

Friedman and Divisia Monetary Measures

William A. Barnett

University of Kansas, Lawrence, Kansas

Center for Financial Stability, New York City

IC² Institute, University of Texas at Austin

Working paper version of chapter prepared for Robert Cord and Dan Hammond (eds.), *Milton Friedman: Contributions to Economics and Public Policy*, Oxford University Press, forthcoming.

Abstract: This paper explores the relationship between Milton Friedman's work and the work on Divisia monetary aggregation, originated by William A. Barnett. The paradoxes associated with Milton Friedman's work are largely resolved by replacing the official simple-sum monetary aggregates with monetary aggregates consistent with economic index number theory, such as Divisia monetary aggregates. Demand function stability becomes no more of a problem for money than for any other good or service. Money becomes relevant to monetary policy in all macroeconomic traditions, including New Keynesian economics, real business cycle theory, and monetarist economics.

Research and data on Divisia monetary aggregates are available for over 40 countries throughout the world from the online library within the Center for Financial Stability's (CFS) program, *Advances in Monetary and Financial Measurement*. This paper supports adopting the standards of monetary data competency advocated by the CFS and the International Monetary Fund (2008, pp. 183-184).

Keywords: Divisia monetary aggregates, demand for money, monetarism, index number theory.

Contributor Biography: William A. Barnett is the Oswald Distinguished Professor at the University of Kansas, Director of the Center for Financial Stability in NY City, and Senior Research Fellow at the IC² Institute of the University of Texas at Austin. He is Founder and Editor of the Cambridge University Press journal, *Macroeconomic Dynamics*, and the Emerald Press monograph series, *International Symposia in Economic Theory and Econometrics*. He is the Founder and President of the Society for Economic Measurement and originator of the Divisia monetary aggregates. His book with Paul Samuelson, *Inside the Economist's Mind*, has been translated into six languages. His MIT Press book, *Getting It Wrong: How Faulty Monetary Statistics Undermine the Fed, the Financial System, and the Economy*, won the American Publishers Award for Professional and Scholarly Excellence (the PROSE Award) for the best book published in economics during 2012.

Friedman and Divisia Monetary Measures William A. Barnett

“This [simple summation] procedure is a very special case of the more general approach. In brief, the general approach consists of regarding each asset as a joint product having different degrees of ‘moneyness,’ and defining the quantity of money as the weighted sum of the aggregated value of all assets, the weights for individual assets varying from zero to unity with a weight of unity assigned to that asset or assets regarded as having the largest quantity of ‘moneyness’ per dollar of aggregate value. The procedure we have followed implies that all weights are either zero or unity.

The more general approach has been suggested frequently but experimented with only occasionally. We conjecture that this approach deserves and will get much more attention than it has so far received.”

[Friedman and Schwartz, (1970, pp. 151—152)]

1. Introduction

The fact that simple sum monetary aggregation is unsatisfactory has long been recognized, and there has been a steady stream of attempts at weakening the implied perfect substitutability assumption by constructing weighted average monetary aggregates. See, for example, Hawtrey (1930), Gurley (1960, p. 7—8), Friedman and Meiselman (1963, p. 185), Kane (1964), Ford and Stark (1967), Chetty (1969, 1972), Friedman and Schwartz (1970 p. 151—52), Steinhauer and Chang (1972), Lee (1972), Bisignano (1974), Moroney and Wilbratte (1976), and Barth, Kraft, and Kraft (1977).¹

Since those weighted averages were not directly based upon aggregation or index number theory, there is a continuum of such potential “ad hoc” weighting

¹ Belongia (1995) provides a survey of work on weighted monetary aggregates.

schemes, with no basis in economic theory to prefer one over the others.

Economic aggregation and index number theory were first applied to monetary aggregation by Barnett (1980), who constructed monetary aggregates based upon Diewert's (1976) class of 'superlative' quantity index numbers with user cost pricing. Barnett's 1980 paper, formally derived from intertemporal economic optimization theory, marks the beginning of modern monetary aggregation theory.

Since simple sum aggregation is clearly inappropriate at high levels of aggregation, most economists have rarely placed much faith in the broader aggregates. An exception was Milton Friedman, who rejected M1 and became a strong advocate of M2. Nevertheless, Friedman and Schwartz, (1970, pp. 151-152) clearly described the problem with high level simple sum aggregates in the quotation at the top of this article.

I had not been aware of that quotation, when I published my 1980 paper. I found out from a surprising phone call. Shortly after the paper appeared, I received a phone call from my old friend, Arnold Zellner, at the University of Chicago. He told me that he had had lunch with Friedman at the Hoover Institution, and Friedman wanted Zellner to ask me to cite the above quotation from the Friedman and Schwarz book. I do not know why Friedman felt the need for a middle man. He could have called me directly, and I would have been more than happy to cite him on the subject. Clearly Friedman and Schwartz had acknowledged the problem's existence and the need for its solution.

In fact, there is a long footnote to that quotation, listing many of Friedman's students who had worked on that problem in their dissertations at the University of Chicago. Evidently trying to solve that problem was a major issue in the well-known monetary workshop for his PhD students. The fact that they had not been able to solve the problem was not a result of lack of focus or will.² The reason was that the user cost price of monetary assets had not yet been derived until Barnett (1978) published his derivation, and the gap between index number theory and aggregation theory had not yet been closed, prior to Diewert's (1976) work on superlative index numbers, unifying aggregation and index number theory.

Nevertheless, Friedman had anticipated the nature of the user cost price of monetary assets in his classic work, Friedman (1956), in which he recognized the relevancy of the opportunity cost of holding money and the fact that velocity is not a constant but rather depends upon interest rates. This correct conclusion was a break from the early Cambridge equation approach, which postulated a constant velocity. But for policy purposes, he advocated focusing only on the long run, under the assumption that the interest rate on capital is at its nearly constant "natural level." Hence for Federal Reserve policy, he did advocate the Cambridge equation, depending only on permanent income, with the velocity function evaluated with its arguments set at their long run natural values, viewed as constant. He argued that interest rate and velocity variations over the business

² They had recognized that money is a joint product in Friedman and Schwartz (1969).

cycle were normal parts of the economy's transient response and should not motivate attempts at countercyclical policy, which he viewed as often counterproductive, because of long lags in policy response and in the transmission mechanism.

When I derived the foundations of modern monetary aggregation and index number theory and began computing and supplying the Divisia monetary aggregates, I was on the staff of the Federal Reserve Board. I had the tools available that I needed. While Friedman and his students were ahead of their time in their focus, they did not have the tools they needed.

Of course I was very flattered that Friedman would want me to cite his work as supporting mine and as showing how much time and effort he and his students had put into trying to solve the problem. In Barnett (1982) I cited Friedman and Schwartz (1970) and provided the above quotation. I also have done so repeatedly in subsequent publications, such as Barnett and Serletis (2000, p. 127), Barnett and Chauvet (2011a, p. 11; 2011b, p. 18), and Barnett (2012, p. 44).

Friedman was often criticized for his skeptical views about the Federal Reserve, even when he was visiting at the Federal Reserve. While I was on the staff of the Federal Reserve Board, I heard how this developed. He was on the semiannual panel of economic advisors to the Federal Reserve Board. At one meeting, he stood up, said the Fed was not listening to him, left the room, and

never returned. The Federal Reserve would have liked him to continue in that roll, since as a critic he was viewed far more favorably than Karl Brunner, who was really disliked. The guards at the entrances to the Federal Reserve building were given orders never to permit Brunner in the building. There were no such orders to keep out Friedman, but he never did return.

I have a somewhat unusual view of Friedman's criticisms of Federal Reserve policy. I do not believe that he was too distrustful of the Federal Reserve. On the contrary, I argue that his occasional misjudgments on monetary policy were caused by excessive trust in the Federal Reserve. He regularly reached conclusions based on the Federal Reserve's official simple sum monetary aggregates, which he should not have trusted.

In contrast, during an airport conversation, I asked Brunner whether he was upset about being barred from the Federal Reserve Board building. He smiled at me, and said, "Bill, nothing has done more for my career."

2. The Problem

By equally weighting components, simple sum aggregation can badly distort an aggregate. If one wished to obtain an aggregate of transportation vehicles, one would never aggregate by simple summation over the physical units, of, say, subway trains and roller skates. Instead one could construct a quantity index using weights that depend upon the values of the different modes of transportation. As a further example of this point, suppose the money supply were

measured by the Federal Reserve's former broadest simple-sum aggregate, L . It included most of the national debt of short and intermediate maturity. All of that debt could be monetized (bought and paid for with freshly printed currency) without increasing either taxes or L , since the public would simply have exchanged component securities in L for currency. However, the Divisia index over the components of L would not treat this transfer as an exchange of 'pure money' for 'pure money.' Instead Divisia L would likely rise at about the same rate as the inflation in prices that would result from this action. In short, Divisia L would behave in a manner consistent with Friedman's frequently published views about the neutrality of money, while simple sum L would not.

At the opposite extreme, many problems associated with policies that target low level aggregates, such as $M1$, result from the inability of those aggregates to internalize pure substitution effects occurring within the economy's transactions technology, because the aggregation is over such a small subset of the factors of production in that transactions technology. Friedman's early rejection of $M1$ is consistent with that theory.

The traditionally constructed high level aggregates (such as $M2$, former $M3$, and former L) implicitly view distant substitutes for money as perfect substitutes for currency. Rather than capturing only part of the economy's monetary services, as $M1$ does, the broad aggregates swamp the included monetary services with heavily weighted nonmonetary services. Nevertheless, the need remains for an

aggregate that captures the contributions of all monetary assets to the economy's flow of monetary services.

3. The Divisia Quantity Index

3.1. The Definition

In this section, I define the Divisia monetary quantity index and describe the procedure for computing it.

Before the Divisia index formula can be computed, the user cost price of each component monetary asset must be computed, since the price of the services of a durable, such as money, is its user cost. The complete formula for the user cost price of a monetary asset was derived in Barnett (1978, 1980). However, a simplification is possible. All of the factors in the formula for the i th asset's user cost, π_i , can be shown to cancel out of the Divisia index except for $R - r_i$, where for any given time period, r_i is the own rate of return on asset i and R is the expected maximum holding period yield available in the economy during that period.

The growth rate of the Divisia quantity index is a weighted average of the growth rates of the component quantities. The weights are the corresponding user-cost-evaluated *value shares*. Since those value shares represent the contributions of the components to expenditure on the services of all of the components, use of those shares as weights for the corresponding component

quantity growth rates makes intuitive sense but are derived from a formal mathematical proof, not from *ad hoc* economic intuition.³

It is important to recognize that the user costs *are not the weights*, but rather are the *prices* used along with all of the quantities in computing the weights. Each weight depends upon all prices and all quantities. In fact, if the own price elasticity of demand of asset *i* is greater than one, then changes in the asset's own rate of return will induce changes in the asset's weight in the same direction. In the case of Cobb-Douglas utility, the expenditure shares, and therefore the Divisia growth rate weights, are independent of prices, while in general the direction in which the weights move, when own interest rates move, depends upon whether the own price elasticity of demand is greater than or less than 1.0.

In addition, it is important to recognize that the shares are not weights on levels, but on growth rates. Solving the Divisia differential equation for the level of the aggregate shows that levels of aggregates are deeply nonlinear line integrals and cannot be expressed as weighted averages of component levels. Most misunderstandings of the Divisia monetary aggregates result from lack of understanding of that fact.

It can be shown that the simple sum aggregates are a special case of the Divisia aggregates. If all own rates of return on all monetary assets are the same, then the growth rates of the Divisia indices reduce to the growth rates of the

³ For a presentation of the general theory of Divisia aggregation, see Balk (2005; 2008, chapter 6).

corresponding simple sum aggregates. The implied assumption of always equal own rates could be justified, if all component monetary assets were perfect substitutes. In fact, many decades of accumulated empirical research, reported in the published literature, overwhelmingly suggest that substitutability among different monetary assets is low, and of course all component own rates of return are not the same, except when all interest rates have been driven down to the zero rate of return on currency. In short, simple sum monetary aggregation can be justified as an approximation, only when financial markets are in a liquidity trap, as has been the case in the United States during and immediately after the Great Recession.⁴

3.2. The Effects of Interest Rate Changes

If the interest rate on a component monetary asset is changed (resulting in a change in the user cost), asset holders will respond by substituting towards the assets with relatively decreased user costs (i.e., increased own yield). The Divisia monetary quantity index will change only if the approximated economic quantity aggregate changes. The economic quantity aggregate can be a utility, distance, or production sub-function, such as a transactions technology, weakly separable from the economy's structure, to satisfy the theoretical existence condition for an aggregator function. Hence the aggregate will change only if the change in relative

⁴ By "liquidity trap," I mean a situation in which nominal interest rates are near their lower bound of zero. I do not mean to suggest that the central bank loses its ability to influence liquidity, when interest rates are near zero. The latter was shown not to be the case by Brunner and Meltzer (1968).

prices results in a change in the aggregator function, or, equivalently, in the monetary asset service flow, as perceived by the economic agent. Equivalently, the index number will change only if the change in the interest rate has an income effect. Any quantity index number constructed from index number theory will perfectly internalize pure substitution effects, which, by definition, occur at constant levels of monetary service flows. Hence, the aggregate will not change, when it should not change. The simple sum aggregate, on the other hand, does not internalize pure substitution effects. Interest rate changes cause shifts in a simple sum aggregate, even when there has been no change in monetary service flows.

The conclusions reached in the previous paragraph are produced by economic index number theory. But it should be observed that the exact same Divisia monetary index can be acquired without the use of economic theory. The statistical theory of index numbers, advocated by Henri Theil (1967) and others, can produce the same index from stochastic reasoning.⁵ In short, the expenditure shares, which are all between 0 and 1, are viewed as probabilities, and the index's growth rates are produced as a statistical mean of the component growth rates with those probabilities. The resulting Divisia index is then interpreted in terms of statistical sampling theory.

⁵ See Barnett and Serletis (2000, pp. 172-173).

4. Graphical Comparisons of the Divisia and Simple Sum Aggregates

The extensive available graphical comparisons between the behavior of the Divisia and simple sum monetary aggregates are scattered over various sources. In this section, some of the more interesting graphical comparisons from those sources are collected together and discussed relative to the views of Milton Friedman.

The first charts of the behavior of the Divisia aggregates appeared in Barnett (1980) and Barnett and Spindt (1979). One of those charts is reproduced below as Figure 1. That figure contains plots of the velocity of the Federal Reserve's former M3 and of M3⁺, which roughly corresponds to the aggregate later called *L* and now called M4 by the Center for Financial Stability in New York City. Velocity is plotted in each of those cases with the monetary aggregate alternatively computed as a simple sum or as a Divisia quantity index. With M3⁺ the Laspeyres quantity index also is used. These figures are of particular importance, since they focus on the years that produced the view of money demand structural breaks and instability, with particular emphasis on 1974.

The velocity of the Divisia aggregates can be seen to follow a path that closely resembles that of the interest rate cycle from 1968 to 1978, as displayed in Figure 2. Hence velocity appears to be a stable function of the opportunity-cost interest rate. In addition, the Laspeyres quantity index moves much more closely to the Divisia index than to the simple sum index. By contrast, the velocity of the

simple sum aggregates can be seen to trend downwards in a manner that violates theoretical views regarding the behavior of velocity during periods of rising interest rates and inflationary expectations. Substitution between money and higher yielding assets (disintermediation) appears to go in the wrong direction.

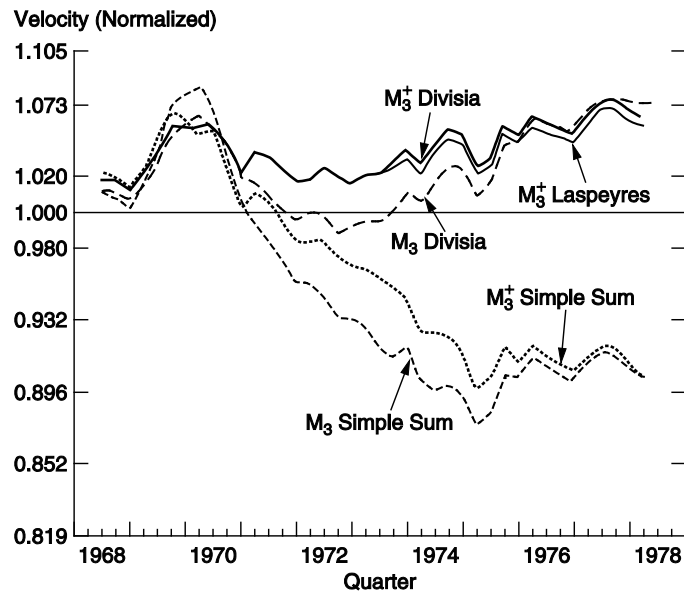


Figure 1: Seasonally adjusted normalized velocity during the 1970s.

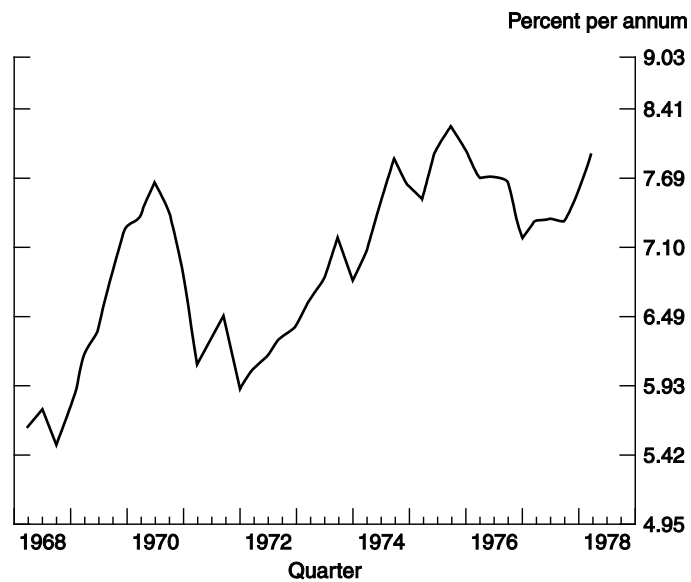


Figure 2: Interest rates during the 1970s: 10 year government bond rate.

Most of the concern about economic “paradox” in the 1970s was focused on 1974, when it was believed that there was a sharp structural shift in money markets in the form of a shift that could not be explained by interest rate movements. Figure 3 displays a source of that concern, taken very seriously by the Federal Reserve Board staff at that time. In figure 3, we have plotted velocity against a bond rate, rather than against time. As is evident from Figure 3, there appears to be a dramatic shift downwards in velocity in 1974, and clearly that shift cannot be explained by interest rates, since it happened suddenly at a time when there was little change in interest rates. But observe that this result was acquired using simple sum M3. Figure 4 displays the same cross plot of velocity against an interest rate, but with M3 computed as its Divisia index. Observe that velocity no longer is constant, either before or after 1974. But there is no structural shift, since the variations in velocity correlate with interest rates. In fact the plot not only has no discontinuous jump at a particular interest rate, but is nearly along a straight (linear) line. Also note that the line slopes in the correct direction: velocity goes up when interest rates go up, as is consistent with demand for money theory.

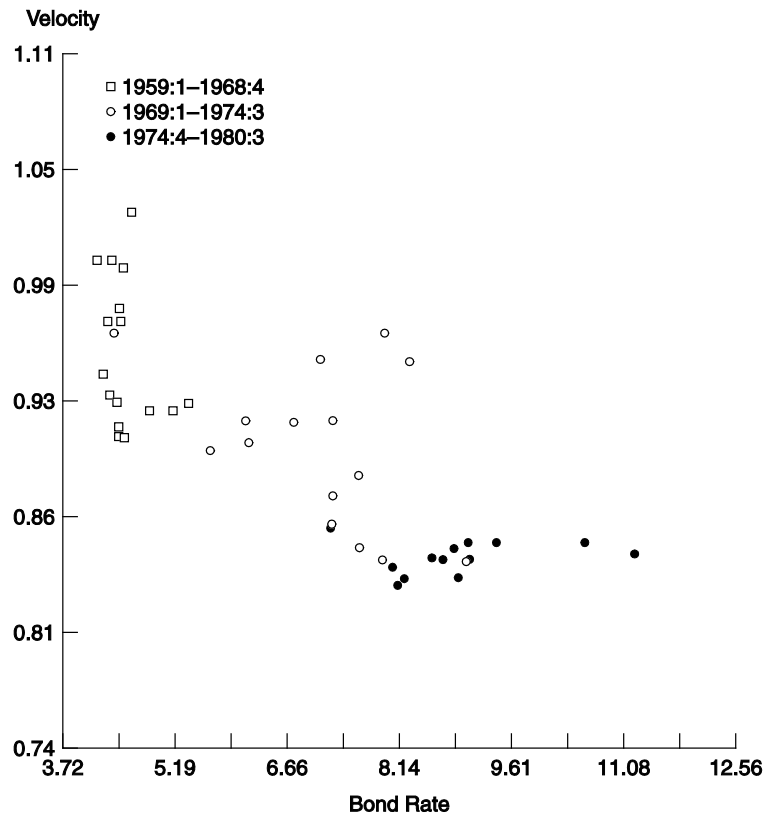


Figure 3: Simple sum M3 velocity versus interest rate: Moody's AAA corporate bond rate, quarterly, 1959.1-1980.3

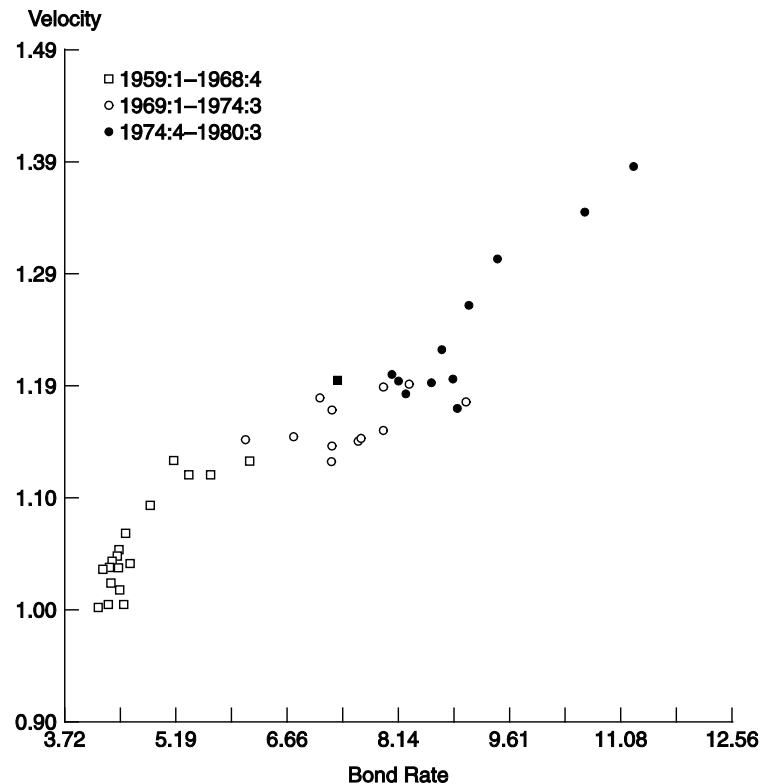


Figure 4: Divisia M3 velocity versus interest rate: Moody's AAA corporate bond rate, quarterly, 1959.1-19980.3

At that time, the claims of unstable demand for money and unstable velocity function, both denied by Milton Friedman, grew in influence. Milton Friedman was right, as was known by the Federal Reserve Board's model manager, Jerry Enzler, but was not revealed to the public. The hypothesis was tested carefully by PAVB Swamy and Peter Tinsley on the Federal Reserve Board's staff. They produced a stochastic coefficient model of the Goldfeld (1973) money demand equation, used at that time in the published research on unstable money demand and used in the Board's macroeconometric model. Swamy and Tinsley ran the test twice: once with simple sum M2 on the left hand side, and once with Divisia M2 on the left

hand side. No changes were made to the right hand side of that equation. The conclusion was that money demand was unstable with simple sum money but stable with Divisia money. See Barnett, Offenbacher, and Spindt (1984). These results were known to Jerry Enzler.

About a year after I had left the Federal Reserve Board's staff for my first academic position, at the University of Texas at Austin, I received an astonishing telephone call from a former Federal Reserve Board staff economist. He informed me that Jerry Enzler had had a serious automobile accident and was on permanent disability leave. His replacement discovered that Jerry had changed the official simple sum M2 aggregate to the corresponding Divisia M2 aggregate in the Board's quarterly macroeconomic model. Jerry had told no one that he had made that change. More information about those events can be found in my book, Barnett (2012).

There were analogous concerns about the supply side of money function. The reason is evident from Figure 5, which plots the base multiplier against a bond rate's deviation from trend. The base multiplier is the ratio of a monetary aggregate to the monetary base. There is good reason for looking at that ratio. The Federal Reserve's open market operations change the Federal Reserve's balance sheet. Those changes, in turn, change the "monetary base," also called "high powered money," or outside money. Changes in the monetary base work themselves through the banking system and result in a change in the money

supply. That was viewed as a “transmission mechanism” for monetary policy to the money supply, and thereby to inflation and the economy. The ratio of the money supply to the monetary base should exceed 1.0, since bank reserves back a larger amount of bank deposits by customers, since only a fraction of the total deposits need be held as reserves. The controllability of the money supply was viewed as depending heavily upon the ability to anticipate the ratio of the money supply to the monetary base. That ratio, also called a “multiplier,” need not be constant to assure controllability of the money supply, but should best be stably related to interest rates, so the multiplier can be anticipated based upon current interest rates. It was believed that open market operations then could be used to influence the money supply through changes in the monetary base, which is directly under the control of the Federal Reserve through its open market operations.

In Figure 5, the monetary aggregate is again the simple sum M3, as computed and officially provided by the Federal Reserve at that time. The base multiplier (adjusted for trend) is plotted against an interest rate. Observe the dramatic structural shift, which is not explained by interest rates. Prior to 1974, the data are along a curved line, in fact a parabola. After 1974 the data is along an intersecting straight line. But again this puzzle was produced by the simple sum monetary aggregate. In Figure 6, the same plot is provided, but with the monetary aggregate changed to Divisia M2. The structural shift is gone.

A systematic comparison of Divisia versus simple sum monetary aggregation during those years of growing skepticism about money demand stability was conducted initially by Barnett, Offenbacher, and Spindt (1984) and subsequently, over a longer time span, by Belongia (1996), who wrote in his paper's abstract:

“Inferences about the effects of money on economic activity may depend importantly on the choice of a monetary index because simple-sum aggregates cannot internalize pure substitution effects. This hypothesis is investigated by replicating five recent studies that have challenged an aspect of the 'conventional wisdom' about the effects of money on aggregate activity. In four of the five cases, the qualitative inference in the original study is reversed when a simple-sum monetary aggregate is replaced by a Divisia index of the same asset collection.”⁶

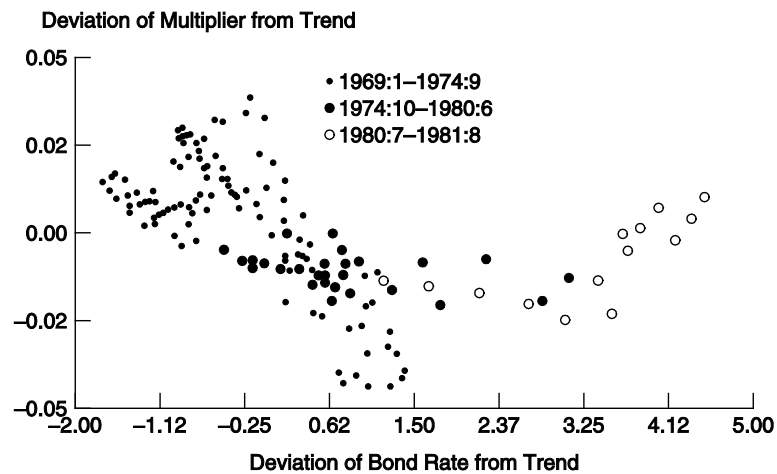


Figure 5: Simple-sum M3 base multiplier versus interest rate: deviation from time trend of Moody's Baa corporate bond rate, monthly 1969.1-1981.8.

⁶ Hendrickson (2013) reached similar conclusions in replications of published studies, but with the simple sum aggregates replaced by the comparable Divisia measures.

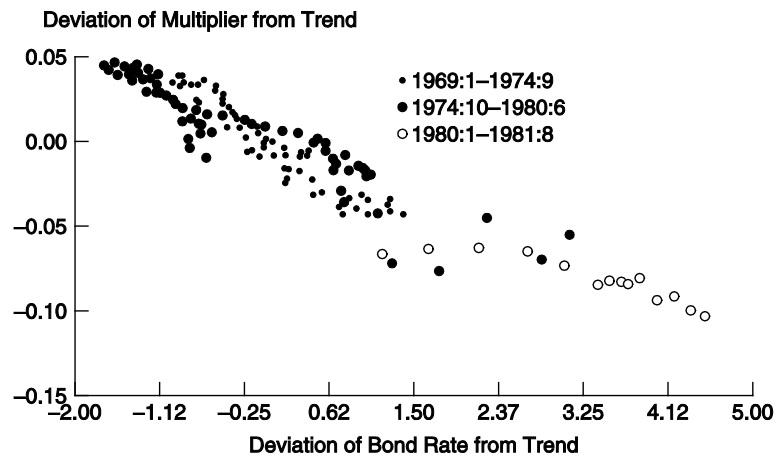


Figure 6: Divisia M3 monetary aggregate base multiplier versus deviation from time trend of Moody's Baa corporate bond interest rate, monthly 1969.1-1981-8.

5. End of the Monetarist Experiment: 1983-1984

As stated above, Friedman was not overly critical of the Federal Reserve, but rather too trusting of the Fed's data. In contrast, his successor at the University of Chicago, Robert Lucas (2000, p. 270) wrote, "I share the widely held opinion that M1 is too narrow an aggregate for this period, and I think that the Divisia approach offers much the best prospects for resolving this difficulty." Particularly puzzling was Friedman's continued confidence in Federal Reserve monetary aggregates following the end of the Monetarist Experiment.⁷ At that time, he became very vocal with his prediction that there had just been a huge surge in the growth rate of the money supply, and that the surge would work its way through the economy to produce a new inflation. He further predicted that there would subsequently be an overreaction by the Federal Reserve, plunging the economy back down into a recession. He published this view repeatedly in the media in various magazines and newspapers, with the most visible being his *Newsweek* article, "A Case of Bad Good News," which appeared on p. 84 on September 26, 1983. I have excerpted some of the sentences from that *Newsweek* article below:

"The monetary explosion from July 1982 to July 1983 leaves no satisfactory way out of our present situation. The Fed's stepping on the brakes will appear to have no immediate effect. Rapid recovery will continue under the impetus of earlier monetary growth. With its historical shortsightedness, the Fed will be tempted to step still harder on the brake – just as the failure of rapid monetary growth in late

⁷ Other puzzles include Friedman's inconsistent switches between simple sum M1 and M2 during 1981-1985, when offering policy prescriptions and critiques, as documented in Nelson's (2007, pp. 162-166) history of Friedman's views.

1982 to generate immediate recovery led it to keep its collective foot on the accelerator much too long. The result is bound to be renewed stagflation – recession accompanied by rising inflation and high interest rates . . . The only real uncertainty is when the recession will begin.”

But on exactly the same day, September 26, 1983, I published a very different view in my article, “What Explosion?” on p. 196 of *Forbes* magazine. The following is an excerpt of some of the sentences from that article:

“People have been panicking unnecessarily about money supply growth this year. The new bank money funds and the super NOW accounts have been sucking in money that was formerly held in other forms, and other types of asset shuffling also have occurred. But the Divisia aggregates are rising at a rate not much different from last year’s . . . the ‘apparent explosion’ can be viewed as a statistical blip.”

Milton Friedman would not have taken such a strong position without reason.

His reason is evident from Figure 7, acquired from the St. Louis Fed’s web site.⁸

The percentage growth rates in that figure are not annualized, so should be multiplied by 12 to acquire the approximate annualized growth rates. Notice the large spike in growth rate, which increases to nearly 36% per year. But that solid line is produced from simple sum M2, which was greatly overweighting the sudden new availability of super-NOW accounts and money market deposit accounts.

There was no spike in the Divisia monetary aggregate, represented by the dashed line.

⁸ The St. Louis Federal Reserve Bank has played a historic role in supplying Divisia monetary aggregates for the United States. See Anderson and Jones (2011). A newer source, including broad Divisia monetary aggregates, is the Center for Financial Stability in New York City. See Barnett, Liu, Mattson, and van den Noort (2013) and <http://centerforfinancialstability.org>.

If the huge surge in the money supply had happened, then inflation would surely have followed, unless money were extremely non-neutral even in the long run --- a view held by very few economists. But there was no inflationary surge and no subsequent recession.

Subsequent to the work of Friedman on monetary economics, there has been a dramatic increase in substitutes for money associated with “shadow banking.” Long before the evolution of shadow banking assets, Keynes recognized the relevancy of broad monetary measures.⁹ Paradoxically, the response of the Federal Reserve to money market innovations has been to remove the entire negotiable money market from its monetary aggregates, by discontinuing its broad aggregates, M3 and L, since simple sum aggregation over substitutes for money excessively weight substitutes for money. In contrast, Divisia monetary aggregates can dynamically incorporate properly weighted substitutes for money as they evolve, and as made available to the public by the Center for Financial Stability in its broadest monetary aggregates, M3 and M4.

⁹ Given that the assets in the Keynes (1936) model consisted of just aggregated money and aggregated bonds, it is hard to think that he conceived of the relevant aggregate as anything but a broad one. That conclusion would also be consistent with his discussion of the role of what he called “the financial circulation” in Keynes (1930). On this point, also see Tim Congdon (2011, pp. 28 and 83). I am indebted to David Laidler for pointing this out to me.

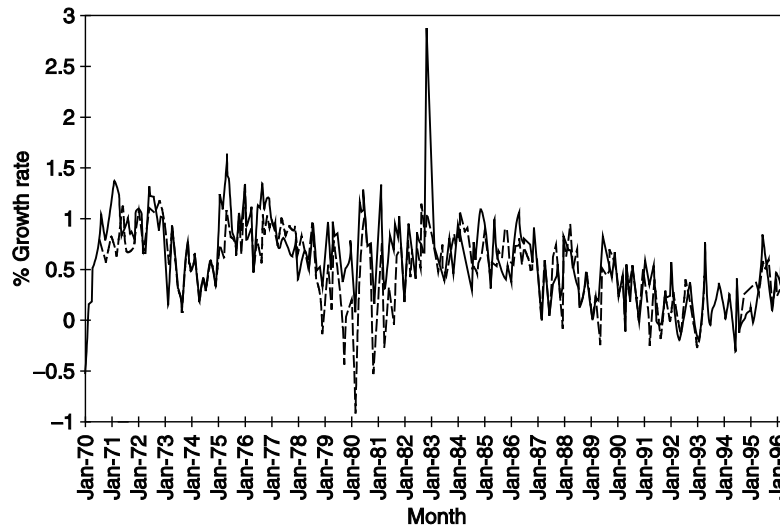


Figure 7: Monetary Growth Rates, 1970-1996, from St. Louis Federal Reserve's Database, FRED. The growth rates are not annualized.

6. Optimal Aggregation

Having briefly surveyed the source of misunderstandings about money demand and money supply, I now outline the formal procedure for correct monetary aggregation, as consistent with the fields of index number and aggregation theory and with Barnett (1982). In economic theory, the optimal aggregate is acquired through a three-stage selection procedure. The selection procedure at each stage is distinct and the stages cannot be interchanged in order. As we shall see, the deficiencies in the official aggregates result from the fact that the third-stage criteria alone have been used to select the aggregates, although those criteria are inappropriate within either of the two prior stages.

6.1. Stage 1: Selection of Admissible Component Groupings

The first stage in the selection of an optimal aggregate is the determination of all theoretically admissible sets of components assets. Aggregates then can be constructed over only those sets. To acquire those admissible sets, we first must acquire the separable subsets of the set of all monetary components.

We now define the conditions for a separable subset, which we shall call a separable component group.

Condition 1: Let C be a subset of the set of all monetary assets, M , so that C is itself a set of component monetary assets. Then C is a separable component group, if and only if the marginal rate of substitution between any two assets in C is independent of the quantity of any good or asset not in C .

Condition 1 is necessary and sufficient for a subfunction of the assets in C alone to be factorable out of utility or production functions. The value of that factored subfunction, which is called the economic aggregate, depends only upon the assets in C and is independent of the quantities consumed or held of any other goods or assets. Without condition 1, no stable preferences or transactions technology can exist over elements of C alone, and hence goods or assets not in C will act as missing shift variables in the demand for any aggregates constructed over assets in C alone.

We call condition 1 the Existence Condition, since it defines the condition under which an economic aggregate exists in aggregation theory. However, existence alone is not sufficient for an aggregate to behave like an elementary

good. To acquire that result, we need the following stronger condition, which we call the Consistency Condition.

Condition 2. An admissible component group, C , is a consistent component group, if the elasticity of substitution between any component asset in the group and any good or asset not in the group is independent of the good or asset that is not in the group.¹⁰

Condition 2 implies condition 1, but the converse is not true. Every consistent component group is separable, but not every separable component group is consistent. Although some aspects of aggregation theory can be applied with only satisfaction of condition 1, condition 2 substantially simplifies interpretation and use of an aggregate.¹¹ Condition 2 is acquired by imposing linear homogeneity on the economic aggregate that condition 1 assures exists over the assets in C . It would be very curious indeed if linear homogeneity of an aggregate failed; in such a case, the growth rate of the aggregate would differ from the growth rates of the components, even if all components were growing at the same rate.

Neither condition 1 nor condition 2 has meaning unless the concept of substitutability among assets has meaning. We need the existence of a utility function or production function containing monetary assets (with or without other goods) as arguments. The existence of a transactions technology would suffice.

¹⁰ Condition 2 defines a linearly homogeneous, weakly separable block in aggregation theory.

¹¹ Examples using only condition 1 can be found in Barnett (1980) and Barnett (1981, chap. 7).

If our objective were to explain why people hold money, then entering monetary assets enter utility or production functions would be assuming away the problem. However for our purposes, we can assume that money has positive value in equilibrium. Under that assumption, it has been proved in general equilibrium theory that money must enter into a derived utility or production function.¹²

Hence conditions 1 and 2 can be defined relative to that derived function.¹³

In order for an aggregate to serve the role of ‘money’ in the economy, we would expect the components to satisfy another restriction. In addition to satisfying condition 2, the components should satisfy the following condition, which we call the *Recursiveness condition*.¹⁴

Condition 3. The components of each aggregate must include currency (legal tender) and must not include any good or asset that is not a ‘monetary asset.’

Although aggregation theory provides no definition of ‘monetary asset,’ the identity of the ‘money market’ is no less widely understood than the identity of the durables market or the recreational goods market. In attaching a name to an

¹² See Arrow and Hahn (1971) and Quirk and Saposnik (1968, p. 97). A widely used special case of the general equilibrium result arises if money acquires its usefulness by entering a transactions constraint. In that case the derived utility function is the Lagrangian containing both the original elementary utility function (which does not contain money) and the constraint defined by the transactions technology. See Phelps and Spinnewyn (1982), whose approach applies regardless of whether money does or does not yield interest.

¹³ There is still the problem of aggregating over individual economic agents (households and firms). For a survey of that literature, see Barnett (1981, pp. 306-7). For a newer approach to the problem, see Barnett (1979). For a well-known directly relevant approach, see Muellbauer (1975) and Phelps and Spinnewyn (1982). For recent surprisingly favorable empirical results on aggregation over economic agents, see Varian (1983).

¹⁴ The condition results in a recursively nested functional structure for the aggregator function in aggregation theory. See Barnett (1977, 1980).

aggregate, such as food or money, a prior definition of the components' domain must be selected.

Aggregation theory itself does not dictate use of condition 3. In principle, we would lose nothing by dropping condition 3. However in practice the number of component groups that would satisfy condition 2 is likely to be large. Although the condition 2 component groups would always contain the 'best' group (or groups) for any particular purpose, the empirical research needed to choose among such a large class of component groups could be difficult and expensive. Condition 3 applies conventional views from monetary theory to restrict further the number of admissible component groups. In particular, condition 3 restricts the domain of possible components to 'monetary assets' and requires the collection of admissible component groups to be nested about 'hard core money,' defined here by its legal tender property. Researchers applying the procedures outlined in this paper could tighten or loosen the restrictiveness of condition 3 as needed to accommodate the scope (i.e., ambitiousness) of the research, with the limiting (most ambitious) case being elimination of condition 3 entirely.

The component groups that satisfy both conditions 2 and 3 comprise a family of completely nested sets. Hence aggregation is perfectly recursive as we pass to increasingly high levels of aggregation over those nested groups. We shall call a set of components that satisfies both conditions 2 and 3 a *consistent recursive component group*.

When we seek an aggregate to serve the role of ‘money’ in the economy, as would be acquired for a monetary target or for the variable ‘money’ in a model, we shall require the components of the aggregate to comprise a consistent recursive component group. If we were seeking only an indicator, the prior imposition of condition 3 would generally not be justified, and we could perhaps even do without condition 2. We therefore say that a set of components that is separable is *admissible for an indicator*.¹⁵ We define a set of components that is also consistent and recursive to be *admissible for a monetary variable*. Since the primary objective of this paper is determination of the optimal monetary variable, indicators will not be explored in detail. In the monetary asset case, I derived the precise test for condition 2 in Barnett (1980) and in Barnett (1981, chap. 7).¹⁶

I summarize the results of this section with a statement of the solution to the stage 1 problem in monetary aggregation:

Step 1. Determine those sets of monetary assets that satisfy both conditions 2 and 3.

¹⁵ For further discussion on the subject of monetary indicators, see Brunner and Meltzer (1967, 1969) and Hamburger (1970).

¹⁶ The component groups used in constructing the Federal Reserve Board’s official monetary aggregates were not acquired by testing for either condition 1 or condition 2. Some of those component groups are probably inadmissible either for indicators or for monetary variables. For example, Series E bonds and nonnegotiable, consumer-type, small time deposits are probably not elements of any admissible monetary asset component group, although they may be elements of an admissible group of components for an intermediate-term-bond aggregate.

6.2. Stage 2: Selection of an Index Number Formula

Having determined the admissible component groupings, we next must determine a formula to be used for computing the aggregates over the components of each admissible group. In the Federal Reserve Board's current official aggregates, the formula is simple summation of the component quantities. The aggregation formula is called the aggregator function, and the necessary and sufficient conditions for a linear aggregator function are the following:

Condition 4. The aggregator function is linear if and only if the component goods (or assets) are perfect substitutes.

We call condition 4 the *Linearity Condition*. Clearly the components of the monetary assets are not perfect substitutes. Hence the aggregates cannot be computed from simple summation or from any other linear formula. When components are not perfect substitutes, a nonlinear aggregator function is needed. The aggregate acquired from the nonlinear function is called the 'economic aggregate.'¹⁷

To use a nonlinear aggregator function directly, we would have to specify the form of the function and then estimate its parameters. Alternatively index number theory provides parameter-free approximations to the economic aggregates of aggregation theory. Economic quantity aggregates depend upon

¹⁷ If \mathbf{m} is the vector of components and if f is the aggregator function, then $f(\mathbf{m})$ is the economic aggregate.

quantities and unknown parameters, but not prices. However quantity *index numbers* depend upon quantities and prices, but not unknown parameters.

Hence quantity index numbers dispense with unknown parameters by introducing prices. Diewert (1976) has defined the class of 'superlative' index numbers, which provide very high quality approximations to economic aggregates.

In constructing a superlative monetary quantity index number, the selection of the index number from among those in the superlative class is of little importance, since all of the index numbers within the class move very closely together. However, they all jointly diverge from the simple sum index.

As a result, the simple sum index is entirely disreputable in the index number literature. Fisher (1922) found that the simple sum index was the very worst index that he was able to devise. The simple sum index was the only index that possessed both of the bad properties that he defined: bias and freakishness, and he found the size of the bias to exceed 35 percent of the level of the index. Fisher (1922, p. 29) observed that "the simple arithmetic average produces one of the very worst of index numbers, and if this book has no other effect than to lead to the total abandonment of the simple arithmetic type of index number it will have served a useful purpose." Fisher (1922, p. 36) concluded that "the simple arithmetic [index] should not be used under any circumstances."

My work on monetary aggregates has usually applied the Divisia index formula. That index has the most easily understood form of any of the index

numbers in Diewert's superlative class and hence is potentially the most useful to policymakers.

The selection of the prices to be used in the index number formula is important. If user cost (or "rental") prices are used in an aggregate of durables, such as monetary assets, then the resulting quantity index measures the *service flow* generated by the component durable goods. The formula for the user cost price of a monetary asset was derived by Barnett (1978, 1980).²⁷

I summarize the conclusions of this section with a statement of the solution to the stage 2 problem in monetary aggregation:

Step 2. Using user cost prices for each component asset, compute a superlative quantity index over each of the admissible groups of component assets acquired from step 1.

6.3. Stage 3: Selection of the Optimal Level of Aggregation

Having completed step 2, we have a hierarchy of nested aggregates. We now must select among them.

Each element of our hierarchy of monetary quantity index numbers approximates a perfect economic aggregate. In aggregation theory every household and firm in the economy can be shown to behave *as if* a consistent aggregate is an elementary good.¹⁸ Hence no information about the economy is lost

¹⁸ The solution to each economic agent's full decision is the same as the solution to a multistage decision. The consistent aggregates are the variables in each stage of the multistage budgeting procedure except for the last stage. The variables in the last stage are the elementary goods from the original full decision. See Green (1964, theorem 4).

by using a consistent aggregate. Since the component monetary quantities and interest rates are not likely to be final targets of policy, we have nothing to lose by using the highest level aggregate among those that are in the admissible hierarchy.

The question now arises as to whether we have anything to lose by using an admissible aggregate *other than* the highest level aggregate, and we do indeed. By using a lower level aggregate, we are omitting factors of production from the economy's transactions technology. Furthermore, as the level of aggregation decreases, the number of price aggregates contained in the demand function increases.¹⁹ As a result, modeling the demand for the aggregate becomes more difficult; and if any of the explanatory variables is omitted, the function will appear to shift in an unstable fashion. However the economic aggregates of aggregation theory can be shown to internalize pure substitution effects *perfectly*.

In short, the highest level admissible aggregate from step 2 provides a properly weighted measure of the total service flow of all of the money market's separable components. We now can state the final step in the procedure for choosing the optimal monetary aggregate:

Step 3. Select the highest level aggregate from among those generated by step 2.

¹⁹ This result follows from duality theory. There is a price aggregate, which is dual to the quantity aggregate, over each group of components satisfying condition 2. The demand for automobiles depends upon the price of aggregated automobiles, fuel, and housing. But the price of BMW X3s depends upon the price of other small SUVs and individual grades of gasoline.

However, at this point it may be wise to hedge our bets. While aggregation theory does tell us to use only the nested admissible aggregates provided by step 2, my advocacy of the highest of those aggregates on theoretical grounds depends upon a nontrivial assumption: equilibrium in the market for every component asset. As Cagan (1982) has observed, the economy's disequilibrium dynamics can affect differently the information content of aggregates at different levels of aggregation.

Aggregates generated by step 2 at each level of aggregation will properly measure the total service flow produced by the components, regardless of whether interest rates are market determined or are set by regulation, and regardless of whether the markets for components are or are not in equilibrium. Hence the aggregates produced by step 2 remain the valid aggregate monetary variables. But the validity of step 3 depends upon an equilibrium assumption and is therefore conclusive only in the long run.²⁰

The importance of the theoretical conclusion (step 3) should be an increasing function of the weight that we place upon the objective of controlling the long-run (equilibrium) inflation rate. I conclude this section with a statement of the short-run disequilibrium alternative to step 3.

²⁰ However it is perhaps unlikely that this problem would be serious with aggregates satisfying conditions 1 and 2, since the markets for those components are among the best and fastest adjusting in the economy. Nevertheless the components of the current official aggregates include some very illiquid assets having long maturities and heavy early redemption penalties. Hence until the components are properly regrouped in accordance with step 1, step 3 cannot be viewed as conclusive on theoretical grounds alone.

Step 3a. From *among the admissible aggregates* provided by step 2, select the one that empirically works best in the application in which the aggregate is to be used.

Observe that step 3a *cannot* be accomplished without prior completion of steps 1 and 2. However, the Federal Reserve Board's official aggregates are constructed from research that related *solely* to step 3a. Step 1 was never conducted, and step 2 was replaced by simple sum aggregation. As emphasized by Mason (1976), Friedman and Schwartz (1970) contained the same violations of scientific methodology, which insists that measurement must precede use to avoid circularity. Nevertheless, while Friedman and Schwartz (1970) did not conduct formal tests for blockwise weak separability of component clusterings, Swofford (1995) did and found support for the groupings used by Friedman and Schwartz (1970).

Step 1 is needed to assure the existence of behaviorally stable aggregates over the component groups, and step 2 is needed to assure that the computed aggregates provide high quality approximations to the unknown behaviorally stable aggregates.³¹ By violating step 2, the official aggregates cannot approximate behaviorally stable aggregates; and by using components that need not satisfy step 1, the official aggregates permit all forms of spurious and unstable relationships during the application of step 3a. In short, the procedure used in selecting the official aggregates virtually eliminates the possibility of imputing meaning or

replicable explanatory or predictive ability to the movements of the aggregates, since the economy cannot act as if the official aggregates are variables in the economy's structure.

Movements of the official aggregates could have stable relationships with actual structural variables, such as the inflation rate, only if the aggregates' components were perfect substitutes, so that step 2 would be satisfied, and if step 1 were satisfied by a combination of chance and superior judgment. Clearly the violation of step 2 is more serious than the violation of step 1, since the perfect substitutability condition needed for satisfaction of step 2 with simple sum aggregation could not have been satisfied by chance. In addition, if step 2 had been applied, the possibility would have increased that the component groupings selected from step 3a would have satisfied step 1.

7. Conclusions

Misperceptions about the stability of money demand have played a major role in the profession's move away from the views of Milton Friedman on monetary policy. But current views about money demand instability are not based on accepted methodology in the literature on consumer demand systems.

I have contributed substantially to the literature on consumer demand systems modeling. See, e.g., the reprints of my many publications in that area in the books Barnett and Serletis (2000), Barnett and Binner (2004), and Barnett and Chauvet (2011b). I am the originator of the Müntz-Szatz series

seminonparametric approach (Barnett and Jonas (1983), Barnett and Yue (1988), Barnett, Geweke, and Yue (1991)) and the Laurent series approach (Barnett (1983)) and the generalized hypocycloidal demand model (Barnett (1977)). I am also the originator of aggregation theoretic foundations for the Rotterdam demand systems model (Barnett (1979)). The consumer demand modeling literature insists upon use of microeconomic foundations along with the relevant aggregation and index number theory. The resulting systems of equations usually are nonlinear and estimated jointly. The views on money demand equation instability are based on research making no use of any of the accepted methodologies in consumer demand system modeling and estimation.

Economists working on consumer demand systems modeling are well aware of the fact that acquiring stable systems requires state-of-the-art modeling and estimation. My experience using that approach with money demand have consistently shown that money demand is no more difficult to estimate stably than demand for any other good. See, e.g., Barnett (1983). Others familiar with the literature on modeling and estimating consumer demand systems have similarly reached the same conclusions, over and over again for decades. See, for example, Hendrickson (2013), Serletis (2007), and Serletis and Shahmoradi (2005, 2006, 2007). The contrary literature fails to use data or modeling approaches meeting elementary standards of the modern literature on consumer demand systems

modeling and is thereby over a half century behind the state of the art in modeling demand for any other good or service.

The eminent new Keynesian economist, Julio Rotemberg, has contributed in major ways to the literature on monetary aggregation theory through his own publications [Rotemberg (1991), Poterba and Rotemberg (1987), and Rotemberg, Driscoll, and Poterba (1995)] and has acknowledged the following in his endorsement quotation on the back cover of Barnett (2012): “This book first makes you care about monetary aggregation and then masterfully shows you how it should be done.” That book won the American Publishers Award for Professional and Scholarly Excellence (the PROSE Award) for the best book published in economics during 2012.

Indeed money does matter in New Keynesian models, when there are financial shocks to the economy, as has become a major focus of research in recent years. Keating and Smith (2013a) have shown that augmenting a typical Taylor rule with a reaction to Divisia money growth improves welfare, when the financial sector is a source of shocks driving the economy. This result contrasts with the literature arguing that optimal monetary policy can be implemented in New Keynesian models without reference to a monetary aggregate. However, that literature makes the very strong assumption that the central bank perfectly observes the natural rate of interest and the output gap in real time. By eschewing those assumptions, the policy rule Keating and Smith (2013a) propose is

realistically implementable by central banks by including the non-parametric Divisia monetary aggregate within the policy rule.

Keating and Smith (2013b) have further shown that Friedman's constant money growth rate rule is likely to result in indeterminacy for most parameter values, when implemented with a simple-sum monetary aggregate. The indeterminacy stems from simple-sum's error in tracking the true monetary aggregate. This indeterminacy is resolved, if the rule is implemented with the Divisia monetary aggregate. They show that a similar result is obtained when considering inflation-targeting interest-rate rules reacting to money growth. In particular, the addition of a reaction to simple-sum money growth creates indeterminacy. On the other hand, inflation-targeting interest-rate rules reacting to Divisia growth satisfy a novel type of Taylor principle --- reacting more than 1:1 to Divisia growth is sufficient for determinacy.

Belongia and Ireland (2013) similarly used a New Keynesian model in their analysis of the potential importance of Divisia monetary aggregates in policy.²¹ Their paper extends a New Keynesian model to include roles for currency and deposits as competing sources of liquidity services demanded by households. According to their paper's abstract,

“it shows that, both qualitatively and quantitatively, the Barnett critique applies: While a Divisia aggregate of monetary services tracks the true monetary aggregate almost perfectly, a simple-sum measure often behaves

²¹ Peter Ireland is a member of the Shadow Open Market Committee.

quite differently. The model also shows that movements in both quantity and price indices for monetary services correlate strongly with movements in output following a variety of shocks. Finally, the analysis characterizes the optimal monetary policy response to disturbances that originate in the financial sector.”

Recently, many New Keynesian economists have emphasized the possible merits of nominal GDP targeting in monetary policy. Although not necessarily advocating nominal GDP targeting, Belongia and Ireland (2013b) and Barnett, Chauvet, and Leiva-Leon (2013) have found that Divisia monetary aggregates should play an important role, if nominal GDP targeting were to be adopted. Alternatively, advocates of Friedman’s preference for monetary targeting will find support for Divisia monetary targeting in Serletis and Rahman (2013).²²

Divisia monetary aggregates are not only relevant to New Keynesian models, but more relevant than commonly believed to classical real business cycle models, as found by Serletis and Gogas (2014). They conclude:

“King, Plosser, Stock, and Watson (1991) evaluate the empirical relevance of a class of real business cycle models with permanent productivity shocks by analyzing the stochastic trend properties of postwar U. S. data. . . . We revisit the cointegration tests in the spirit of King et al. (1991), using improved monetary aggregates whose construction has been stimulated by the Barnett critique. We show that previous rejections of the balanced growth hypothesis and classical

²² Personally I am not an advocate of any particular policy rule, but rather of competent measurement within any policy approach. But I do think it paradoxical that economists favoring a monetary growth intermediate target have often preferred a simple sum monetary aggregate, while simultaneously and inconsistently preferring an inflation-rate final target measured as a chained Fisher ideal or Laspeyres index, rather than a simple sum or arithmetic average price index.

money demand functions can be attributed to monetary aggregation issues.”

A large literature, not dependent on New Keynesian, monetarist, or real business cycle theory, now exists on the relationship between Divisia money, inflation, and output.²³ In the years immediately following the Great Recession, the emphasis of the Center for Financial Stability (CFS) on Divisia monetary aggregates has been far more informative than the Federal Reserve’s emphasis on interest rates. See, e.g., Barnett (2012) and Barnett and Chauvet (2011a). During those years, interest rates have been nearly constant at approximately zero, while bank reserves and monetary policy have been the most volatile and aggressive in the history of the Federal Reserve System. Much credit for the role of the CFS in providing valuable aggregation-theoretic economic data to the public should go to Steve Hanke at Johns Hopkins University and Lawrence Goodman, the President of the CFS. Following reading and commenting extensively on the original manuscript of Barnett (2012), Hanke put me in touch with Goodman, who invested heavily in the Divisia database as a public service. The rest is history --- of the best kind.

The time to reconsider the views of Milton Friedman on monetary policy are long overdue, but with properly measured monetary aggregates, in accordance with the standards of competency advocated by the International Monetary Fund

²³ An overview of much of that literature can be found in Barnett (2012), Barnett and Binner (2004), Barnett and Chauvet (2011a,b), Barnett and Serletis (2000), Belongia (1996), Belongia and Ireland (2003a,b,c), Serletis (2007), Serletis and Gogas (2014), and Serletis and Shahmoradi (2006).

(2008, pp. 183-184). Initial constructive steps in that direction have been taken by

Belongia and Ireland (2013c), who find:

“Fifty years ago, Friedman and Schwartz presented evidence of pro-cyclical movements in the money stock, exhibiting a lead over corresponding movements in output, found in historical monetary statistics for the United States. Very similar relationships appear in more recent data. To see them clearly, however, one must use Divisia monetary aggregates in place of the Federal Reserve’s official, simple-sum measures.”

References

Anderson, R. G. and B. E. Jones, 2011, "A Comprehensive Revision of the Monetary Services (Divisia) Indexes for the United States," *Federal Reserve Bank of St. Louis Review*, September/October, 325-359.

Arrow, K. J. and G. H. Hahn, 1971. *General Competitive Analysis*, San Francisco: Holden Day.

Balk, Bert, 2005. "Divisia Price and Quantity Indices: 80 Years After," *Statistica Neerlandica* 59, 119-158.

Balk, Bert, 2008. *Price and Quantity Index Numbers: Models for Measuring Aggregate Change and Difference*, New York: Cambridge University Press.

Barnett, W. A., 1977. "Recursive Subaggregation and a Generalized Hypocycloidal Demand Model," *Econometrica* 45, 1117-1136. Reprinted in Barnett, W. A. and J. M. Binner (eds), 2004. *Functional Structure and Approximation in Econometrics*, Amsterdam: Elsevier, chapter 11, 233-255.

Barnett, W. A., 1978. "The User Cost of Money," *Economics Letters* 1, 145-149. Reprinted in Barnett, W. A. and A. Serletis (eds), 2000. *The Theory of Monetary Aggregation*, Amsterdam: Elsevier, chapter 1, 6-10.

Barnett, W. A., 1979. "Theoretical Foundations for the Rotterdam Model," *Review of Economic Studies* 46, 109-130. Reprinted in Barnett, W. A. and J. M. Binner (eds), 2004. *Functional Structure and Approximation in Econometrics*, Amsterdam: Elsevier, chapter 1, 9-39.

Barnett, W. A., 1980. "Economic Monetary Aggregates: An Application of Aggregation and Index Number Theory," *Journal of Econometrics* 14, 11-48. Reprinted in Barnett, W. A. and A. Serletis (eds), 2000. *The Theory of Monetary Aggregation*, Amsterdam: Elsevier, chapter 2, 11-48.

Barnett, W. A., 1981. *Consumer Demand and Labor Supply: Goods, Monetary Assets, and Time*, Amsterdam: Elsevier.

Barnett, W. A., 1982. "The Optimal Level of Monetary Aggregation," *Journal of Money, Credit, and Banking* 14:687-710. Reprinted in Barnett, W. A. and A. Serletis (eds), 2000. *The Theory of Monetary Aggregation*, Amsterdam: Elsevier, chapter 7, 125-159.

- Barnett, W. A., 1983. "New Indices of Money Supply and the Flexible Laurent Demand System," *Journal of Business and Economic Statistics* 1, 7-23. Reprinted in Barnett, W. A. and A. Serletis (eds), 2000. *The Theory of Monetary Aggregation*, Amsterdam: Elsevier, chapter 16, 325-359.
- Barnett, W. A., 2012. *Getting It Wrong: How Faulty Monetary Statistics Undermine the Fed, the Financial System, and the Economy*, Boston: MIT Press.
- Barnett, W. A. and J. M. Binner (eds), 2004. *Functional Structure and Approximation in Econometrics*, Amsterdam: Elsevier.
- Barnett, W. A. and M. Chauvet, 2011a. "How Better Monetary Statistics Could Have Signaled the Financial Crisis," *Journal of Econometrics* 161, March, 6-23.
- Barnett, W. A. and M. Chauvet (eds), 2011b. *Financial Aggregation and Index Number Theory*, Singapore: World Scientific.
- Barnett, W. A., M. Chauvet, and D. Leiva-Leon, 2013. "Real-Time Nowcasting of Nominal GDP," *Journal of Econometrics*, forthcoming.
- Barnett, W. A. and A. B. Jonas, 1983. "The Müntz-Szatz Demand System---An Application of a Globally Well-Behaved Series Expansion," *Economics Letters* 11, 337-342. Reprinted in Barnett, W. A. and J. M. Binner (eds), 2004. *Functional Structure and Approximation in Econometrics*, Amsterdam: Elsevier, chapter 8, 159-164.
- Barnett, W. A. and A. Serletis (eds), 2000. *The Theory of Monetary Aggregation*, Amsterdam: Elsevier.
- Barnett, W. A. and P. A. Spindt, 1979. "The Velocity Behavior and Information Content of Divisia Monetary Aggregates," *Economics Letters* 4, 51-57.
- Barnett, W. A. and P. Yue, 1988. "Semi-parametric Estimation of the Asymptotically Ideal Model: The AIM Demand System," *Advances in Econometrics* 7, 229-251. Reprinted in Barnett, W. A. and J. M. Binner (eds), 2004. *Functional Structure and Approximation in Econometrics*, Amsterdam: Elsevier, chapter 9, 165-185.
- Barnett, W. A., J. Geweke, and P. Yue, 1991. "Semi-nonparametric Bayesian Estimation of the Asymptotically Ideal Model: The AIM Demand System," in: W. A. Barnett, J. Powell, and G. Tauchen (eds.) *Nonparametric and Semiparametric Methods*, Cambridge University Press, Cambridge, U.K., 127-173. Reprinted in

Barnett, W. A. and J. M. Binner (eds), 2004. *Functional Structure and Approximation in Econometrics*, Amsterdam: Elsevier, chapter 10, 187-232.

Barnett, W. A., E. K. Offenbacher, and P. A. Spindt, 1984. "The New Divisia Monetary Aggregates," *Journal of Political Economy* 92, 1049-1087. Reprinted in Barnett, W. A. and A. Serletis (eds), 2000. *The Theory of Monetary Aggregation*, Amsterdam: Elsevier, chapter 17, 360-388.

Barnett, W. A., J. Liu, R. S. Mattson, and J. van den Noort, 2013. "The New CFS Divisia Monetary Aggregates: Design, Construction, and Data Sources," *Open Economies Review* 24, 101-124.

Barth, J. R., A. Kraft, and J. Kraft, 1977. "The 'Moneyness' of Financial Assets," *Applied Economics* 9, 51-61.

Belongia, M. T., 1995. "Weighted Monetary Aggregates: a Historical Survey," *Journal of International and Comparative Economics*, Winter, 877-114.

Belongia, M. T., 1996, "Measurement Matters: Recent Results from Monetary Economics Reexamined," *Journal of Political Economy* 104, 1065-1083.

Belongia, M. T. and P. N. Ireland, 2013a. "The Barnett Critique after Three Decades: A New Keynesian Analysis," *Journal of Econometrics*, forthcoming.

Belongia, M. T. and P. N. Ireland, 2013b. "A 'Working' Solution to the Question of Nominal GDP Targeting," Boston College Working Papers in Economics 802, Boston College Department of Economics.

Belongia, M. T. and P. N. Ireland, 2013c. "Instability: Monetary and Real," Boston College Working Papers in Economics 830, Boston College Department of Economics.

Bisignano, J., 1974. "Real Money Substitutes," Discussion Paper #17, Federal Reserve Bank of San Francisco.

Brunner, K. and A. H. Meltzer, 1967. "The Meaning of Monetary Indicators," in George Horwich (ed.), *Monetary Process and Policy: A Symposium*, Homewood: Richard D. Irwin.

Brunner, K. and A. H. Meltzer, 1968. "Liquidity Traps for Money, Bank Credit, and Interest Rates," *Journal of Political Economy* 76: 1-37.

- Brunner, K. and A. H. Meltzer, 1969. "The Nature of the Policy Problem," in: K. Brunner (ed.), *Targets and Indicators of Monetary Policy*, San Francisco: Chandler, 1-26.
- Cagan, P., 1982. "The Choice Among Monetary Aggregates as Targets and Indicators for Monetary Policy," *Journal of Money, Credit and Banking* 14, 661-686.
- Chetty, K. V., 1969. "On Measuring the Nearness of Near-Moneys," *American Economic Review* 59, 270-281.
- Chetty, K. V., 1972. "On Measuring the Nearness of Near-Moneys: Reply," *American Economic Review* 62, 226-229.
- Congdon T., 2011. *Money in a Free Society*, London and New York, Encounter Books.
- Diewert, W. E., 1976. "Exact and Superlative Index Numbers," *Journal of Econometrics* 4, 115-145.
- Fisher, D., 1992. "Money-Demand Variability: A Demand Systems Approach," *Journal of Business and Economic Statistics* 10, 143-152.
- Ford, J. L. and F. Stark, 1967. *Long and Short Term Interest Rates*, New York: Augustus M. Kelley.
- Friedman, M., 1956, "The Quantity Theory of Money--A Restatement" in *Studies in the Quantity Theory of Money*, M. Friedman, ed., Chicago, IL: University of Chicago Press.
- Friedman, M. and D. I. Meiselman, 1963. "The Relative Stability of Monetary Velocity and the Investment Multiplier in the United States, 1897-1958." In *Commission on Money and Credit, Stabilization Policies*, Englewood Cliffs, N.J.: Prentice Hall.
- Friedman, M. and A. J. Schwartz, 1969. "The Definition of Money: Net Wealth and Neutrality as Criteria," *Journal of Money, Credit and Banking* 1, 1-14.
- Friedman, M. and A. J. Schwartz, 1970. *Monetary Statistics of the United States: Estimates, Sources, Methods, and Data*, New York: Columbia University Press (for the NBER).

- Goldfeld, S., 1973. "The Demand for Money Revisited," *Brookings Papers on Economic Activity* 3, 577-638.
- Green, H. A. J., 1964. *Aggregation in Economic Analysis: An Introductory Survey*, Princeton, N. J.: Princeton University Press.
- Gurley, J. G., 1960. "Liquidity and Financial Institutions in the Postwar Economy," Study Paper #14, In *The Study of Employment, Growth, and Price Levels*, U.S. Government Printing Office, Washington, D.C.
- Hamburger, M. J., 1970. "Indicators of Monetary Policy: The Arguments and the Evidence," *American Economic Review Papers and Proceedings* 60, 32-39.
- Hawtrey, R. G., 1930. "Money and Index Numbers," *Journal of the Royal Statistical Society* 93, 64-85.
- Hendrickson, J. R., 2013, "Redundancy or Mismeasurement? A Reappraisal of Money," *Macroeconomic Dynamics*, forthcoming.
- International Monetary Fund (2008). "Divisia Money," *Monetary and Financial Statistics: Compilation Guide*, International Monetary Fund Publications Services, Washington, D.C., 183-184.
- Kane, E. J., 1964. "Money as a Weighted Aggregate," *Zeitschrift fur Nationalokonomie* 25, 221-243.
- Keating, J. and Smith, L., 2013a. "Price Versus Financial Stability: A Role for Money in Taylor Rule?" Working Paper 201307, University of Kansas, Department of Economics.
- Keating, J. and Smith, L., 2013b. "Determinacy and Indeterminacy in Monetary Policy Roles with Money," Working Paper 201310, University of Kansas, Department of Economics.
- Keynes, J. M., 1930. *A Treatise on Money*. Macmillan, London.
- Keynes, J. M., 1936. *The General Theory of Employment, Money and Interest*, Macmillan, London.
- King, R. G., C. I. Plosser, J. H. Stock, and M. W. Watson, 1991. "Stochastic Trends and Economic Fluctuations," *American Economic Review* 81, 819-840.

- Lee, T. H., 1972. "On Measuring the Nearness of Near Monies: Comment," *American Economic Review* 62, 217-220.
- Lucas, R. E., 2000. "Inflation and Welfare," *Econometrica* 68, 247-274.
- Mason, W. E., 1976. "The Empirical Definition of Money: A Critique," *Economic Inquiry* 14, 525-538.
- Moroney, J. R. and B. J. Wilbratte, 1976. "Money and Money Substitutes," *Journal of Money, Credit, and Banking* 8, 181-198.
- Muellbauer, J. N., 1975. "Aggregation, Income Distribution and Consumer Demand," *Review of Economic Studies* 62, 524-544.
- Nelson, 2007. "Milton Friedman and U. S. Monetary History: 1961-2006," *Federal Reserve Bank of St. Louis Review* 89, 153-182
- Quirk, J. D. and R. Saposnik, 1968. *Introduction to General Equilibrium Theory and Welfare Economics*, New York: McGraw-Hill.
- Phlips, L. and F. Spinnewyn, 1982. "Rational Versus Myopia in Dynamic Demand Systems." In *Advances in Econometrics*, R. L. Basmann and G. F. Rhodes (eds.), Greenwich, CT: JAI Press, 3-33.
- Poterba, J. M. and J. J. Rotemberg, 1987. "Money in the Utility Function: An Empirical Implementation," in W. A. Barnett and K. J. Singleton (eds), *New Approaches to Monetary Economics*, Cambridge: Cambridge University Press, 219-240.
- Rotemberg, J. J., 1991. "Commentary: Monetary Aggregates and Their Uses." In: M. T. Belognia (ed.), *Monetary Policy on the 75th Anniversary of the Federal Reserve System*, Boston: Kluwer Academic 232-44.
- Rotemberg, J. J., J. C. Driscoll, and J.M. Poterba, 1994, "Money, Output, and Prices: Evidence from a New Monetary Aggregate," *Journal of Business and Economic Statistics* 13: 67-83.
- Serletis, A., 2007. *The Demand for Money: Theoretical and Empirical Approaches*, 2nd edition, Springer Publishing, New York.

Serletis, A. and P. Gogas, 2014. "Divisia Monetary Aggregates, the Great Ratios, and Classical Money Demand Functions," *Journal of Money, Credit and Banking* 46, 229-241.

Serletis, A. and S. Rahman, 2013. "The Case for Divisia Money Targeting," *Macroeconomic Dynamics* 17, 1638-1658.

Serletis, A. and A. Shahmoradi, 2005. "Semi-nonparametric Estimates of the Demand for Money in the United States," *Macroeconomic Dynamics* 9: 542-559.

Serletis, A. and A. Shahmoradi, 2006. Velocity and the Variability of Money Growth: Evidence from a VARMA, GARCH-M Model, *Macroeconomic Dynamics* 10: 652-666.

Serletis, A. and A. Shahmoradi, 2007. Flexible Functional Forms, Curvature Conditions, and the Demand for Assets, *Macroeconomic Dynamics* 11: 455-486.

Steinhauer, L. and J. Chang, 1972. "On Measuring the Nearness of Near-Moneys: Comment," *American Economic Review* 62, 221-225.

Swofford, J., 1995. "A Revealed Preference Analysis of Friedman and Schwartz Money," *Journal of Money, Credit and Banking* 27, 154-164.

Theil, H., 1967. *Economics and Information Theory*, Amsterdam: Elsevier.

Varian, H. R., 1982, "Non-Parametric Tests of Consumer Behavior," *Review of Economic Studies* 50, 99-110.