Do You Mind if I Round?: Eliminating the Penny
A Structural Analysis*

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Abstract

For decades, economists have debated the price-rounding effect on the economy if the penny is eliminated. Deviating from the bulk of the literature, which typically considers case-studies with empirical simulations and data manipulation, I evaluate a multiple household, deterministic model with endogenous currency production. My findings suggest that the elimination of the smallest unit of currency has a “nickel-and-dime” effect on the economy, regardless of the rounding policy. This structural model is constructed and calibrated to emulate a “worst-case scenario”, but it is also robust to the empirical results in the literature.

JEL Classification: E41, E42, E61, E62, E63, E64

Keywords: Penny, Rounding Policy, Deterministic Model, Treasury Policy, Currency, Multiple Household Model

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1 Introduction

Using a multiple household, deterministic model, I find that the abandonment of the smallest unit of currency will have a trivial effect on the economy. This model analyzes the direct, indirect, distributional, and dynamic aspects of such a policy revision, finding that the frivolous nature of these results spans all four dimensions. In the past, studies have been conducted to investigate the overall effect of such an elimination, particularly direction and magnitude of price rounding. Many of these investigations have used menu item simulations with the most common of rounding policies, while others have applied these policies to actual transaction data. The results of these studies have been controversial, citing net rounding-up in some instances and net rounding-down in other instances. This paper diverges from the simulation-based empirical methods and answers a rather simple question: how will the overall economy react to this type of change in Treasury policy? We find that the direction of price rounding does not matter, as even the most exaggerated magnitudes affect the economy in a paltry fashion. This model is robust to the estimates found in the literature, allowing me to bridge the gap between their empirical work and this theoretical insight. All in all, the worst-case scenario produces a cost of just over three dollars per person, per year; with more realistic examples producing even less significant effects.

1.1 Background

There has been growing support in the United States for the elimination of the penny from the coinage system. Many other countries, including the United States, have already passed policies like this. For example, the United States government eliminated the half-cent coin in 1857, an action that would be analogous to eliminating the dime today. As far as I can tell, there were no serious consequences to this action. Recently, the Canadian government has moved to phase out its one cent coin, citing increasing costs of production and marginal utilization of the medium. Other countries, such as Australia and New Zealand, have also transitioned from their smallest denominated coins rather seamlessly, with New Zealand doing it twice since 1990. Proponents of the move believe that the production cost of the penny is too high for its lack of use, claiming that it costs taxpayers millions of dollars a year. Opponents claim that the elimination of the penny, and the rounding policies that follow, will implicitly establish a severe, regressive “rounding tax” that will drive up prices and cause overall welfare to fall.

One of the basic concepts of maintaining a stable, working monetary system is that the value of the commodity money as its base material must be less than the face value.\(^1\) In 2012, the US mint estimated that it cost roughly $0.016 on average for the raw materials to produce a penny, and $0.02 overall.\(^2\) The same report also estimates that the US government generated -$58 million in seigniorage from the penny in 2012 and has generated negative seigniorage from the penny for the past seven years. While this paper focuses mostly on the penny, the trend we see here is not exclusive to this coin. Nickels have the same problem as well, requiring $0.0829 in raw materials and $0.1009 overall to produce one nickel. The fact that these coins cost more to produce than their face value can have some bad consequences. By Gresham’s law, and many of its derivatives, these coins are in danger of being exported or melted down.\(^3\) For example, if the value of the raw

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1 See Cipolla (1956), Smith and Sargent (1997), and Sargent and Velde (1999) for further discussion.
2 These figures were taken from the United States Mint 2012 Annual Report.
3 See Rolnick and Weber (1986).
materials used to produce the penny continue to rise, at what point will a black market emerge for melting the coins down and selling them as their base materials? Of course, conducting such action is illegal in accordance with United States Code.\textsuperscript{4}

The other side of this “coin” is that, if the penny is eliminated, there will be a significant disturbance in the economy from the ensuing rounding policies. Much of this centers around the possibility of prices being rounded upwards, costing consumers millions of dollars every year. Other studies suggest that, since the nickel’s face value is less than the value of its materials, eliminating the penny would actually cost the government more since demand would shift from the penny to the nickel, though these reports may be subject to bias.\textsuperscript{5} Also, since any rounding policy would only apply to cash transactions, the elimination of the penny would be a regressive policy, hurting the poor more than the rich since they use cash and coin as their primary medium of exchange.

The literature on this topic is fairly thin. One main cause of this is that most of the nations that have undergone this transition were emerging economies recovering from massive inflation, making the data unreliable. Those nations that consider this policy without a hyperinflation episode are few, and their occurrences are few and far between. For example, policy makers in Australia decided to phase out their one and two cent coins in 1992. New Zealand implemented similar policy twice, doing away with their one cent coin in 1990 and their five cent coin in 2006. Even though we have some examples of this policy, in both scenarios global contagion caused increased volatility in both economies.\textsuperscript{6} Thus, there are many roadblocks to conducting reliable econometric tests. So this paper takes a theoretical approach to see if, and to what degree, the elimination of the penny will affect the economy. I propose a deterministic, structural framework with two heterogeneous representative households, a producer, a banking sector, and a government sector that uses taxes and worn out currency to produce new currency. Using this simple model, I show the direct, indirect, distributional, and dynamic effects of eliminating small currency denominations and find that eliminating the penny will have varying effects, but that these effects are too small to matter in the long run. Steady state levels do adjust after the policy change, but the extent of this adjustment relative to the magnitude of the policy change I propose is negligible at best. We examine the effects of a policy change that essentially adds a half cent to the value of every dollar, which is orders of magnitude larger than even the most extreme cases in the literature.\textsuperscript{7} Even with an extremely large change in policy, the effects are very small.

1.2 Literature Review

As was mentioned above, the literature on this topic is fairly thin, but still controversial. Treasury policies in other nations typically round purchases made with currency to the next-smallest denomination, i.e. with transactions ending in $0.01, $0.02, $0.06, and $0.07 being rounded down and those ending in $0.03, $0.04, $0.08, and $0.09 begin rounded up here in the United States.

The current literature on this topic boils down to two simple methods: a simulation-based hypothetical approach, and an empirical-based, data-manipulation approach. The former is used

\textsuperscript{4} See Title 18, Part 1, Chapter 17, Section 331 as well as United States Mint Press Release: December 14, 2006.

\textsuperscript{5} See Bosco and Davis (2012). This report was commissioned by Jarden Zinc Products, which claims to be “North America’s leading plated coin blank producer.” This report may be biased towards not eliminating the penny.

\textsuperscript{6} “Black Monday” events sparked a US recession in the early 1990s and the global financial crisis beginning in 2007 were felt by both nations either during or immediately after their transition periods.

\textsuperscript{7} See Whaples (2007) and Lombra (2001).
by Lombra (2001, 2007) as well as Chande and Fisher (2003). Lombra bases his simulations on the menu of a convenience store chain, estimating the cost of transactions of three items or less. His results find that the prices of 60-93% of those simulated purchases were rounded up, with 50-83% of them being paid for with cash. This implies an annual cost to consumers of roughly $318-$818 million each year. Chande and Fisher, on the other hand conduct similar simulations and find that the distribution of the hundredths place approaches uniformity as more items are purchased per transaction. This implies that the rounding effect may be very small in a big picture sense, or even negative. These results, and those of Lombra (2007) include the use of sales tax, whereas Lombra (2001) does not. Distributionally, Lombra estimates that upwards of 9.5% of consumers in the United States do not have any kind of transactions account, implying that this policy has the potential to be extremely regressive. The lack of transaction accounts is important because only those purchases made with cash will be rounded, while those paid with other types of money will not be. Thus, poorer households without transactions accounts or credit cards will be subjected to this policy change more than richer households.

The second method, used by Whaples (2007), considers actual transactions data from a convenience store chain, rounding those prices in accordance with the proposed policy. Upon review of these estimates, he finds that the “rounding tax” is, on average, slightly negative, finding only one State with a positive result. In all cases, whether positive or negative, these results were not significantly different from zero. Thus, he finds that the net rounding effect of such a policy will be effectively null.

2 The Mechanism at Work: A Two-Period Model

Before we can understand how the economy will react as a whole, we need to explore the mechanism involved. Specifically, I am interested in how the households will react to the change, allowing me to make simplifying assumptions such as constant government spending and tax rates. We can then use these results to make inferences about the larger economy. For example, if the net effect of the policy change is that prices are rounded down, I find that consumption levels rise and currency holdings fall. While government spending is constant in this model, the increase in consumption and the decrease in the need for new currency implies that there would be downward pressure on overall government expenditures if this was not the case. Thus, the elimination of the penny is welfare-improving.

2.1 The Model

Here I present a two-agent, two-period endowment economy with currency production. The first agent is a representative household which receives an endowment each period and has the ability to invest and consume in the first period. In the second period, the household uses all of the savings and investments, along with the second period endowment, to purchase consumption goods. The second agent is the government, which takes in a sales tax from the household and produces the currency used in transactions. After addressing the model, I consider a change in the treasury’s policy that will effect the purchasing power of the currency held by the household.
The Household  The household maximizes the discounted sum of its utility, where its utility function for each period is given by

\[ U_t = \ln c_t - \frac{\chi c_t}{\mu n_t} \]

for period \( t = \{1, 2\} \), where \( c \) denotes real consumption, \( \chi \) and \( \mu \) are parameters representing preferences and the Treasury policy, respectively, and \( n \) represents the stock of currency holdings for the household given by the following law of motion

\[ n_t = n_t^p + (1 - \sigma)n_{t-1}. \tag{1} \]

To simplify the matter, I set \( n_0 = 0 \), that is, it starts with zero currency holdings. The term \( n_t^p \) represents the newly printed currency in period \( t \). The parameter \( \sigma \), represents the rate at which currency wears out each period. The household receives an endowment of \( y_1 \) in the beginning of the first period and has the ability to consume and invest in government bonds \( b \). Combining these characteristics gives me the first-period budget constraint for the household,

\[ \tau c_1 + b + n_1^p = y_1, \tag{2} \]

where \( \tau \) is the gross consumption tax rate levied on the household by the government. We choose the sales tax over a lump sum tax for its distortionary and regressive properties. Choosing this will ensure the maximum overall effects, if such effects exist, giving me the extremes off which to base my judgement.

In the second period, the household once again receives an endowment \( y_2 \) as well as receiving the return on the bonds purchased in the first period and acquiring more currency for purchases in the second period. Thus, the household’s budget constraint in the second period is as follows:

\[ \tau c_2 + n_2^p = y_2 + rb, \tag{3} \]

where \( r \) represents the gross interest rate on the government bonds.

The Government  Here I consider a government which produces units of government spending \( g_t \) as well as currency for the economy \( n_t^p \). In order to finance these expenditures, the government takes in the sales tax revenue from the household, collects the worn out currency in the economy, and sells bonds. It costs the government \( \zeta > 0 \) to produce each unit of new currency. Thus, the government’s real budget constraints for each period are

\[ g_1 + \zeta n_1^p = b + (\tau - 1)c_1 + \Delta n_1 + \sigma n_0 \]

and

\[ g_2 + rb + \zeta n_2^p = (\tau - 1)c_2 + \Delta n_2 + \sigma n_1, \]

where \( \Delta n_t \) represents the seignorage income from a change in the amount of currency in the economy.

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\(^8\) For simplicity, I’ll refer to it as the depreciation rate of currency in the future, though this may be a slight abuse of the language.
2.2 Comparative Statics: Treasury Policy Changes

Here, I analyze the effects of a change in the Treasury policy parameter $\mu$. Notice that I model this policy parameter so that, if prices are rounded down overall, the parameter will increase, making currency more valuable. While this model is simple and only two periods, the inclusion of currency into the model makes it too complex to solve by hand. Thus, I have to calibrate the parameters and solve for the variables before acquiring numerical results for the comparative statics problem.

Parameter Calibration

This model contains six parameters which are calibrated to match moments in the data or values commonly used in the literature. First, I normalize total output $y$ to be unity. This is purely for simplification of the model. We then calibrate the exogenous government spending $g$ to be 0.20. This matches the mean quarterly ratio of federal government current expenditures to nominal gross domestic product in the United States between 1947:Q1 and 2013:Q1, which is 0.1971. We also calibrate the net interest rate in the model $r$ to be 1.04, which matches the average one-year Treasury constant maturity rate between January 1987 and May 2013. We calibrate $\beta$, which is the personal discount factor, to 0.995, which matches much of the literature cited in this paper. Our currency depreciation parameter $\sigma$ is calibrated to 0.40 for two reasons. First, it’s large enough to ensure that the government will have to produce new currency in the second period. Second, one-dollar bills in the United States last an average of about 18 months, whereas larger bills and coins last much longer. Since there are considerably more one-dollar bills in circulation than any other paper denominations, this calibration seems to be a reasonable estimate of the average depreciation of all paper currency. The calibration of the tax rate $\tau$ is ten-percent. This sales tax rate is mostly ad hoc and larger than most state and local rates, but since this is the only form of taxes in the model, letting this be a little larger than the data suggests covers the fact that the overall amount of taxes paid is much larger than that suggested by the model. The last two parameters $\chi$ and $\zeta$ are calibrated to ensure that the levels of consumption in each period sum to approximately 0.65.\(^9\) The average quarterly ratio of real personal consumption expenditure to real gross domestic product from 1947:Q1 to 2013:Q1 is 0.6547, meaning that a calibration of this type matches this moment in the data. With this in mind, I calibrate the values to $\chi = 0.10$ and $\zeta = 0.90$.

Numerical Results for the Model

Using the parameter values outlined above, I can solve for the variables in the model. Using these values, along with the parameter values, I can also numerically solve the comparative statics problem. Since the initial condition is that the Treasury continues to produce the penny, I consider the changes to the variables to a Treasury policy change when $\mu = 1$, meaning that there is no rounding policy. Table (1) presents these results. Notice that, as expected, consumption in both periods will move proportionally with the Treasury policy parameter via an income effect. This means that, if a policy change results in a net round-down situation ($\mu$ increases), then consumption will rise in both periods. The effect on the second period’s currency holdings is inversely related to the policy parameter, which is fairly intuitive since the value of currency increases when prices are rounded down, allowing consumers to purchase more consumption goods with less currency. The effect on the first period’s currency holdings is less certain. This value hovers around zero, moving slightly one way or the other with adjustments to

\(^9\) Of course, the summation includes discounting the second period’s consumption by $\frac{1}{r}$, which is how the total output for the two periods is summed.
Table 1: Comparative Statics Results: $\mu$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Sign</th>
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<tbody>
<tr>
<td>$c_1$</td>
<td>Period 1 Consumption</td>
<td>+</td>
</tr>
<tr>
<td>$c_2$</td>
<td>Period 2 Consumption</td>
<td>+</td>
</tr>
<tr>
<td>$n_1$</td>
<td>Period 1 Currency Holdings</td>
<td>/−</td>
</tr>
<tr>
<td>$n_2$</td>
<td>Period 2 Currency Holdings</td>
<td>−</td>
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the tax policy or the production cost of currency. This is also fairly intuitive since the consumer, in this model, carries over the total currency holdings (net depreciation) to the second period. Thus, the effects on the second period’s currency holdings can be expected to be larger in magnitude than that of the first period’s currency holdings.

3 Deterministic Model with Heterogeneous Households

While a two-period model does give us an idea of how the households will react to this change (holding all else constant), the literature raises questions that require a slightly more complex model, allowing for more endogenous adjustment. To address these questions, I present a deterministic model of the economy with multiple, heterogeneous households and a government sector that produces currency. One of the first questions deals with welfare distribution, so I present one wealthy household and one poor household to the model. The former is a typical Ricardian household that has access to the financial markets, allowing it to consume and save in each period. The latter is a “rule-of-thumb” household as in Campbell and Mankiw (1989). This household is also referred to as a “hand-to-mouth” household in that it has no access to financial markets, meaning that all income is consumed immediately.\footnote{For more examples of this type of household, see Mankiw (2000), Weber (2000), and Galí, López-Salido, and Vallés (2007).}

We introduce a banking sector to the economy, which provides an inside money substitute to the currency produced by the government. This government takes in taxes through a sales tax and collects worn out currency to produce the new currency. We also introduce a representative firm to analyze any labor-leisure tradeoffs. This firm uses the labor provided by the households and the public goods produced by the government to produce consumer goods.

3.1 The Heterogeneous Households

The Ricardian Household

The population in this model is divided, consisting of $(1 - \lambda)$ Ricardian households and $\lambda$ “rule of thumb” households, where $\lambda \in [0, 1]$. Following the literature, I’ll denote the Ricardian household with an “$o$”, such that a variable $x$ that belongs to the optimizing household is referred to by $x^o$. With a discount factor $\beta$, this household maximizes an intertemporal problem with contemporaneous utility function

$$U_t^o = \ln c_t^o - \eta h_t^o - \frac{1}{\chi} \left( \frac{c_t^o P_t}{M_t^o} \right)^\chi,$$

\footnote{For more examples of this type of household, see Mankiw (2000), Weber (2000), and Galí, López-Salido, and Vallés (2007).}
where $c_t^r$, $h_t^r$, $P_t$, and $M_t^o$ denote real consumption expenditures, hours worked, the price level, and the nominal monetary services aggregate, respectively. The monetary services aggregate is a CES function that aggregates the purchasing abilities of outside money, denoted by nominal currency holdings $N_t^o$, and inside money, denoted by nominal deposits $D_t$ in the following manner

$$M_t^o = \left[ \xi^\theta (\mu N_t^o)^{\frac{\theta-1}{\theta}} + (1 - \xi)^{\theta} D_t^{\frac{\theta-1}{\theta}} \right]^{\frac{1}{\theta}}$$

where $\xi \in (0, 1)$ is the distribution parameter as described by Arrow, Chenery, Minhas, and Solow (1961), $\theta > 0$ represents the elasticity of substitution between currency holdings and deposits in providing monetary services, and $\mu$ represents a Treasury policy parameter that embodies the price rounding effect on currency purchases. Thus a net rounding down of prices would be represented by an increase in $\mu$. As a base for further analysis, the pre-policy change value is set to $\mu = 1$.

The third part of expression (4) is a shopping time friction, which is a positive function of the consumption good and a negative function of the purchasing medium. This expression is used frequently in the literature.

To encompass the ideas of a money multiplier and creation of inside money, I allow the Ricardian household to save and borrow. In each period, it earns its wage income $W_t h_t^r$, receives its investment income via principle payments on government discount bonds purchased last period $B_{t-1}$ and interest payments on deposits $r_{t-1}^d D_{t-1}$, and takes out loans from the representative bank $L_t$. Here I consider $W_t$ to be the nominal wage rate set by the representative firm, and $r_t^d$ to be the gross nominal interest rate on deposits set by the representative bank. It then takes this income and uses it for consumption, investment in new discount bonds, savings, repayment of debt from last period, and holdings of newly printed currency $N_t^{o,p}$, where

$$N_t^o = N_t^{o,p} + (1 - \sigma) N_{t-1}^o.$$ 

Currency depreciates at rate $\sigma$, which implies that the government needs to collect this currency and replace it with new currency. Thus, the government needs to print currency in order to keep up with both the change in the amount demanded as well as replace the depreciated currency. With this in mind, the budget constraint of the Ricardian household becomes

$$\tau_t c_t^r P_t + \frac{B_t}{r_t} + D_t + N_t^{o,p} + r_{t-1}^d L_{t-1} = W_t h_t^r + B_{t-1} + r_{t-1}^d D_{t-1} + L_t,$$

where $r_t^d$ is the gross interest rate on loans and $\tau_t$ is the gross sales tax rate.

**The Hand-to-Mouth Household** This household has no access to financial markets, implying that it consumes all of its labor income in each period. Thus, it only has to decide how many hours to work each period. As in the literature, I denote the rule-of-thumb household’s variables with an “$r$” superscript. This household therefore maximizes its contemporaneous utility function each period, given by

$$U_t^r = \ln c_t^r - \eta h_t^r - \frac{1}{\chi} \left( \frac{c_t^r P_t}{M_t^o} \right)^\chi,$$

11 See Belongia and Ireland (2012) and Ireland (2012) for more uses of this functional form in this type of scenario. For a deeper look into these types of monetary aggregates and their usefulness in microeconomic and macroeconomic theory, see Barnett (1978, 1980, 1990) and Barnett, Offenbacher, and Spindt (1984).
where each of the variables are analogous to those of the Ricardian household. In this case, however, the monetary services aggregate collapses to

\[ M^r_t = \xi^r \mu N^r_t \]

since this household cannot access financial markets for deposits. Since this household consumes all its labor income, the budget constraint is simply

\[ \tau^r_t c^r_t P^r_t = W^r_t h^r_t \]  \hspace{1cm} (5)

and all labor income is converted to currency for consumption purposes, giving me \( \mu N^r_t = W^r_t h^r_t \).

3.2 The Representative Bank

The representative bank’s primary purpose is to provide a substitute for the Ricardian household’s medium of exchange. It’s secondary purpose is to facilitate multiple deposit creation and the money multiplier that follows. Each period, the bank takes in new deposits and payments on matured loans from last period. It also issues new loans, makes interest payments on last period’s deposits, and incurs a linear deposit creation cost \( x^d D_t \), which causes a wedge in the saving/borrowing process. Thus the profits \( \Pi_b^t \) for the representative bank are given as

\[ \Pi_b^t = D_t - L_t + r^t_{t-1} L_{t-1} - r^d_{t-1} D_{t-1} - x^d D_t. \]

On top of this budget constraint, the bank is also subject to reserve requirements by the monetary authority. In accordance with profit maximization, the bank wants to hold zero excess reserves, holding only those that are required. Thus, I have the equilibrium condition \( L_t = (1 - \omega) D_t \) for all \( t = 0, 1, \ldots \); where \( \omega \in [0, 1] \) represents the required reserves ratio.

3.3 The Representative Firm

The representative firm produces real output \( y_t \) using the aggregate labor hours from the households \( h_t \) and the real public goods produced by the government \( g_t \).\(^\text{13}\) The firm pays out a single wage rate \( W_t \), so it does not care where the labor hours come from, maximizing its profits \( \Pi^f_t \) by choosing this aggregate, given by \( h_t = (1 - \lambda) h^q_t + \lambda h^r_t \). The production function is constant-returns-to-scale Cobb-Douglas, such that \( y_t = h^\alpha_t g^{1-\alpha}_t \) and \( \alpha \in (0, 1) \). The profit function of the representative firm is

\[ \Pi^f_t = P_t y_t - W_t h_t. \]

3.4 The Government and Monetary Authority

The government in this model not only produces the public good \( G_t \), but also prints new currency \( N^p_t \). To do so, the government must collect the sales tax from the households, borrow from

\(^{12}\) Combining this with equation (5) gives me an implicit Clower (1967) constraint, which seems to be redundant considering I include the shopping time friction, but this allows me to ensure that the only decision this household makes is one of labor hours.

\(^{13}\) Notice that lower case letters represent the real values of their corresponding upper-case variables.
the households, and remove the depreciated currency from the economy. With this in mind, the government’s budget constraint becomes

\[(\tau_t - 1)c_t P_t + \frac{B_t}{r_t} + \Delta N_t + \sigma N_{t-1} = G_t + B_t + \zeta N^P_t,\]

where \(\Delta N_t\) represents the seignorage from the increase in the aggregate currency levels, \(\zeta > 0\) represents the cost of printing new currency, and \(c_t\) represents real aggregate consumption. Aggregate consumption and currency levels are given by \(c_t = (1 - \lambda)c^o_t + \lambda c^r_t\) and \(N_t = (1 - \lambda)N^o_t + \lambda N^r_t\), respectively. The tax policy used by the government follows the simple rule

\[\ln \left( \frac{\tau_t}{\tau} \right) = \varphi_{\tau} \ln \left( \frac{\tau_{t-1}}{\tau} \right) + \varphi_y \ln \left( \frac{y_t}{y_{t-1}} \right),\]

where \(\tau\) is the steady state level for the tax rate and \(\varphi_{\tau}\) and \(\varphi_y\) are necessarily positive parameters. The use of a cyclical tax rate ensures that the economy does not diverge in one direction or another. The autoregressive nature of this fiscal policy rule is considered due to the fact that tax policies generally don’t change often or by very much at any particular time. The monetary authority follows a Taylor (1993) type interest rate rule with smoothing

\[\ln \left( \frac{r_t}{r} \right) = \rho_r \ln \left( \frac{r_{t-1}}{r} \right) + \rho_\pi \ln \left( \frac{\pi_t - 1}{\pi} \right),\]

where \(\rho_r\) and \(\rho_\pi\) are positive parameters and \(r\) and \(\pi\) are steady state values for the bond rate and the inflation rate, respectively.

4 Results

Here I present some of the findings in the model. We start with the calibration of the parameter values, which are set using both the literature and historical data. We then provide some graphical representations of the model dynamics after a foreseen, permanent change to the Treasury policy parameter \(\mu\). We consider both rounding up and rounding down scenarios, increasing the value of \(\mu\) from unity to 1.005 and decreasing it to 0.995. Specifically, I consider a situation where every dollar gains/loses a half cent in value. This is much, much larger than any of the simulations in the literature suggest, but I consider this larger value for expositional purposes.\(^{14}\) In addition, a larger value will show us what happens to the economy if this change results in a massive, permanent shock to the economy. One can think of this value as a best-case/worst-case scenario analysis.

4.1 Calibration

Whenever possible, I use quarterly data from 1987:1–2006:4 to encompass the end of the Great Moderation up to just before the recent financial crisis. Some parameters are set following past literature values. For example, I set \(\chi = 5\), \(\xi = 0.20\), and \(\theta = 0.50\) following Ireland (2012), which...
uses the same functional forms for the monetary services aggregate and the shopping time friction. Considering Lombra (2001), who suggests that 9.5% of households don’t have access to transaction accounts, I set \( \lambda = 0.095 \). Other variables are set to fit simple intuition. For example, I calibrate \( \eta = 3 \) because the typical work day is eight hours, or one third of the day. Setting \( \eta \) to this value gives me labor hours close to \( h = \frac{1}{3} \). We also set \( \alpha = 0.80 \), assuming that public goods produced by the government do not add much to the production process, which leans heavily on labor hours. Another parameter that can be set fairly easily is the steady state inflation rate. We set this to \( \pi = 1.005 \), which implies an annual inflation rate of around two percent, the implied inflation target. Considering general data trends found at the Federal Reserve Bank of Richmond, I assume that the typical unit of currency will wear out every five years or so, implying that \( \sigma = 0.05 \). The same holds true for seigniorage data from the Board of Governors of the Federal Reserve System, implying that \( \zeta = 0.30 \), a value that ensures that producing currency yields positive seigniorage in general.\(^{15}\)

The rest of the parameters have been estimated using the data. We calibrate \( \omega = 0.035 \) to match the average ratio of the St. Louis Adjusted Monetary Base less currency in circulation and excess reserves to savings deposits at commercial banks. This is an estimate of the average required reserves ratio during the period 1987:Q1–2006:Q4. For the same time period, I estimate the value of \( x^d \) by considering a ratio of a deposit rate estimate to the bank prime loan rate. The deposit rate estimate is the over-time average of the mean of the 6-month certificate of deposit secondary market rate and the rate on money market mutual fund accounts. This gives me an arbitrary short-term interest rate that lies somewhere between the highest and lowest available rates on differing types of interest bearing deposits. Doing so implies that \( x^d \) should be set to 0.01. The Ricardian household’s discount factor \( \beta \) is set by considering the average effective federal funds rate over my calibrated value for inflation. This gives me a value of \( \beta = 0.99 \). The parameter values in the fiscal policy rule are set with a simple linear regression. Here I regress the values of the federal government current tax receipts-to-personal consumption expenditures ratio against its owned lagged value and the GDP growth rate. Doing so gives me parameter values \( \phi_\tau = 0.9009 \) and \( \phi_y = 0.0074 \). This implies that the tax rate does not react strongly to outside forces, which would coincide with the idea that tax rates are mostly exogenous. The values for the monetary policy rule were calibrated in the same fashion. Regressing the effective federal funds rate against its own lag and a one-period lag of the inflation rate gives me parameter values of \( \rho_\tau = 0.96 \) and \( \rho_\pi = 0.06 \).

4.2 Deterministic Shock Simulations

Since this paper considers fiscal policy, any policy change is announced in a particular period, but there is a one year (four period) lag before the policy comes into effect. Figure (1) shows the impulse responses for aggregate consumption, aggregate currency holdings, the inflation rate, and government spending after a positive, permanent shock to the Treasury policy parameter, i.e. prices are rounded down and \( \mu \) increases to 1.005. Each of the panels in the figure contains two vertical lines. These lines coincide with the announcement and implementation of the policy change, respectively. We chose the aggregate values for each because the individual household impulse response functions are nearly identical in shape, so a broad, macroeconomic view represents a good

\(^{15}\) This value for \( \zeta \) is probably larger than it should be, but a larger value will again give me a worst-case-scenario if it costs the government large amounts to produce the currency. We are also focusing more on the effects of coin costs and not that of paper currency costs, which are dramatically larger, in a relative sense.
As can be seen in the figure, the effects of this relatively large shock are minor. Upon the announcement of the new policy, the households realize that their steady state consumption levels will increase. Therefore, in an effort to smooth their consumption, they deviate from their Euler equations and begin increasing consumption immediately (panel 1). Due to the persistence in the tax level though, this transition is slow and does not reach the new steady state level before implementation of the new policy. In order to reach these higher levels of consumption, they need more currency and deposits, as can be seen by the upwards drift in the second panel. This increase in consumption, currency, and deposits puts upward pressure on inflation (panel 3), but is quickly corrected by monetary policy. As for the government sector of the economy, the balanced budget assumption and the increased need for currency force the government to divert resources from traditional spending to currency production. This is exacerbated by the fact that output remains relatively unchanged, causing a crowding out effect that forces spending to fall at
a faster rate.

**Policy Implementation** Upon implementation of the new policy, the currency holdings of the households become more valuable. This income effect causes consumption to spike slightly and currency holdings to fall. This increase in consumption and the fall in seignorage revenues cause government spending to fall temporarily at implementation. In a reversal of the announcement period, the fall in currency holdings causes inflation to fall immediately, but is corrected by monetary policy in the next period. The increase in the value of currency also causes deposits to fall by a substitution effect. After the initial impact of implementation, the households continue to transition to the new steady state level of consumption. This increase causes an increased need for currency, which again combine to push government spending down as in the announcement period.

If I reverse the net effect and follow the rounding direction estimated by Lombra (2001) ($\mu = 0.995$), I get nearly symmetric results, despite the inherent non-linearity of the model, which coincides with his findings. In this scenario, welfare would fall, government spending would increase, and there would be upward pressure on inflation. However, due to monetary policy and an inflation target that is maintained, inflation is contained and the spike is even neutralized, a situation not considered in the literature.

Overall, we can see in Table (3) that the long-run effects of this permanent shock are very small. This table shows the initial and resulting steady-state values for the model, along with the percentage change in each of them. We see that, for either a positive or negative rounding result, most of the variables in the model will settle in close to their initial values. The largest shift in any variable is in the currency holdings, but even this adjustment is by less than a half of one percent in the aggregate. If we were to look at the data, this adjustment would imply that the currency component of $M1$ would increase or decrease by around $3.6$ million in the first quarter of 2013, or a little over a penny per person. Thus, the long-run effects of this policy, even after this large shock, would be insignificant.

### 4.3 Welfare Analysis: Consumption Equivalent Variation

In this section, I conduct a welfare analysis using consumption equivalent variation (CEV). In keeping with Lombra (2001), I consider the first five years after the policy announcement, where the policy is announced at the beginning of the first year and implemented at the beginning of the second year. To evaluate the CEV, I first consider the situation in which the policy doesn’t change, i.e. the penny is not eliminated. We then consider the utility function maximized by each household where policy does change. Adding my measure for welfare gain or loss, I have the following utility functions,

$$\max_{i = \{o, r\}} \beta^t \left[ \ln c^t_i (1 + \phi^t_w) - \eta h^t_i - \frac{1}{\chi} \left( \frac{c^t_i}{m^t_i} \right)^{\chi} \right],$$

for $i = \{o, r\}$, where $\phi^t_w$ and $\phi^t_r$ are the equivalent variation measurements for each of the respective households.\(^\dagger\) While the poor household is constrained to the point that it can only make contemporaneous decisions, it discounts time in the same fashion as the rich household. Considering the

\(^\dagger\) Negative values for these measurements imply that the household is better off after the policy change, whereas positive values indicate that the household is worse off.
values depicted in Figure (1), and comparing these maximized utility values with those considering only the initial steady state values, I find that, in the case of a net-round down situation, both households are better off, though the poor household is considerably more. The results are provided in Table (2). They show what percentage of additional consumption is needed each quarter to make the household indifferent between the two situations. The last row of the table considers the aggregated CEV using the same method as with the other terms in the model, \( \phi_w^a = (1 - \lambda)\phi_w^o + \lambda\phi_w^r \). Considering that the rich household has access to financial markets, it is intuitive that its results are less volatile than that of the poor household. Since the poor household is completely constrained and cannot access financial markets, variations in the value of its currency holdings will cause larger equivalency requirements. Thus, we see that the progressive/regressive nature of this “rounding tax” depends on the direction of the rounding. If the net result is that we round down, we find that the welfare gap closes slightly. If we round up, on the other hand, then the gap would widen.

**Comparing to Past Results** We can now take the values from Table (2) and see how they compare to the results found in the literature, finding that my model is quite robust. For example, Lombra (2001) claims that these changes will cost consumers no less than $1.5 billion over a five-year period. Using data on personal consumption expenditures (measured in billions of chained 2009 dollars during 1990:Q1–1999:Q4) I find that, if prices are rounded up at this estimated rate, consumers will lose an average of $118 million per quarter, or about $2.36 billion over an average five-year period, which is in his estimated range.\(^{17}\) This averages out to about a $1.60 cost to each consumer per year.\(^{18}\) If I consider the values estimated in Whaples (2007), and conduct the same test as above, I find that the average consumer would benefit by approximately 0.04 cents per year. Thus, my model fits the results in the literature quite well. In any case, the cost or benefit to each consumer in these events is negligible.

### Table 2: Welfare Analysis: Consumption Equivalent Variation

<table>
<thead>
<tr>
<th>Measure</th>
<th>Positive Shock ((\mu = 1.005))</th>
<th>Negative Shock ((\mu = 0.995))</th>
<th>Lombra (2001) ((\mu = 0.99828))</th>
<th>Whaples (2007) ((\mu = 1.0000455))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rich CEV, (\phi_w^o)</td>
<td>-0.0441%</td>
<td>0.0444%</td>
<td>0.0152%</td>
<td>-0.0004%</td>
</tr>
<tr>
<td>Poor CEV, (\phi_w^r)</td>
<td>-0.7458%</td>
<td>0.7562%</td>
<td>0.2589%</td>
<td>-0.0069%</td>
</tr>
<tr>
<td>Aggregate CEV, (\phi^a)</td>
<td>-0.1108%</td>
<td>0.1120%</td>
<td>0.0384%</td>
<td>-0.0010%</td>
</tr>
</tbody>
</table>

\(^{17}\) $2.36 billion measured in 2009 dollars is approximately equivalent to $1.89 billion measured in 2000 dollars, the publication year of the menu costs in Lombra’s simulations. Thus, applying his results to my model yields nearly the same conservative values he finds.

\(^{18}\) In calculating these values, I multiply the values found in Table (2) by the quarterly personal consumption expenditures and take the average. We then take this value and divide it by the average population at the time.

5 **Concluding Remarks**

This paper deviates from the literature through the use of a structural model. In doing so, I find that the effects of eliminating a nation’s smallest unit of currency are insignificant, even when considering shocks that are many times larger than those suggested in the literature. While I consider the larger shock for expositional purposes, the model is robust to shocks which correspond

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to those past estimates. Future inquiries can build on these results by relaxing some of my base assumptions, such as a balanced government budget, or by considering a more intricate currency production process. The simplicity and robustness of this model, however, makes it a good starting point for future debate on a growing issue. As prices continue to rise gradually, the production and distribution of the penny will become increasingly burdensome for the US government and taxpayers, implying a need for policy change. The governments of Canada, New Zealand, Australia, and others; facing the same pressures; resorted to discontinuation of their smallest denominations. What will the US government decide to do?
References


Table 3: Steady State Analysis\textsuperscript{a}

<table>
<thead>
<tr>
<th>Variable</th>
<th>Initial Value</th>
<th>Round Down\textsuperscript{b}</th>
<th>Round Up\textsuperscript{c}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Value</td>
<td>Change (%)</td>
</tr>
<tr>
<td><strong>Rich Household</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption, $c_r^t$</td>
<td>0.2876</td>
<td>0.2878</td>
<td>0.0778</td>
</tr>
<tr>
<td>Currency, $n_r^t$</td>
<td>0.0599</td>
<td>0.0597</td>
<td>-0.2490</td>
</tr>
<tr>
<td>Deposits, $d_r$</td>
<td>0.6069</td>
<td>0.6069</td>
<td>0.0004</td>
</tr>
<tr>
<td>Loans, $l_r$</td>
<td>0.5856</td>
<td>0.5856</td>
<td>0.0004</td>
</tr>
<tr>
<td>Hours, $h_r^o$</td>
<td>0.3326</td>
<td>0.3326</td>
<td>0.0004</td>
</tr>
<tr>
<td><strong>Poor Household</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption, $c_p^t$</td>
<td>0.2964</td>
<td>0.2967</td>
<td>0.0749</td>
</tr>
<tr>
<td>Currency, $n_p^t$</td>
<td>0.3323</td>
<td>0.3307</td>
<td>-0.4988</td>
</tr>
<tr>
<td>Hours, $h_p^t$</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>Government</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spending, $g_t$</td>
<td>0.0382</td>
<td>0.0380</td>
<td>-0.5947</td>
</tr>
<tr>
<td>Curr. Prod., $n_p^o$</td>
<td>0.0043</td>
<td>0.0043</td>
<td>-0.3409</td>
</tr>
<tr>
<td><strong>General Economy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output, $y_t$</td>
<td>0.4146</td>
<td>0.4146</td>
<td>0.0003</td>
</tr>
<tr>
<td>Inflation, $\pi_t$</td>
<td>1.0050</td>
<td>1.0050</td>
<td>0.0000</td>
</tr>
<tr>
<td>Interest Rate, $r_t$</td>
<td>1.0152</td>
<td>1.0152</td>
<td>0.0000</td>
</tr>
<tr>
<td>Wage Rate, $w_t$</td>
<td>0.9970</td>
<td>0.9970</td>
<td>-0.0001</td>
</tr>
</tbody>
</table>

\textsuperscript{a} Recall that the shock I consider here is orders of magnitude larger than those estimated in the literature. We do this for expositional purposes.

\textsuperscript{b} Treasury policy parameter $\mu$ set to 1.005.

\textsuperscript{c} Treasury policy parameter $\mu$ set to 0.995.