserve ratio τ_t , assume it varies exogenously according to the AR(1) process (in logs),

$$\ln(\tau_t) = (1 - \rho_\tau)\ln(\bar{\tau}) + \rho_\tau \ln(\tau_{t-1}) + \varepsilon_t^\tau \quad \varepsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau). \quad (\text{A.3})$$

An increase in τ_t can therefore be interpreted as a reserves demand shock, as opposed to a change in policy.

The financial firm chooses L_t and D_t in order to maximize period profits

$$\max_{L_t, D_t} r_t^L L_t - r_t^D D_t - L_t + D_t - x_t D_t$$

subject to the balance sheet constraint (A.2). Since the loan and deposits markets are perfectly competitive, substituting the balance-sheet constraint into the profit function and imposing zero results in the loan-deposit spread,

$$r_t^L - r_t^D = \tau_t(r_t^L - 1) + x_t. \quad (\text{A.4})$$

This confirms that if banks choose to hold more reserves or become relatively less productive, the consumer will have to pay a higher interest rate on their loans relative to the rate they receive on their deposits.

A.4 Central Bank Policy

As is standard in New-Keynesian economies, the system of equations is under determined without a specification of monetary policy. Interest-rate rules are typically used to describe central bank

policy since Taylor's (1993) seminal paper,

$$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi^r \tilde{\pi}_t + \phi_y^r \tilde{Y}_t + \varepsilon_t^r \quad (\text{A.1})$$

where we have augmented the rule slightly to include a policy disturbance (which is white noise) and allow for the possibility that the central bank chooses to smooth movements in interest rates.

However, we may also describe central bank policy by a money growth rule. In this economy, there are multiple monetary assets, N_t and D_t . This raises the question of which monetary aggregate to control. One popular index produced by most central banks is the simple-sum aggregate.

Definition 3. *The growth rate of the nominal Simple-Sum monetary aggregate is defined by*

$$\ln(\mu^{SS}) = \ln\left(\frac{N_t + D_t}{N_{t-1} + D_{t-1}}\right) + \ln(\pi_t) \quad (\text{A.2})$$

However, this popular aggregate lacks many desirable features one would want in an index number. In particular, the simple-sum aggregate implicitly assumes N_t and D_t are perfect, one for one, substitutes. This is only true in the extreme cases that the true aggregate has only one asset (specified by (A.5)) $\nu = 1$, $\nu = 0$ or these assets enter in a linear fashion $\omega \rightarrow \infty$.

An alternative to the simple sum index is the Divisia Monetary Aggregate proposed by Barnett (1980). This index has been extensively studied over the last 30 years and has been shown to outperform the simple-sum alternative in both empirical and theoretical applications.²¹ Simply put, the Divisia index number is better suited to track the growth rate of the monetary service flow than its simple-sum counterpart.

Definition 4. *The growth rate of the nominal Divisia monetary aggregate is defined by*

$$\ln(\mu_t^{Divisia}) = \left(\frac{s_t^N + s_{t-1}^N}{2}\right) \ln\left(\frac{N_t}{N_{t-1}}\right) + \left(\frac{s_t^D + s_{t-1}^D}{2}\right) \ln\left(\frac{D_t}{D_{t-1}}\right) + \ln(\pi_t) \quad (\text{A.3})$$

where s_t^N and s_t^D are the expenditure shares of currency and interest bearing deposits respectively

²¹ For more general research examining the Divisia monetary aggregate's properties relative to alternative simple-sum measures see the following works. At paper length Barnett and Chauvet (2011b); Belongia (1996) and at book length Barnett and Singleton (1987); Belongia and Binner (2000); Barnett and Serletis (2000); Barnett and Chauvet (2011a); Barnett (2012).

defined by

$$s_t^N = \frac{u_t^N N_t}{u_t^N N_t + u_t^D D_t} = \frac{(r_t^L - 1)N_t}{(r_t^L - 1)N_t + (r_t^L - r_t^D)D_t} \quad (\text{A.4})$$

$$s_t^D = \frac{u_t^D D_t}{u_t^N N_t + u_t^D D_t} = \frac{(r_t^L - r_t^D)D_t}{(r_t^L - 1)N_t + (r_t^L - r_t^D)D_t}. \quad (\text{A.5})$$

By weighting the growth rate of the individual assets (with time-varying weights) the Divisia monetary aggregate is able to successfully account for changes in the composition of the aggregate which may have no impact on the overall aggregate. This superior accuracy places the Divisia index amongst Diewart's (1976) class of *superlative* index numbers, meaning the Divisia index has the ability track any linearly homogenous function (with continuous second-derivatives) up to second-order accuracy. Due to this superior accuracy, we choose to define the money growth rule in terms of the Divisia monetary aggregate (augmented to include a white noise monetary policy shock).

$$\tilde{\mu}_t^{Divisia} = \rho_\mu \tilde{\mu}_t^{Divisia} - \phi_\pi^\mu \tilde{\pi}_t - \phi_y^\mu \tilde{Y}_t - \varepsilon_t^\mu \quad (\text{A.6})$$

A.5 Market Clearing

It is now possible to define the equilibrium conditions which close the model. Equilibrium in the final goods market requires that the accounting identity

$$Y_t = C_t + x_t \frac{D_t}{P_t} + \frac{\phi}{2} [\pi_t - 1]^2 Y_t \quad (\text{A.1})$$

holds. Equilibrium in the money market, equity market and bond market requires that at all times

$$M_t = \frac{M_{t-1}}{\pi_t} + W_t h_t - C_t + T_t \quad (\text{A.2})$$

$$s_t(i) = s_{t-1}(i) = 1 \quad (\text{A.3})$$

$$B_t = B_{t-1} = 0 \quad (\text{A.4})$$

respectively. Finally, imposing the symmetry among the intermediate goods producing firms requires that in equilibrium

$$Y_t(i) = Y_t \tag{A.5}$$

$$h_t(i) = h_t \tag{A.6}$$

$$P_t(i) = P_t \tag{A.7}$$

$$F_t(i) = F_t \tag{A.8}$$

$$Q_t(i) = Q_t. \tag{A.9}$$

A.6 The Log-Linear System

The large system of 17 variables²² $N_t, D_t, M_t^A, W_t, h_t, C_t, L_t, \Lambda_t^1, \Lambda_t^2, \Lambda_t^3, r_t^D, r_t^L, r_t, \pi_t, Y_t, \mu_t^{Divisia}, \mu_t^{Simple-Sum}$ can be condensed down to a much smaller 5-equation New-Keynesian model (when appended with a money demand equation and the definition of the Simple-Sum aggregate) focusing on the typical macro time-series of interest. In what follows variables with a bar over them denote steady-state values and variables with a tilde denote log-deviations from steady-state.

The IS Curve

The dynamic IS curve is the log-linearized version of the household's Euler equation. In particular, combine equations (A.6) and (A.18) and take note of the stochastic process defined for a_t in (A.3).

$$\tilde{C}_t \approx \mathbb{E}_t[\tilde{C}_{t+1}] - \frac{1}{\theta_c} (\tilde{r}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]) + \left(\frac{1 - \rho_a}{\theta_c} \right) \tilde{a}_t \quad (\text{A.1})$$

This equation relates consumption to the real interest rate, similar to a typical IS curve. However, the appearance of expected future consumption makes this equation dynamic.

The Demand for Money and Monetary Aggregation

Now I seek a log-linear demand for the monetary aggregate. Two points are worth noting before proceeding to the linearization of the household's first order conditions. First, define the user costs of monetary assets, N_t and D_t . Specifically, the user costs appear naturally as the price of monetary assets according to the familiar optimality condition from microeconomics which dictates, at an optimum, equating the marginal rate of substitution of currency for deposits to the ratio of the price of currency to the price of deposits.

$$\frac{\frac{\partial u_t}{\partial N_t}}{\frac{\partial u_t}{\partial D_t}} = \frac{\frac{\partial u_t}{\partial M_t^A} \frac{\partial M_t^A}{\partial N_t}}{\frac{\partial u_t}{\partial M_t^A} \frac{\partial M_t^A}{\partial D_t}} = \frac{\frac{\Lambda_t^1 - \Lambda_t^3}{\Lambda_t^2}}{\frac{\Lambda_t^1 - \Lambda_t^3 r_t^D}{\Lambda_t^2}} = \frac{\Lambda_t^3 r_t^L - \Lambda_t^3}{\Lambda_t^3 r_t^L - \Lambda_t^3 r_t^D} = \frac{\frac{r_t^L - 1}{r_t^L}}{\frac{r_t^L - r_t^D}{r_t^L}} \equiv \frac{u_t^N}{u_t^D} \quad (\text{A.2})$$

Second, we can also derive the exact price dual to the true quantity aggregate in a similar fashion. Instead of considering the price of each monetary assets individually, consider the optimality condition the monetary aggregate must satisfy. For simplicity, consider the marginal rate of substitution

²²And 17 equations, (A.6)-(A.11), (A.18)-(A.20), (A.2), (A.7), (A.2), (A.4), (A.2), (A.3), (A.1), and either (A.1) or (A.6).

of the monetary aggregate for consumption. Since the price of the consumption good is normalized to 1, the result will denote the price of the monetary aggregate.

$$\frac{\frac{\partial u_t}{\partial M_t^A}}{\frac{\partial u_t}{\partial C_t}} = \frac{\Lambda_t^2}{\Lambda_t^1} = \left[\nu(u_t^N)^{(1-\omega)} + (1-\nu)(u_t^D)^{(1-\omega)} \right]^{\frac{1}{1-\omega}} \equiv \frac{r_t^L - r_t^A}{r_t^L} = u_t^A \quad (\text{A.3})$$

The first equality follows from equations (A.6) and (A.8), the household's first order conditions for C_t and M_t^A respectively. The second equality follows from solving equation (A.9) for D_t , equation (A.10) for N_t and substituting the resulting expressions into equation (A.5). We define this expression to be u_t^A , or the opportunity cost of holding the aggregate monetary asset M_t^A .

The resulting aggregate user cost (A.3) as defined by Barnett (1978) is of the same form as the individual component user costs in equation (A.2). It can be verified that u_t^A is in fact the true price dual to the true monetary aggregate in equation (A.5) since it satisfies Fisher's factor reversal test.

$$M_t^A u_t^A = u_t^N N_t + u_t^D D_t \quad (\text{A.4})$$

Equation (A.4) states that the true quantity index times the true price index equals total expenditures, in this sense, (A.3) is the exact price dual to (A.5).

Demand for the Monetary Aggregate

Using the relationships defined in the above equations allows us to easily define a typical log-linear demand curve for the monetary aggregate. Combining (A.6) and (A.8) and taking logs,

$$\tilde{M}_t^A = \frac{\theta_c}{\theta_m} \tilde{C}_t - \frac{1}{\theta_m} \tilde{u}_t^A + \frac{1}{\theta_m} \tilde{v}_t. \quad (\text{A.5})$$

Although this expression is compact, it is useful to further expand $\tilde{u}_t^A$ in terms of $\tilde{r}_t^L$.

$$\begin{aligned} \tilde{u}_t^A &\approx \left[\frac{\partial \ln(u_t^A)}{\partial \ln(u_t^N)} \right] \frac{\partial \ln(u_t^N)}{\partial \ln(r_t^L)} \tilde{r}_t^L + \left[\frac{\partial \ln(u_t^A)}{\partial \ln(u_t^D)} \right] \left[\frac{\partial \ln(u_t^D)}{\partial \ln(r_t^L)} \tilde{r}_t^L + \frac{\partial \ln(u_t^D)}{\partial \ln(\tau_t)} \tilde{\tau}_t + \frac{\partial \ln(u_t^D)}{\partial \ln(x_t)} \tilde{x}_t \right] \\ &= \left[\bar{s}^N \frac{1}{\bar{r}^L - 1} + \bar{s}^D \frac{(\bar{\tau} - \bar{x}) \bar{r}^L}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \right] \tilde{r}_t^L + \bar{s}^D \frac{\bar{\tau}(\bar{r}^L - 1)}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \tilde{\tau}_t + \bar{s}^D \frac{\bar{x}}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \tilde{x}_t \end{aligned} \quad (\text{A.6})$$

where

$$\bar{s}^N = \frac{\nu \bar{u}^{N(1-\omega)}}{\nu \bar{u}^{N(1-\omega)} + (1-\nu) \bar{u}^{D(1-\omega)}} \quad \text{and} \quad \bar{s}^D = 1 - \bar{s}^N.$$

Combining (A.5) and (A.6) we have the following demand curve for the monetary aggregate:

$$\tilde{M}_t^A \approx \eta_c \tilde{C}_t - \eta_r \tilde{r}_t^L - \eta_\tau \tilde{\tau}_t - \eta_x \tilde{x}_t + \eta_v \tilde{v}_t,$$

where

$$\begin{aligned}\eta_c &= \frac{\theta_c}{\theta_m} \\ \eta_r &= \frac{1}{\theta_m} \left[\bar{s}^N \frac{1}{\bar{r}^L - 1} + \bar{s}^D \frac{\bar{\tau} - \bar{x}}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \right] \\ \eta_\tau &= \frac{1}{\theta_m} \left[\bar{s}^D \frac{\bar{\tau}(\bar{r}^L - 1)}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \right] \\ \eta_x &= \frac{1}{\theta_m} \left[\bar{s}^D \frac{\bar{x}}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \right] \\ \eta_v &= \frac{1}{\theta_m}.\end{aligned}$$

Accuracy Properties of Divisia and Simple-Sum Aggregates

It is useful at this point to define the accuracy properties of the Divisia and Simple-Sum monetary aggregates in the context of the linearized economy. Two results emerge from this section. First, a log-linear approximation of the Divisia aggregate track the growth rate of the true monetary aggregate without error. The same can not be said for the simple-sum aggregate. The simple-sum aggregate and the growth rate of the true monetary aggregate differ at first order endogenously due to the loan rate and exogenously due to financial sector disturbances.

Divisia Monetary Aggregate

First notice, if we combine the component user costs and the aggregate user costs, defined in (A.2) and (A.3) respectively, with the demand for the component assets from the household's first order condition, equations (A.9) and (A.10), we can specify the factor demands for the components of the CES aggregate.

$$N_t = \nu M_t^A \left(\frac{u_t^A}{u_t^N} \right)^\omega \tag{A.7}$$

$$D_t = (1 - \nu) M_t^A \left(\frac{u_t^A}{u_t^D} \right)^\omega \tag{A.8}$$

Now using these in the definition of the Divisia monetary aggregate we have the following useful expression for the difference between the growth rates of the Divisia monetary aggregate and the true monetary aggregate.

$$\begin{aligned}
& \ln(\mu_t^{Divisia}) - \ln\left(\frac{M_t^A}{M_{t-1}^A}\right) \\
&= \omega \left[\Delta \ln(u_t^A(u_t^N, u_t^D)) - \frac{1}{2} (s_t^N + s_{t-1}^N) \Delta \ln(u_t^N) - \frac{1}{2} (s_t^D + s_{t-1}^D) \Delta \ln(u_t^D) \right] \\
&= E^D(\ln(u_t^N), \ln(u_{t-1}^N), \ln(u_t^D), \ln(u_{t-1}^D))
\end{aligned}$$

Now perform a first order Taylor approximation around the deterministic steady state (denoted by a subscript 'ss').²³

$$\begin{aligned}
& \ln(\mu_t^{Divisia}) - \Delta \ln(M_t^A) \\
&\approx \left[\frac{\partial E^D}{\partial \ln(u_t^N)} \right]_{ss} \tilde{u}_t^N + \left[\frac{\partial E^D}{\partial \ln(u_t^D)} \right]_{ss} \tilde{u}_t^D + \left[\frac{\partial E^D}{\partial \ln(u_{t-1}^N)} \right]_{ss} \tilde{u}_{t-1}^N + \left[\frac{\partial E^D}{\partial \ln(u_{t-1}^D)} \right]_{ss} \tilde{u}_{t-1}^D \\
&= \left[\frac{\partial E^D}{\partial \ln(u_t^N)} \right]_{ss} \tilde{u}_t^N + \left[\frac{\partial E^D}{\partial \ln(u_t^D)} \right]_{ss} \tilde{u}_t^D - \left[\frac{\partial E^D}{\partial \ln(u_t^N)} \right]_{ss} \tilde{u}_{t-1}^N - \left[\frac{\partial E^D}{\partial \ln(u_t^D)} \right]_{ss} \tilde{u}_{t-1}^D
\end{aligned}$$

²³Notice we do not include any terms of s_t^N, s_{t-1}^N, s_t^D nor s_{t-1}^D because derivative of the error term, E^D , with respect to these variables are zero since $\Delta \ln(\bar{u}^N) = \Delta \ln(\bar{u}^D) = 0$

Finally, notice that

$$\begin{aligned} \left[\frac{\partial E^D}{\partial \ln(u_t^N)} \right]_{ss} &= \omega \left[\frac{\partial \ln(u_t^A)}{\partial \ln(u_t^N)} - \frac{1}{2}(s_t^N + s_{t-1}^N) \right]_{ss} \\ &= \omega \left[\frac{\nu (\bar{u}^N)^{1-\omega}}{\nu (\bar{u}^N)^{1-\omega} + (1-\nu) (\bar{u}^D)^{1-\omega}} - \frac{\nu (\bar{u}^N)^{1-\omega}}{(\bar{u}^A)^{1-\omega}} \right] = 0 \end{aligned}$$

and

$$\begin{aligned} \left[\frac{\partial E^D}{\partial \ln(u_t^D)} \right]_{ss} &= \omega \left[\frac{\partial \ln(u_t^A)}{\partial \ln(u_t^D)} - \frac{1}{2}(s_t^D + s_{t-1}^D) \right]_{ss} \\ &= \omega \left[\frac{(1-\nu) (\bar{u}^D)^{1-\omega}}{\nu (\bar{u}^N)^{1-\omega} + (1-\nu) (\bar{u}^D)^{1-\omega}} - \frac{(1-\nu) (\bar{u}^D)^{1-\omega}}{(\bar{u}^A)^{1-\omega}} \right] = 0. \end{aligned}$$

Summarizing this more compactly, we have that up to first order,

$$\ln(\mu_t^{Divisia}) \approx \ln\left(\frac{M_t^A}{M_{t-1}^A}\right). \quad (\text{A.9})$$

Therefore, in all the simulations, the growth rate of the Divisia monetary aggregate equals that of the true monetary aggregate.

Simple-Sum Monetary Aggregate

Proceeding in a similar fashion as we did for the Divisia monetary aggregate, substitute the component factor demands into the definition of the simple-sum monetary aggregate.

$$\begin{aligned} \ln(\mu_t^{SS}) - \Delta \ln(M_t^A) &= \omega \Delta \ln(u_t^A(u_t^N, u_t^D)) + \Delta \ln(\nu(u_t^N)^{-\omega} + (1-\nu)(u_t^D)^{-\omega}) \\ &\equiv E^{SS}(\ln(u_t^N), \ln(u_t^D), \ln(u_{t-1}^N), \ln(u_{t-1}^D)). \end{aligned}$$

Now take a first-order Taylor expansion of the right hand side around the deterministic steady-state, denoted with a subscript ‘ss’.

$$\begin{aligned}
& E^{SS}(\ln(u_t^N), \ln(u_t^D), \ln(u_{t-1}^N), \ln(u_{t-1}^D)) \\
& \approx \left[\frac{\partial E^{SS}}{\partial \ln(u_t^N)} \right]_{ss} \tilde{u}_t^N + \left[\frac{\partial E^{SS}}{\partial \ln(u_t^D)} \right]_{ss} \tilde{u}_t^D + \left[\frac{\partial E^{SS}}{\partial \ln(u_{t-1}^N)} \right]_{ss} \tilde{u}_{t-1}^N + \left[\frac{\partial E^{SS}}{\partial \ln(u_{t-1}^D)} \right]_{ss} \tilde{u}_{t-1}^D \\
& = \left[\frac{\partial E^{SS}}{\partial \ln(u_t^N)} \right]_{ss} \tilde{u}_t^N + \left[\frac{\partial E^{SS}}{\partial \ln(u_t^D)} \right]_{ss} \tilde{u}_t^D - \left[\frac{\partial E^{SS}}{\partial \ln(u_t^N)} \right]_{ss} \tilde{u}_{t-1}^N - \left[\frac{\partial E^{SS}}{\partial \ln(u_t^D)} \right]_{ss} \tilde{u}_{t-1}^D \\
& = \left[\frac{\partial E^{SS}}{\partial \ln(u_t^N)} \right]_{ss} \Delta \tilde{u}_t^N + \left[\frac{\partial E^{SS}}{\partial \ln(u_t^D)} \right]_{ss} \Delta \tilde{u}_t^D \\
& = \omega \left[\frac{\partial \ln(u_t^A)}{\partial \ln(u_t^N)} - \frac{\partial \ln(\nu(u_t^N)^{-\omega} + (1-\nu)(u_t^D)^{-\omega})}{\partial \ln(u_t^N)} \right]_{ss} \Delta \tilde{u}_t^N \\
& + \omega \left[\frac{\partial \ln(u_t^A)}{\partial \ln(u_t^D)} - \frac{\partial \ln(\nu(u_t^N)^{-\omega} + (1-\nu)(u_t^D)^{-\omega})}{\partial \ln(u_t^D)} \right]_{ss} \Delta \tilde{u}_t^D \\
& = \omega \left[s^N - \frac{\nu(\bar{u}^N)^{-\omega}}{\nu(\bar{u}^N)^{-\omega} + (1-\nu)(\bar{u}^D)^{-\omega}} \right] \Delta \tilde{u}_t^N \\
& + \omega \left[s^D - \frac{(1-\nu)(\bar{u}^D)^{-\omega}}{\nu(\bar{u}^N)^{-\omega} + (1-\nu)(\bar{u}^D)^{-\omega}} \right] \Delta \tilde{u}_t^D \\
& = \omega \left[\frac{\nu(\bar{u}^N)^{1-\omega}}{\nu(\bar{u}^N)^{1-\omega} + (1-\nu)(\bar{u}^D)^{1-\omega}} - \frac{\nu(\bar{u}^N)^{-\omega}}{\nu(\bar{u}^N)^{-\omega} + (1-\nu)(\bar{u}^D)^{-\omega}} \right] \Delta \tilde{u}_t^N \\
& + \omega \left[\frac{(1-\nu)(\bar{u}^D)^{1-\omega}}{\nu(\bar{u}^N)^{1-\omega} + (1-\nu)(\bar{u}^D)^{1-\omega}} - \frac{(1-\nu)(\bar{u}^D)^{-\omega}}{\nu(\bar{u}^N)^{-\omega} + (1-\nu)(\bar{u}^D)^{-\omega}} \right] \Delta \tilde{u}_t^D
\end{aligned}$$

Let $\alpha = \bar{u}^D / \bar{u}^N \in (0, 1)$, then we have

$$\begin{aligned}
& E^{SS}(\ln(u_t^N), \ln(u_t^D), \ln(u_{t-1}^N), \ln(u_{t-1}^D)) \\
& \approx \omega \left[\frac{\nu(\alpha)^{\omega-1}}{\nu(\alpha)^{\omega-1} + (1-\nu)} - \frac{\nu(\alpha)^\omega}{\nu(\alpha)^\omega + (1-\nu)} \right] \Delta \tilde{u}_t^N \\
& + \omega \left[\frac{(1-\nu)}{\nu(\alpha)^{\omega-1} + (1-\nu)} - \frac{(1-\nu)}{\nu(\alpha)^\omega + (1-\nu)} \right] \Delta \tilde{u}_t^D \\
& = \omega \left[\frac{\nu(\alpha)^{\omega-1}(\nu(\alpha)^\omega + (1-\nu)) - \nu(\alpha)^\omega(\nu(\alpha)^{\omega-1} + (1-\nu))}{(\nu(\alpha)^\omega + (1-\nu))(\nu(\alpha)^{\omega-1} + (1-\nu))} \right] \Delta \tilde{u}_t^N \\
& + \omega \left[\frac{(1-\nu)(\nu(\alpha)^\omega + (1-\nu)) - (1-\nu)(\nu(\alpha)^{\omega-1} + (1-\nu))}{(\nu(\alpha)^\omega + (1-\nu))(\nu(\alpha)^{\omega-1} + (1-\nu))} \right] \Delta \tilde{u}_t^D \\
& = \omega \left[\frac{\alpha^\omega[(\alpha)^{-1}\nu(1-\nu) - \nu(1-\nu)]}{(\nu(\alpha)^\omega + (1-\nu))(\nu(\alpha)^{\omega-1} + (1-\nu))} \right] \left[\frac{\partial \ln(u_t^N)}{\partial \ln(r_t^L)} - \frac{\partial \ln(u_t^D)}{\partial \ln(r_t^L)} \right]_{ss} \Delta \tilde{r}_t \\
& - \omega \left[\frac{\alpha^\omega[(\alpha)^{-1}\nu(1-\nu) - \nu(1-\nu)]}{(\nu(\alpha)^\omega + (1-\nu))(\nu(\alpha)^{\omega-1} + (1-\nu))} \right] \left[\frac{\partial \ln(u_t^D)}{\partial \ln(\tau_t)} \right]_{ss} \tilde{\tau}_t \\
& - \omega \left[\frac{\alpha^\omega[(\alpha)^{-1}\nu(1-\nu) - \nu(1-\nu)]}{(\nu(\alpha)^\omega + (1-\nu))(\nu(\alpha)^{\omega-1} + (1-\nu))} \right] \left[\frac{\partial \ln(u_t^D)}{\partial \ln(x_t)} \right]_{ss} \tilde{x}_t \\
& \equiv \psi(\omega) \left[\frac{\bar{r}^L \bar{x}}{(\bar{r}^L - 1)(\bar{\tau}(\bar{r}^L - 1) + \bar{x})} \right] \Delta \tilde{r}_t^L \\
& - \psi(\omega) \left[\frac{\bar{\tau}(\bar{r}^L - 1)}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \right] \Delta \tilde{\tau}_t - \psi(\omega) \left[\frac{\bar{x}}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \right] \Delta \tilde{x}_t.
\end{aligned}$$

Summarizing this error term more compactly, we have:

$$\ln(\mu_t^{SS}) - \ln\left(\frac{M_t^A}{M_{t-1}^A}\right) \approx \psi_r^{SS} \Delta \tilde{r}_t^L - \psi_\tau^{SS} \Delta \tilde{\tau}_t - \psi_x^{SS} \Delta \tilde{x}_t, \quad (\text{A.10})$$

where:

$$\begin{aligned}
\psi(\omega) &= \omega \left[\frac{\alpha^\omega[(\alpha)^{-1}\nu(1-\nu) - \nu(1-\nu)]}{(\nu(\alpha)^\omega + (1-\nu))(\nu(\alpha)^{\omega-1} + (1-\nu))} \right] \\
\psi_r^{SS} &= \psi(\omega) \left[\frac{\bar{r}^L \bar{x}}{(\bar{r}^L - 1)(\bar{\tau}(\bar{r}^L - 1) + \bar{x})} \right] \\
\psi_\tau^{SS} &= \psi(\omega) \left[\frac{\bar{\tau}(\bar{r}^L - 1)}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \right] \\
\psi_x^{SS} &= \psi(\omega) \left[\frac{\bar{x}}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \right].
\end{aligned}$$

Notice that if $\omega \rightarrow \infty$, $\nu \rightarrow 1$ or $\nu \rightarrow 0$ then $\psi(\omega) \rightarrow 0$. Except for these extreme cases, the simple-sum monetary aggregate will fail to accurately track the true monetary aggregate.

New-Keynesian Phillips Curve

The New-Keynesian Phillips Curve, relating inflation to real marginal cost and expected future inflation, emerges from log-linearizing the intermediate goods producing firm's optimal pricing rule in a symmetric equilibrium (where $P_t(i) = P_t \forall i \in [0, 1]$ and $\Pi_t = P_t/P_{t-1}$), (A.7), around the deterministic steady-state.

$$0 = \theta e^{\ln(W_t) - \ln(Z)_t} - \phi \left(e^{2\ln(\Pi_t)} - e^{\ln(\Pi_t)} \right) \quad (\text{A.11})$$

$$+ \beta \phi \mathbb{E}_t \left[e^{\Delta[\ln(\Lambda_t^3) + \ln(Y_t)]} \left(e^{2\ln(\Pi_{t+1})} - e^{\ln(\Pi_{t+1})} \right) \right] \quad (\text{A.12})$$

$$\approx \theta \frac{\bar{W}}{\bar{Z}} (\tilde{W}_t - \tilde{Z}_t) - \phi \tilde{\pi}_t + \beta \phi \mathbb{E}_t [\tilde{\pi}_{t+1}]$$

Now substitute the household's consumption/leisure rule to eliminate the real wage. Also, make use of the fact that (A.7) evaluated in steady-state implies that $\bar{W} = \frac{\theta-1}{\theta}$.

$$\begin{aligned} \tilde{\pi}_t &\approx \frac{(\theta-1)\theta_c}{\phi} [\tilde{C}_t - \tilde{Z}_t] + \beta \mathbb{E}_t [\tilde{\pi}_{t+1}] \\ &= \kappa [\tilde{C}_t - \tilde{Z}_t] + \beta \mathbb{E}_t [\tilde{\pi}_{t+1}], \end{aligned} \quad (\text{A.13})$$

where $\kappa = (\theta-1)\theta_c/\phi$.

Output

The goods market clearing condition is useful for defining output in this economy so that we have a relevant counterpart for the variable in the VAR. Any reasonable calibration will imply in steady-state that output is largely equal to consumption, to be precise however, log-linearizing (A.1) implies,

$$\tilde{Y}_t \approx \frac{\bar{C}}{\bar{Y}} \tilde{C}_t + \left(1 - \frac{\bar{C}}{\bar{Y}} \right) (\tilde{x}_t + \tilde{D}_t). \quad (\text{A.14})$$

A.7 The Complete Log-Linearized Model

The set log-linear equations which we will analyze are given by the following equations. The semi-structural parameters $\eta_c, \eta_r, \eta_\tau, \eta_x, \eta_v, \kappa, \psi_r^{SS}, \psi_\tau^{SS}, \psi_x^{SS}$ and $\bar{s}^N$ and $\bar{s}^D$ are defined in the preceding

sections. Below, we utilize the fact that $\tilde{r}_t^L = \tilde{r}_t$.

Endogenous Variables

$$\tilde{C}_t = \mathbb{E}_t[\tilde{C}_{t+1}] - \frac{1}{\theta_c} (\tilde{r}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]) + \left(\frac{1 - \rho_a}{\theta_c} \right) \tilde{a}_t \quad (\text{A.7.1})$$

$$\tilde{M}_t^A = \eta_c \tilde{C}_t - \eta_r \tilde{r}_t - \eta_\tau \tilde{\tau}_t - \eta_x \tilde{x}_t + \eta_v \tilde{v}_t \quad (\text{A.7.2})$$

$$\tilde{\pi}_t = \kappa [\tilde{C}_t - \tilde{Z}_t] + \beta \mathbb{E}_t[\tilde{\pi}_{t+1}] \quad (\text{A.7.3})$$

$$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi^r \tilde{\pi}_t + \phi_y^r \tilde{Y}_t + \varepsilon_t^r$$

$$\text{or} \quad (\text{A.7.4})$$

$$\tilde{\mu}_t^{Divisia} = \rho_\mu \tilde{\mu}_t^{Divisia} - \phi_\pi^\mu \tilde{\pi}_t - \phi_y^\mu \tilde{Y}_t - \varepsilon_t^\mu$$

$$\tilde{Y}_t = \frac{\bar{C}}{\bar{Y}} \tilde{C}_t + \left(1 - \frac{\bar{C}}{\bar{Y}} \right) \bar{Y} (\tilde{x}_t + \tilde{D}_t) \quad (\text{A.7.5})$$

$$\tilde{u}_t^A = \bar{s}^N \tilde{u}_t^N + \bar{s}^D \tilde{u}_t^D \quad (\text{A.7.6})$$

$$\tilde{u}_t^D = \left[\frac{\bar{\tau} - \bar{x}}{\bar{\tau}(\bar{r} - 1) + \bar{x}} \right] \tilde{r}_t + \left[\frac{\bar{\tau}(\bar{r}^L - 1)}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \right] \tilde{\tau}_t + \left[\frac{\bar{x}}{\bar{\tau}(\bar{r}^L - 1) + \bar{x}} \right] \tilde{x}_t \quad (\text{A.7.7})$$

$$\tilde{D}_t = \tilde{M}_t^A - \omega (\tilde{u}_t^D - \tilde{u}_t^A) \quad (\text{A.7.8})$$

$$\tilde{u}_t^N = \frac{1}{\bar{r} - 1} \tilde{r}_t \quad (\text{A.7.9})$$

$$\tilde{N}_t = \tilde{M}_t^A - \omega (\tilde{u}_t^N - \tilde{u}_t^A) \quad (\text{A.7.10})$$

$$\tilde{\mu}_t^{Divisia} = \tilde{M}_t^A - \tilde{M}_{t-1}^A + \tilde{\pi}_t \quad (\text{A.7.11})$$

$$\tilde{\mu}_t^{SS} = \tilde{M}_t^A - \tilde{M}_{t-1}^A + \tilde{\pi}_t + \psi_r^{SS} \Delta \tilde{r}_t - \psi_\tau^{SS} \Delta \tilde{\tau}_t - \psi_x^{SS} \Delta \tilde{x}_t \quad (\text{A.7.12})$$

$$\tilde{\pi}_t = p_t - p_{t-1} \quad (\text{A.7.13})$$

Exogenous Variables

$$\tilde{a}_t = \rho_a \tilde{a}_{t-1} + \varepsilon_t^a \quad (\text{A.7.14})$$

$$\tilde{v}_t = \rho_v \tilde{v}_{t-1} + \varepsilon_t^v \quad (\text{A.7.15})$$

$$\tilde{Z}_t = \rho_Z \tilde{Z}_{t-1} + \varepsilon_t^Z \quad (\text{A.7.16})$$

$$\tilde{\tau}_t = \rho_\tau \tilde{\tau}_{t-1} + \varepsilon_t^\tau \quad (\text{A.7.17})$$

$$\tilde{x}_t = \rho_x \tilde{x}_{t-1} + \varepsilon_t^x \quad (\text{A.7.18})$$

Combining equations (A.7.1) - (A.7.4), depending on the choice of policy rules (rules that don't react to output) is an identified system in the sense that there are as many equations as endogenous

variables. If output is included, then the system grows by 1 equation. We analyze the larger system so that axillary variables such as the simple-sum monetary aggregate, reserves and the monetary base can be analyzed in the context of the model.

A.8 Calibration

Before dynamics of monetary policy shocks can be studied, values must be assigned to the model's parameters. In what follows, we describe the approach to choosing values for the parameters, following Kydland and Prescott's (1982) calibration strategy. The choice of parameters is concisely summarized in Table 7 below.

Steady-State Parameters

The model has 11 deep structural parameters, β , η , θ_c , θ_m , $\bar{v}$, ω , ν , θ , ϕ , $\bar{\tau}$ and $\bar{x}$, which pin down the steady-state. Values for these parameters are chosen to match U.S. quarterly data running from 1967 to 2007. Beginning with the household's preference parameters, the model implies the benchmark interest rate equals the rate on 1 period bonds. Using data from the Center for Financial Stability (CFS), the average quarterly gross benchmark interest rate equals 1.02, implying $\beta = .98$. Calibrating $\eta = 1.7$ implies the steady-state share of time spent working equals 1/3. Setting $\theta_c = 1$ implies log-utility. The average ratio of simple-sum M2 to personal consumption expenditures, $\frac{N+D}{C} = 3.3$, the average value in the data when we set $\bar{v} = .04$. The CRRA parameter for monetary services pins down the interest semi-elasticity. There is a long-standing literature estimating this parameter across various money-demand specifications, however a more recent estimate is provided by Ireland (2009) who finds that $\eta_r = 1.9$. Matching this estimate, we set $\theta_m = 10$.

An estimate for ω , the elasticity of substitution between currency and deposits, can be found using OLS to estimate the following regression model.

$$\ln \left(\frac{N_t}{M_t^A} \right) = \beta_0 + \beta_1 \ln \left(\frac{u_t^N}{u_t^A} \right) + \varepsilon_t$$

Just as for the VAR model, the data for components and user costs are from the CFS. The resulting estimates are $\hat{\beta}_0 = -.17$ and $\hat{\beta}_1 = -.4956$ which implies that $\omega = .5$. Setting $\nu = .2566$ calibrates the steady state share $\frac{\bar{N}}{\bar{N}+\bar{D}} = .164$, the average of currency to simple-sum M2 over 1967 to 2007.

On the production side of the economy, the model can not identify θ and ϕ independently, but

instead we set the ratio $\frac{(\theta-1)\theta_c}{\phi} = .025 = \kappa$, the estimated slope of the NKPC from Rotemberg and Woodford (1997) who match the dynamic responses from an equilibrium model to an estimated VAR monetary policy IRF. As for the production of financial assets, $\bar{\tau} = .02$ matches the average reserves ratio from 1967 to 2007 using data from the Federal Reserve Bank of St.Louis on bank reserves and non-currency components of M2. Finally, setting $\bar{x} = 0.006$ implies the annualized steady-state spread, $\bar{r}^L - \bar{r}^D = .027$, the average annualized spread between the benchmark interest rate and the own-rate of return on the non-currency component of M2 using data from the CFS.

To verify the logic of the calibration, the coefficient determining the Simple-Sum's error term, ψ_r^{SS} (See Equation (A.10)), is estimated in Keating and Smith (2012), where they find that $\hat{\psi}_r^{SS} = 5.6362$ with standard error of $S.E. = .6467$. The implied value of ψ_r^{SS} in the steady state of this model, given the above calibration, is slightly larger than 5 confirming the above parameterization of the steady-state is consistent with the data along this additional dimension.

Table 7: Calibration Results

Parameter		Value
Discount Rate:	β	0.98
Disutility of Work:	η	1.70
Consumption CRRA:	θ_c	1.00
Monetary Services CRRA:	θ_m	10
Money Demand Shock:	v	0.04
CES Monetary Aggregate:	ν	0.25
CES Monetary Aggregate:	ω	0.5
Final Goods CES:	θ	2.25
Cost of Price Adjustment:	ϕ	50
Reserves Demand Shock:	$\bar{\tau}$	0.02
Banking Productivity Shock:	$\bar{x}$	0.0062
Preference Shock:	ρ_a	0.9579
Money Demand Shock:	ρ_v	0.9853
Technology Shock:	ρ_Z	0.9853
Reserves Demand Shock:	ρ_τ	0.9843
Banking Productivity Shock:	ρ_x	0.9535
Preference Shock:	σ_a	0.0188
Money Demand Shock:	σ_v	0.0088
Technology Shock:	σ_Z	0.0098
Reserves Demand Shock:	σ_τ	0.0319
Banking Productivity Shock:	σ_x	0.1549
Monetary Policy Shock:	σ_r	0.0025

Dynamic Parameters

The model has a remaining set of parameters which do not enter into the steady-state of the model's equations, but instead describe the central bank's behavior and the statistical properties of the exogenous state variables, $\tilde{a}_t$, $\tilde{v}_t$, $\tilde{Z}_t$, $\tilde{\tau}_t$, $\tilde{x}_t$ and ε_t^i where $i \in \{r, \mu, d\}$.

The IS, money-demand, technology and monetary policy shocks are standard in the DSGE literature and have been estimated in similar models. In particular, Ireland (2004) model's these variables as stationary AR(1) processes and estimates their autocorrelation parameters and standard deviations using FIML methods. Although the policy parameters are also estimated, the purpose of this study is largely to focus on the dynamics of monetary policy shocks under various rules. Hence in what follows we vary rules and parameters.

The financial shock processes have not been estimated in the context of a monetary New-Keynesian model. Fortunately though, both series can be recovered using observable variable and the model's equations. Specifically, the ratio of deposits (non-currency components of M2) held as reserves can be recovered using the St. Louis Fed's adjusted reserves series. The demeaned logged series has estimated parameters, $\rho_\tau = .98$ and $\sigma_\tau = .0038$. Then using this series along with the benchmark interest rate and the own rate of return on non-currency components of M2 and equation (A.4) allows us to back-out a time series for x_t . The demeaned logged series has estimated parameters, $\rho_x = .95$ and $\sigma_x = .0363$.