

**Fellow's Opinion Article**

**Tastes and Technology: Curvature is not Sufficient for Regularity**

by

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**Abstract:**

In specifications of tastes and technology, econometricians often impose curvature globally, but monotonicity only locally or not at all. In fact monotonicity rarely is even mentioned in that literature. But without satisfaction of both curvature and monotonicity, the second order conditions for optimizing behavior fail, and duality theory fails. The resulting first order conditions, demand functions, and supply functions become invalid. Although unconstrained specifications of technology are more likely to produce violations of curvature than monotonicity, I believe that induced violations of monotonicity become common, when curvature alone is imposed. Hence the now common practice of equating regularity with curvature is not justified.

**JEL Classification Codes:** C51, C3, C13

**Keywords:** technology, regularity, curvature, production, flexible

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## 1. Introduction

In the literature on modeling tastes and technology, it has become common to impose curvature globally, but not monotonicity. The reason is that curvature is often violated when not imposed. Monotonicity is less often violated, when neither curvature nor monotonicity is imposed. Imposition of curvature alone has become especially common with applications of the generalized quadratic (formerly called generalized McFadden) specification for technology. Following that widespread practice, Barnett, Kirova, and Pasupathy (1995) globally imposed only curvature, when they estimated the multiproduct generalized quadratic model for financial intermediaries and the single product generalized quadratic model for manufacturing firms. They imposed monotonicity only at a single central data point in each case.

Subsequently Barnett and Pasupathy (forthcoming) checked monotonicity carefully with their previously estimated financial intermediary model. They found that isoquants, although always satisfying the imposed curvature, often had positive slopes at one or both ends of the isoquants. The positive slopes of isoquants demonstrate violations of monotonicity. These problems arose within the range of the data in most two dimensional sections of the isoquants. Although it is never pleasant to have to reveal serious defects in one's own prior published research, I believe that this problem is relevant to a large percentage of the currently popular research that similarly globally imposes only curvature.

## 2. Financial Intermediary Results

Barnett and Pasupathy (forthcoming) found many extreme regularity violations in their multiproduct financial intermediary model. In addition to monotonicity violations, they encountered nonunique isoquants and complex valued solutions. They also identified the possibility that violations of monotonicity could induce violations of curvature, even when curvature itself has been imposed globally by the usual procedure. This latter paradox is possible, since the model is weakly separable, with an inner aggregator function nested within an outer technology. If monotonicity of the outer function is violated, that violation could reverse the curvature of the inner aggregator function, even if curvature was imposed globally on both the inner and outer functions. In short, the common procedure of imposing curvature on inner and outer functions in weakly nested specifications of technology need not assure satisfaction of curvature of the composite function.<sup>i</sup>

Since the financial intermediary results were more heavily compromised than the manufacturing firm results, Barnett and Pasupathy (forthcoming) reported only the results of their regularity explorations for the financial intermediary model.

### 3. Manufacturing Firm Results

The financial intermediary model contains many unusual characteristics, including multiple outputs, risk aversion of firms, and generalized method of moments estimation. Furthermore, we found other irregularities along with the monotonicity violations. It is natural under these circumstances to wonder whether the adverse results with the financial intermediary model are typical and should induce researchers to take monotonicity more seriously, when curvature is imposed. As a result, my other coauthor, Milka Kirova, and I decided to produce the isoquants of the far less unusual manufacturing firm model. Two dimensional sections of those isoquants are displayed in Figure 1 of this article.

It is important to observe that the generalized quadratic model is parsimonious, in the sense that the model's flexibility cannot be retained if monotonicity is imposed globally along with curvature. But imposition of monotonicity locally at one point does not compromise the model's flexibility. Since we imposed monotonicity locally as well as curvature globally, there were no remaining degrees of freedom that could have been used to impose monotonicity globally without loss of flexibility. With no remaining degrees of freedom, the model's global regularity properties are an inherent implication of the model's estimated elasticities. If the estimated elasticities are reasonable and are typical of what frequently can be expected in other studies, then the induced curvature properties are necessarily also typical and highly relevant to what can be expected in other studies.

Although the results with the financial intermediary model are especially troublesome, the results with the manufacturing firm model are very typical. As reported in Barnett, Kirova, and Pasupathy (1995), the estimated elasticities with the manufacturing firm model are entirely reasonable and in no way unusual. In addition, we did not encounter nonunique isoquants, complex valued solutions, risk aversion of firms, curvature reversals of composite functions, or any other results that could be viewed as untypical of the model's normal properties. Hence the isoquant properties displayed in Figure 1 can often be expected from applications of the generalized quadratic model, when elasticities are within the neighborhood of the estimates reported by Barnett, Kirova, and Pasupathy (1995). This conclusion applies regardless of how the elasticity estimates were acquired: whether by generalized method of moments, maximum likelihood, or any other estimation procedure subject to global curvature and local monotonicity.

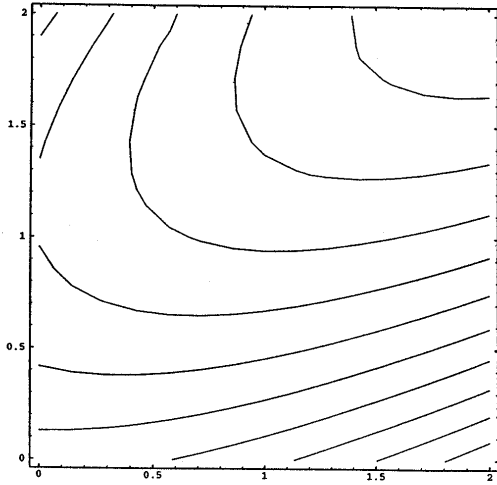
Observe the frequently positive slopes at ends of isoquants in Sections 1, 2, and 3 of Figure 1, as well as the global maximum that appears in Section 4. Clearly monotonicity violations exist within large regions of the data space. Since we plotted the range of the isoquants only within the region of the data, our unpleasant conclusions apply within a large percentage of that region.

#### **4. Conclusion**

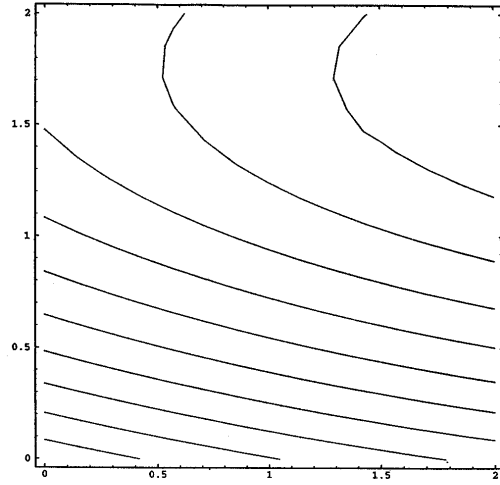
Since unconstrained models are far more likely to violate curvature than monotonicity, the literature on imposing curvature globally and monotonicity locally has followed a logical and potentially constructive direction. But it is my view that imposition of curvature may increase the frequency of monotonicity violations. Hence equating curvature alone with regularity, as has become disturbingly common in this literature, does not seem to me to be justified.

For decades, econometricians have been searching for a model that would permit flexibility and global regularity to be attained simultaneously with a parsimonious model having a finite number of parameters. In my opinion, availability of that capability still has not been demonstrated.<sup>ii</sup> The burden of proof should be on those who believe otherwise, especially when the results of monotonicity checks have not been reported.

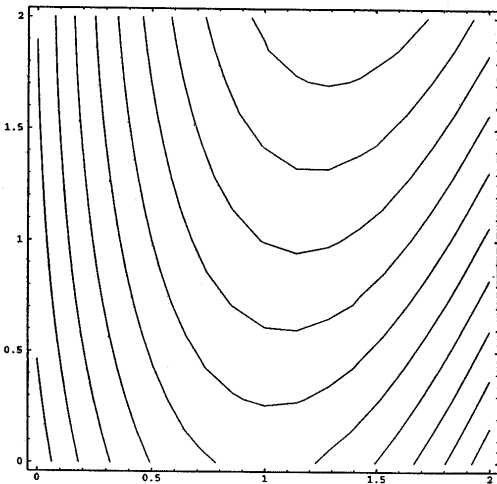
**Section 1: Capital versus Cash**



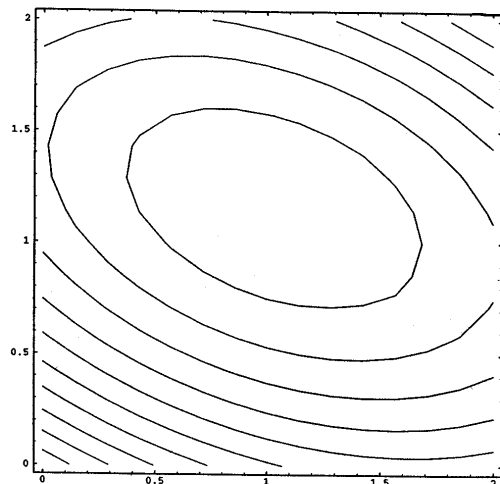
**Section 2: Materials versus Cash**



**Section 3: Labor versus Materials**



**Section 4: Capital versus Labor**



**Figure 1: Estimated Generalize-Quadratic Production Function Isoquant Sections**

## REFERENCES

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## FOOTNOTES

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<sup>i</sup> This phenomenon is analogous to the conversion of concave functions into convex functions by multiplication by  $-1$ .

<sup>ii</sup> The literature on semiparametric inference (e.g., Barnett, Geweke, and Wolfe (1991)) is not part of the literature addressed by this JE Opinion article. That literature has objectives that are not constrained by the goal of parsimonious parameterization.