THE UNIVERSITY OF KANSAS
WORKING PAPERS SERIES IN
THEORETICAL AND APPLIED ECONOMICS

ON USER COSTS OF RISKY MONETARY ASSETS

William A. Barnett
University of Kansas

Shu Wu
University of Kansas

June 2004

THE UNIVERSITY OF KANSAS
WORKING PAPERS SERIES IN THEORETICAL AND APPLIED ECONOMICS
WORKING PAPER NUMBER 200404

On user costs of risky monetary assets

by

William A. Barnett and Shu Wu

University of Kansas, Lawrence, Kansas

June 24, 2004

Forthcoming in the *Annals of Finance*, volume 1, no. 1.

On user costs of risky monetary assets*

William A. Barnett[1] and Shu Wu[2]

1. Department of Economics, University of Kansas, Lawrence, KS 66045, USA
(email: barnett@ku.edu)
2. Department of Economics, University of Kansas, Lawrence, KS 66045, USA
(email: shuwu@ku.edu)

Received: revised:

Summary. We extend the monetary-asset user-cost risk adjustment of Barnett, Liu, and Jensen (1997) and their risk-adjusted Divisia monetary aggregates to the case of multiple non-monetary assets and intertemporal non-separability. Our model can generate potentially larger and more accurate CCAPM user-cost risk adjustments than those found in Barnett, Liu, and Jensen (1997). We show that the risk adjustment to a monetary asset's user cost can be measured easily by its *beta*. We show that any risky non-monetary asset can be used as the benchmark asset, if its rate of return is adjusted in accordance with our formula. These extensions could be especially useful, when own rates of return are subject to exchange rate risk, as in Barnett (2003).

Keywords and Phrases: User costs, Monetary Aggregation, Risk, Pricing kernel, CAPM

JEL Classification Numbers: E41 G12 C43 C22

---

* We thank participants at the 11th Global Finance Annual Conference, Yuqing Huang, and an anonymous referee for helpful comments and suggestions.

Correspondence to: William A. Barnett

1. Introduction

Barnett (1978, 1980, 1997) produced the microeconomic theory of monetary aggregation under perfect certainty, derived the formula for the user cost of monetary assets, and originated the Divisia monetary aggregates to track the theory's quantity and price aggregator functions nonparametrically. The monetary aggregation theory was extended to risk by Barnett (1995) and Barnett, Hinich, and Yue (2000). In producing the Divisia index approximations to the theory's aggregator functions under risk, Barnett, Liu, and Jensen (1997) and Barnett and Liu (2000) showed that a risk adjustment term should be added to the certainty-equivalent user cost in a consumption-based capital asset pricing model (CCAPM). The risk adjustment depends upon the covariance between the rates of return on monetary assets and the growth rate of consumption. Using the components of the usual Federal Reserve System monetary aggregates, Barnett, Liu, and Jensen (1997) showed, however, that the CCAPM risk adjustment is slight and the gain from replacing the unadjusted Divisia index with the extended index is usually small. An overview of the relevant literature is provided in Barnett and Serletis (2000).

The small adjustments are mainly due to the very low contemporaneous covariance between asset returns and the growth rate of consumption. Under the standard power utility function and a reasonable value of the risk-aversion coefficient, the low contemporaneous covariance between asset returns and consumption growth implies that the impact of risk on the user cost of monetary assets is very small. This finding is closely related to those in the well-established literature on the equity premium puzzle [see, e.g., Mehra and Prescott (1985)], in which it is shown that consumption-based asset pricing models with the standard power utility function usually fail to reconcile the observed large equity premium with the low covariance between the equity return and consumption growth. Many different approaches have been pursued in the literature to explain the equity premium puzzle. One successful approach is the use of more general utility functions, such as those with intertemporal non-separability. For example, Campbell and Cochrane (1999) showed that, in an otherwise standard consumption-based asset pricing model, intertemporal non-separability induced by habit formation can produce large time-varying risk premia similar in magnitude to those observed in the data. This suggests that the CCAPM adjustment to the certainty-equivalent monetary-asset user costs can similarly be larger under a more general utility function than those used in Barnett, Liu, and Jensen (1997), who assumed a standard time-separable power utility function.

In this paper, we extend the results in Barnett, Liu, and Jensen (1997) to the case of intertemporal non-separability. We show that the basic result of Barnett, Liu, and Jensen (1997) still holds under a more general utility function. But by allowing intertemporal non-separability, our model can lead to substantial, and we believe accurate, CCAPM risk adjustment, even with a reasonable setting of the risk-aversion coefficient. We believe that the resulting correction is accurate, since the full impact of asset returns on consumption is not contemporaneous, but rather spread over time, as would result from intertemporal non-separability of tastes. This fact has been well established in the finance literature regarding non-monetary assets, and, as we shall see below, the risk adjustment to the rates of return on non-monetary assets plays an important role within the computation of risk adjustments for monetary assets. Hence, even if intertemporal separability from consumption were a reasonable assumption for monetary assets, the established intertemporal non-separability of non-monetary assets from consumption would contaminate the results on monetary assets, under Barnett, Liu, and Jensen's (1997) assumption of the conventional fully intertemporally-separable CCAPM.

We also extend the model in Barnett, Liu, and Jensen (1997) to include multiple risky non-monetary assets, which are the assets that provide no liquidity services, other than their rates of return. This extension is important for two reasons. First, it was shown by Barnett (2003) that the same risk-free benchmark rate cannot be imputed to multiple countries, except under strong assumptions on convergence across the countries. In the literature on optimal currency areas and monetary aggregation over countries within a monetary union, multiple risk-free benchmark assets can be necessary. Even within a single country that is subject to regional regulations and taxation, Barnett's (2003) convergence assumptions for the existence of a unique risk-free benchmark rate may not hold. Under those circumstances, Barnett (2003) has shown that the theory, under perfect certainty, requires multiple benchmark rates of return and the imputation of an additional regional or country subscript to all monetary asset quantities and rates of return, to differentiate assets and rates of return by region or country. Second, the need to compute a unique risk-free benchmark rate of return, when theoretically relevant, presents difficult empirical and measurement problems. We rarely observe the theoretical risk-free asset (in real terms) in financial markets, and hence the risk-free benchmark rate of return is inherently an unobserved variable that, at best, has been proxied. But we show that the relationship between the user cost of a monetary asset and a risky "benchmark" asset's rate of return holds for an arbitrary pair of a monetary and a risky non-monetary asset. Because of this fundamental extension, our results on risk adjustment do not depend upon the existence of a unique risk free rate or the ability to measure such a rate. This important extension remains relevant, even if all monetary asset rates of return are subject to low risk, since relevant candidates for the benchmark asset may all have risky rates of return.

In the asset pricing literature, one of the most popular models is the Capital Asset Pricing Model (CAPM) developed by Sharpe (1964) and Lintner (1965), among others. In CAPM, the risk premium on an individual asset is determined by the covariance of its excess rate of return with that of the market portfolio, or equivalently its *beta*. The advantage of CAPM is that it circumvents the issue of unobservable marginal utility, and the *beta* can easily be estimated. We show in this paper that there also exists a similar *beta* that relates the user cost of an individual monetary asset to the user cost of the consumer's wealth portfolio.

The rest of the paper is organized as follows. In the next section, we specify the representative consumer's intertemporal optimization problem. The main results are contained in Section 3, while Section 4 concludes.

2. Consumer's optimization problem

We use a setup similar to that in Barnett, Liu, and Jensen (1997). We assume that the representative consumer has an intertemporally non-separable general utility function, $U(\boldsymbol{m}_t, c_t, c_{t-1}, \cdots, c_{t-n})$, defined over current and past consumption and a vector of L current-period monetary assets, $\boldsymbol{m}_t = (m_{1,t}, m_{2,t}, \cdots, m_{L,t})'$. The consumer's holdings of non-monetary assets, $\boldsymbol{k}_t = (k_{1,t}, k_{2,t}, ..., k_{K,t})'$, do not enter the utility function, since those assets are assumed to produce no services other than their investment rate of return. To ensure the existence of a monetary aggregate, we further assume that there exists a linearly homogenous aggregator function, $M(\cdot)$, such that U can be written in the form

$$U(\boldsymbol{m}_t, c_t, c_{t-1}, \cdots, c_{t-n}) = V(M(\boldsymbol{m}_t), c_t, c_{t-1}, \cdots, c_{t-n})\,. \tag{2.1}$$

Given initial wealth, W_t, the consumer seeks to maximize her expected lifetime utility function,

$$E_t \sum_{s=0}^{\infty} \beta^s U(\boldsymbol{m}_{t+s}, c_{t+s}, c_{t+s-1}, \cdots, c_{t+s-n})\,, \tag{2.2}$$

subject to the following budget constraints,

$$\begin{aligned} W_t &= p_t^* c_t + \sum_{i=1}^{L} p_t^* m_{i,t} + \sum_{j=1}^{K} p_t^* k_{j,t} \\ &= p_t^* c_t + p_t^* A_t \end{aligned} \tag{2.3}$$

and

$$W_{t+1} = \sum_{i=1}^{L} R_{i,t+1} p_t^* m_{i,t} + \sum_{j=1}^{K} \tilde{R}_{j,t+1} p_t^* k_{j,t} + Y_{t+1}\,, \tag{2.4}$$

where $\beta \in (0,1)$ is the consumer's subjective discount factor, p_t^* is the true cost-of-living index, and $A_t = \sum_{i=1}^{L} m_{i,t} + \sum_{j=1}^{K} k_{j,t}$ is the real value of the asset portfolio. Non-monetary asset j is defined to provide no services other than its gross rate of return, $\tilde{R}_{j,t+1}$, between periods t and $t+1$. Monetary asset i, having quantity $m_{i,t}$, has a gross rate of return $R_{i,t+1}$ between periods t and $t+1$, and does provide monetary services. At the beginning of each period, the consumer allocates her wealth among consumption, c_t, investment in the monetary assets, $\boldsymbol{m}_t$, and investment in the non-monetary assets, $\boldsymbol{k}_t$. The consumer's income from any other sources, received at the beginning of period $t+1$, is Y_{t+1}. If any of that income's source is labor income, then we assume that leisure is weakly separable from consumption of goods and holding of monetary assets, so that the function U exists and the decision exists, as a conditional decision problem. The consumer also is subject to the transversality condition

$$\lim_{s\to\infty} \beta^s p_t^* A_{t+s} = 0 . \tag{2.5}$$

The equivalency of the two constraints (2.3) and (2.4) with the single constraint used in Barnett (1980) and Barnett, Liu, and Jensen (1997) is easily seen by substituting equation (2.3) for period t+1 into equation (2.4) and solving for $p_t^* c_t$.

Define the value function of the consumer's optimization problem to be $H_t = H(W_t, c_{t-1}, \cdots, c_{t-n})$. Assuming the solution to the decision problem exists, we then have the Bellman equation

$$H_t = \underset{(c_t, \boldsymbol{m}_t, \boldsymbol{k}_t)}{Sup} E_t\{U(\boldsymbol{m}_t, c_t, c_{t-1}, \cdots, c_{t-n}) + \beta H_{t+1}\} , \tag{2.6}$$

where the maximization is subject to the budget constraints, (2.3) and (2.4).

The first order conditions can be obtained as

$$\lambda_t = \beta E_t \left[\lambda_{t+1} \tilde{R}_{j,t+1}\, p_t^* / p_{t+1}^* \right] \tag{2.7}$$

and

$$\partial U_t / \partial m_{i,t} = \lambda_t - \beta E_t \left[\lambda_{t+1} R_{i,t+1}\, p_t^* / p_{t+1}^* \right], \tag{2.8}$$

where $U_t = U(\boldsymbol{m}_t, c_t, c_{t-1,} \cdots, c_{t-n})$ and $\lambda_t = E_t(\partial U_t/\partial c_t + \beta\, \partial U_{t+1}/\partial c_t + \cdots + \beta^n\, \partial U_{t+n}/\partial c_t)$. Note that λ_t is the expected present value of the marginal utility of consumption, c_t. In the standard case in which the instantaneous utility function is time-separable, λ_t reduces to $\partial U(\boldsymbol{m}_t, c_t)/\partial c_t$.

3. Risk-adjusted user cost of monetary assets

3.1. The theory

As in Barnett, Liu, and Jensen (1997), we define the contemporaneous real user-cost price of the services of monetary asset i to be the ratio of the marginal utility of the monetary asset and the marginal utility of consumption, so that

$$\pi_{i,t} = \frac{\frac{\partial U_t}{\partial m_{i,t}}}{E_t\left(\frac{\partial U_t}{\partial c_t} + \beta\frac{\partial U_{t+1}}{\partial c_t} + \cdots + \beta^n\frac{\partial U_{t+n}}{\partial c_t}\right)} = \frac{\frac{\partial U_t}{\partial m_{i,t}}}{\lambda_t}. \tag{3.1}$$

We denote the vector of L monetary asset user costs by $\boldsymbol{\pi}_t = (\pi_{1,t}, \pi_{2,t}, \cdots, \pi_{L,t})'$. With the user costs defined above, we can show that the solution value of the exact monetary aggregate, $M(\boldsymbol{m}_t)$, can be tracked accurately in continuous time by the generalized Divisia index, as proved in the perfect certainty special case by Barnett (1980).

Proposition 1. *Let* $s_{i,t} = \frac{\pi_{i,t}m_{i,t}}{\sum_{l=1}^{L}\pi_{l,t}m_{l,t}}$ *be the user-cost-evaluated expenditure share. Under the weak-separability assumption, (2.1), we have for any linearly homogenous monetary aggregator function,* $M(\cdot)$*, that*

$$d\log M_t = \sum_{i=1}^{L} s_{i,t}\, d\log m_{i,t}\,, \tag{3.2}$$

where $M_t = M(\boldsymbol{m}_t)$.

Proof. Under the assumption of weak-separability, we have

$$\frac{\partial U_t}{\partial m_{i,t}} = \frac{\partial V_t}{\partial M_t} \frac{\partial M_t}{\partial m_{i,t}}, \tag{3.3}$$

where $V_t = V(M(\boldsymbol{m}_t), c_t, c_{t-1}, \cdots, c_{t-n})$. By the definition in (3.1) and equation (3.3), it then follows that

$$\frac{\partial M_t}{\partial m_{i,t}} = \pi_{i,t} \left(\lambda_t \Big/ \frac{\partial V_t}{\partial M_t} \right). \tag{3.4}$$

Taking the total differential of $M_t = M(\boldsymbol{m}_t)$ and using the result in (3.4), we obtain

$$dM_t = \left(\lambda_t \Big/ \frac{\partial V_t}{\partial M_t} \right) \sum_{i=1}^{L} \pi_{i,t} dm_{i,t} = \left(\lambda_t \Big/ \frac{\partial V_t}{\partial M_t} \right) \sum_{i=1}^{L} \pi_{i,t} m_{i,t} d \log m_{i,t} . \tag{3.5}$$

On the other hand, because of the linear homogeneity of $M_t = M(\boldsymbol{m}_t)$, it follows from (3.4) that

$$M_t = \sum_{i=1}^{L} \frac{\partial M_t}{\partial m_{i,t}} m_{i,t} = \left(\lambda_t \Big/ \frac{\partial V_t}{\partial M_t} \right) \sum_{i=1}^{L} \pi_{i,t} m_{i,t} . \tag{3.6}$$

The proposition therefore follows by dividing (3.5) by (3.6). +

The exact price aggregate dual, $\Pi_\tau = \Pi(\boldsymbol{\pi}_t)$, to the monetary quantity aggregator function, $M_t = M(\boldsymbol{m}_t)$, is easily computed from factor reversal, $\Pi(\boldsymbol{\pi}_t) M(\boldsymbol{m}_t) = \sum_{i=1}^{L} \pi_{it} m_{it}$, so that

$$\Pi(\boldsymbol{\pi}_t) = \frac{\sum_{i=1}^{L} \pi_{it} m_{it}}{M(\boldsymbol{m}_t)} .$$

In continuous time, the user cost price dual can be tracked without error by the Divisia user cost price index

$$d \log \Pi_t = \sum_{i=1}^{L} s_{i,t} d \log \pi_{i,t} \ .$$

To get a more convenient expression for the user cost, $\pi_{i,t}$, we define the pricing kernel to be

$$Q_{t+1} = \beta \, \lambda_{t+1} / \lambda_t \ . \tag{3.7}$$

Recall that λ_t is the present value of the marginal utility of consumption at time t. Hence Q_{t+1} measures the marginal utility growth from t to $t+1$. For example, if the utility function (2.1) is time-separable, we have that $Q_{t+1} = \beta \frac{\partial U(\boldsymbol{m}_{t+1}, c_{t+1}) / \partial c_{t+1}}{\partial U(\boldsymbol{m}_t, c_t) / \partial c_t}$, which is the subjectively discounted marginal rate of substitution between consumption this period and consumption next period. Clearly Q_{t+1} is positive, as required of marginal rates of substitution.

While the introduction of a pricing kernel is well understood in finance, the intent also should be no surprise to experts in index number theory either. The way in which statistical index numbers, depending upon prices and quantities, track quantity aggregator functions, containing no prices, is through the substitution of first order conditions to replace marginal utilities by functions of relevant prices. This substitution is particularly well known in the famous derivation of the Divisia index by Francois Divisia (1925). In our case, the intent, as applied below, is to replace Q_{t+1} by a function of relevant determinants of its value in asset market equilibriuim. If we use the approximation that characterizes CAPM, then the pricing kernel, Q_{t+1}, would be a linear function of the rate of return on the consumer's asset portfolio, A_t.

Using (3.1) and (3.7), the first order conditions (2.7) and (2.8) can alternatively be written as

$$0 = 1 - E_t(Q_{t+1}\tilde{r}_{j,t+1}), \tag{3.8}$$

$$\pi_{i,t} = 1 - E_t(Q_{t+1}r_{i,t+1}), \tag{3.9}$$

where $\tilde{r}_{j,t+1} = \tilde{R}_{j,t+1}\, p_t^* / p_{t+1}^*$ is the real gross rate of return on non-monetary asset, $k_{j,t}$, and $r_{i,t+1} = R_{i,t+1}\, p_t^* / p_{t+t}^*$ is the real gross rate of return on monetary asset, $m_{i,t}$, which provides the consumer with liquidity service.

Equations (3.8) and (3.9) impose restrictions on asset returns. Equation (3.8) applies to the returns on all risky non-monetary assets, $k_{j,t}$ ($j = 1,\cdots K$), in the usual manner. For monetary assets, equation (3.9) implies that the "deviation" from the usual Euler equation measures the user cost of that monetary asset. To obtain good measures of the user costs of monetary assets, the non-monetary asset pricing within the asset portfolio's pricing kernel, Q_{t+1}, should be as accurate as possible. Otherwise, we would attribute any non-monetary asset pricing errors to the monetary asset user costs in (3.9), as pointed out by Marshall (1997). From the above Euler equations, we can obtain the following proposition.

Proposition 2. *Given the real rate of return,* $r_{i,t+1}$*, on a monetary asset and the real rate of return,* $\tilde{r}_{j,t+1}$*, on an arbitrary non-monetary asset, the risk-adjusted real user-cost price of the services of the monetary asset can be obtained as*

$$\pi_{i,t} = \frac{(1+\omega_{i,t})E_t\tilde{r}_{j,t+1} - (1+\omega_{j,t})E_t r_{i,t+1}}{E_t\tilde{r}_{j,t+1}}, \tag{3.10}$$

where

$$\omega_{i,t} = -Cov_t(Q_{t+1}, r_{i,t+1}) \tag{3.11}$$

and

$$\omega_{j,t} = -Cov_t(Q_{t+1}, \tilde{r}_{j,t+1}). \tag{3.12}$$

Proof. From the Euler equation (3.8), we have

$$1 = E_t Q_{t+1} E_t \tilde{r}_{j,t+1} + Cov_t(Q_{t+1}, \tilde{r}_{j,t+1}), \tag{3.13}$$

and similarly from the Euler equation (3.9), we have

$$\pi_{i,t} = 1 - E_t Q_{t+1} E_t r_{i,t+1} - Cov_t(Q_{t+1}, r_{i,t+1}). \tag{3.14}$$

Note that (3.13) implies that

$$E_t Q_{t+1} = \frac{1 - Cov_t(Q_{t+1}, \tilde{r}_{j,t+1})}{E_t \tilde{r}_{j,t+1}}. \tag{3.15}$$

Hence Proposition 2 follows by substituting $E_t Q_{t+1}$ into (3.14). +

Proposition 2 relates the user cost of a monetary asset to the rates of return on financial assets, which need not be risk free. It applies to an arbitrary pair of monetary and non-monetary assets. In fact, observe that the left hand side of equation (3.10) has no j subscript, even though there are j subscripts on the right hand side. The left hand side of (3.10) is invariant to the choice of non-monetary asset used on the right hand side, as a result of equation (3.15), which holds for all j, regardless of i. This result suggests that we can choose an arbitrary non-monetary financial asset as the "benchmark" asset in calculating the user costs of the financial assets that provide monetary services, so long as we correctly compute the covariance, $\omega_{j,t}$, of the return on the non-monetary asset with the pricing kernel.

Practical considerations in estimating that covariance could tend to discourage use of multiple benchmark assets. In particular, a biased estimate of $\omega_{j,t}$ for a non-monetary asset j would similarly bias the user costs of all monetary assets, if that non-monetary asset were used as the benchmark asset in computing all monetary asset user costs.

Alternatively, if a different non-monetary financial assets were used as the benchmark assets for each monetary asset, errors in measuring $\omega_{j,t}$ for different j's could bias relative user costs of monetary assets.

Nevertheless, in theory it is not necessary to use the same benchmark asset j for the computation of the user cost of each monetary asset i. In particular, we have the following corollary.

Corollary 1. *Under uncertainty we can choose any non-monetary asset as the "benchmark" asset, when computing the risk-adjusted user-cost prices of the services of monetary assets.*

Notice that Proposition 2 doesn't require existence of a risk-free non-monetary asset (in real-terms). Since we rarely observe the rate of return on such an entirely illiquid asset in financial markets, our proposition generalizes the main result in Barnett, Liu, and Jensen (1997) in a very useful manner. Although a unique risk-free totally illiquid investment, having no secondary market, may exist in theory, Corollary 1 frees us from the need to seek a proxy for its inherently unobservable rate of return.

If we were to impose the further assumption of perfect certainty on top of the other assumptions we have made above, there would be only one benchmark asset and all of our results would reduce, as a special case, to those of Barnett (1978,1980). Without risk and with no monetary services provided by benchmark assets, arbitrage would assure that there could be only one benchmark asset.

To see the intuition of Proposition 2, assume that one of the non-monetary assets is risk-free with *gross* real interest rate of r_t^f at time t. Further, as proven by Barnett (1978), the certainty-equivalent user cost, $\pi_{i,t}^e$, of a monetary asset $m_{i,t}$ is

$$\pi_{i,t}^e = \frac{r_t^f - E_t r_{i,t+1}}{r_t^f}. \tag{3.16}$$

From equation (3.8), the first order condition for r_t^f is

$$1 = E_t(Q_{t+1} r_t^f) . \qquad (3.17)$$

Hence we have, from the nonrandomness of r_t^f , that

$$E_t Q_{t+1} = \frac{1}{r_t^f} . \qquad (3.18)$$

Replacing $E_t Q_{t+1}$ in (3.9) with $\frac{1}{r_t^f}$, we then have

$$\pi_{i,t} = \frac{r_t^f - E_t r_{i,t+1}}{r_t^f} + \omega_{i,t} = \pi_{i,t}^e + \omega_{i,t} , \qquad (3.19)$$

where $\omega_{i,t} = -Cov_t(Q_{t+1}, r_{i,t+1})$. Therefore, $\pi_{i,t}$ could be larger or smaller than the certainty-equivalent user cost, $\pi_{i,t}^e$, depending on the sign of the covariance between $r_{i,t+1}$ and Q_{t+1}. When the return on a monetary asset is positively correlated with the pricing kernel, Q_{t+1}, and thereby negatively correlated with the rate of return on the full portfolio of monetary and non-monetary assets, the monetary asset's user cost will be adjusted downwards from the certainty-equivalent user cost. Such assets offer a hedge against aggregate risk by paying off when the full asset portfolio's rate of return is low. In contrast, when the return on a monetary asset is negatively correlated with the pricing kernel, Q_{t+1}, and thereby positively correlated with the rate of return on the full asset portfolio, the asset's user cost will be adjusted upwards from the certainty-equivalent user cost, since such assets tend to pay off when the asset porfolio's rate of return is high. Such assets are very risky.

To calculate the risk adjustment, we need to compute the covariance between the asset return $r_{i,t+1}$ and the pricing kernel Q_{t+1}, which is unobservable. Consumption-based asset pricing models allow us to relate Q_{t+1} to consumption growth through a specific utility function. But most utility functions have been shown not be able to reconcile the aggregate consumption data with the observed stock returns and the interest rate. Kocherlakota (1996) provides an excellent survey on the equity premium puzzle literature. In fact the empirical results from Barnett, Liu, and Jensen (1997) show that the consumption risk adjustments for the user costs of monetary assets are small in many cases under the standard utility function with moderate risk aversion.

With more general utility functions than used in previous empirical studies on monetary aggregation, we can use the theory in this paper to extend the existing empirical studies on the user costs of risky monetary assets, and thereby on the induced risk-adjusted Divisia monetary quantity and user cost aggregates. Campbell and Cochrane (1999), among others, have shown that a habit-formation-based utility function with reasonable risk-aversion coefficient could produce large time-varying risk premia, similar in magnitude to those observed in the data. The risk-adjustment for the user costs of monetary assets is intimately related to the determination of risk premia. As a result, there is reason to believe that intertemporally non-separable utility functions, such as the one in Campbell and Cochrane (1999), can produce larger risk adjustments to the certainty-equivalent user costs than the small adjustments found in Barnett, Liu, and Jensen (1997). The results could be particularly dramatic in open economy applications in which rates of return are subject to exchange rate risk. We are currently conducting an empirical study to implement the theoretical model proposed in this paper.

There is a particularly important reason to use more general and flexible utility functions in computing the user costs of risky monetary assets. As discussed above, to obtain good measures of the user costs of monetary assets, we need to choose the pricing kernel, and hence the utility function, such that the pricing of non-monetary assets within the kernel is as accurate as possible. Otherwise, the user costs of monetary assets would be contaminated by the pricing errors. Since standard utility functions are known to lead to erroneous estimates of non-monetary-asset risk premia, the risk adjustments to the users cost of monetary assets with those utility functions are likely to be much less accurate than those with utility functions that have better empirical performance in matching the observed non-monetary-asset risk premia.

3.2. Approximation to the theory

All of the consumption-based asset pricing models require us to make explicit assumptions about investors' utility functions. An alternative approach, which is commonly practiced in finance, is to approximate Q_{t+1} by some simple function of observable macroeconomic factors that are believed to be closely related to investors' marginal utility growth. For example, the well-known CAPM [Sharpe (1964) and Lintner (1965)] approximates Q_{t+1} by a linear function of the rate of return on the market portfolio. Then the rate of return on any individual asset is linked to its covariance with the market rate of return. Fama and French (1992) include two additional factors, firm size and book-to-market value, and show that the three-factor model is able to capture the cross-sectional variation in average stock returns. Using stock returns, Chen, Roll, and Ross (1986) and Lamont (2000) try to identify macroeconomic variables as priced risk factors. Cochrane (2000) provides detailed discussion on the approximation of the pricing kernel Q_{t+1}.

We show that there also exists a similar CAPM-type relationship among user costs of risky monetary assets, under the assumption that Q_{t+1} is a linear function of the rate of return on a well-diversified wealth portfolio. We believe that this simple specification of the pricing kernel, based upon a long standing tradition in finance, is a reasonable first step in the extension of Barnett, Liu, and Jensen (1997) to the case of intertemporal nonseparability. In the finance literature, the CAPM specification of the pricing kernel results from a special case of a linear factor-model decomposition of the first order conditions, (3.8) and (3.9), under the assumption of quadratic utility or Gaussianity. Deeper specifications of the pricing kernel, as being proposed now in finance, might prove similarly advantageous in future extensions of our research.

Specifically, define $r_{A,t+1}$ to be the share-weighted rate of return on the consumer's asset portfolio, including both the monetary assets, $m_{i,t}$ $(i = 1, \cdots, L)$, and the non-monetary assets, $k_{j,t}$ ($j = 1, \cdots, K$). Then the traditional CAPM approximation to Q_{t+1} mentioned above is of the form $Q_{t+1} = a_t - b_t r_{A,t+1}$, where a_t, and b_t can be time dependent.

Let $\phi_{i,t}$ and $\varphi_{j,t}$ denote the share of $m_{i,t}$ and $k_{j,t}$, respectively, in the portfolio's stock value, so that

$$\phi_{i,t} = \frac{m_{i,t}}{\sum_{l=1}^{L} m_{l,t} + \sum_{j=1}^{K} k_{j,t}} = \frac{m_{i,t}}{A_t}$$

and

$$\varphi_{j,t} = \frac{k_{j,t}}{\sum_{l=1}^{L} m_{l,t} + \sum_{i=1}^{K} k_{i,t}} = \frac{k_{j,t}}{A_t}.$$

Then, by construction, $r_{A,t+1} = \sum_{i=1}^{L} \phi_{i,t} r_{i,t+1} + \sum_{j=1}^{K} \varphi_{j,t} \tilde{r}_{j,t+1}$, where

$$\sum_{i=1}^{L} \phi_{i,t} + \sum_{j=1}^{K} \varphi_{j,t} = 1. \tag{3.20}$$

Multiplying (3.8) by $\varphi_{j,t}$ and (3.9) by $\phi_{i,t}$, we have

$$0 = \varphi_{j,t} - E_t(Q_{t+1} \varphi_{j,t} \tilde{r}_{j,t+1}) \tag{3.21}$$

and

$$\phi_{i,t} \pi_{i,t} = \phi_{i,t} - E_t(Q_{t+1} \phi_{i,t} r_{i,t+1}). \tag{3.22}$$

Summing (3.21) over j and (3.22) over i, adding the two summed equations together, and using the definition of $r_{A,t+1}$, we get

$$\sum_{i=1}^{L} \phi_{i,t} \pi_{i,t} = 1 - E_t(Q_{t+1} r_{A,t+1}) . \tag{3.23}$$

Let $\Pi_{A,t} = \sum_{i=1}^{L} \phi_{i,t} \pi_{i,t} + \sum_{j=1}^{K} \varphi_{i,t} \tilde{\pi}_{j,t}$, where $\tilde{\pi}_{j,t}$ is the user cost of non-monetary asset j. We define $\Pi_{A,t}$ to be the user cost of the consumer's asset wealth portfolio. But the user cost, $\tilde{\pi}_{j,t}$, of every non-monetary asset is simply 0, as shown in (3.8), so equivalently $\Pi_{A,t} = \sum_{i=1}^{L} \phi_{i,t} \pi_{i,t}$. The reason is that consumers do not pay a price, in terms of foregone interest, for the monetary services of non-monetary assets, since they provide no monetary services and provide only their investment rate of return. We can show that our definition of $\Pi_{A,t}$ is consistent with Fisher's factor reversal test, as follows.

Result: *The pair (A_t , $\Pi_{A,t}$) satisfies factor reversal, defined by:*

$$\Pi_{A,t} A_t = \sum_{i=1}^{L} \pi_{i,t} m_{i,t} + \sum_{j=1}^{K} \tilde{\pi}_{j,t} k_{j,t} .$$

Since we know that $\tilde{\pi}_{j,t} = 0$ for all j, factor reversal equivalently can be written as

$$\Pi_{A,t} A_t = \sum_{i=1}^{L} \pi_{i,t} m_{i,t} .$$

The proof of the result is straightforward.

Proof: By the definition of $\phi_{i,t}$, we have $m_{i,t}$ = $\phi_{i,t} A_t$. Hence, we have

$$\sum_{i=1}^{L} \pi_{i,t} m_{i,t} = \sum_{i=1}^{L} \pi_{i,t} \phi_{i,t} A_t$$

$$= A_t \sum_{i=1}^{L} \pi_{i,t} \phi_{i,t} = A_t \Pi_{A,t} ,$$

which is our result. +

Observe that the wealth portfolio is different from the monetary services aggregate, $M(\boldsymbol{m}_t)$. The portfolio weights in the asset wealth stock are the market-value-based shares, while the growth rate weights in the monetary services flow aggregate are the user-cost-evaluated shares.

Suppose one of the non-monetary assets is (locally) risk-free with gross real interest rate r_t^f. By substituting equation (3.16) for $\pi_{i,t}$ into the definition of $\Pi_{A,t}$, using the definition of $r_{A,t+1}$, and letting $r_t^f = \mathrm{E}\,\tilde{r}_{j,t+1}$ for all j, it follows that the certainty equivalent user cost of the asset wealth portfolio is $\Pi_{A,t}^e = \frac{r_t^f - E_t r_{A,t+1}}{r_t^f}$. We now can prove the following proposition.

Proposition 3. *If one of the non-monetary assets is (locally) risk-free with gross real interest rate* r_t^f*, and if* $Q_{t+1} = a_t - b_t r_{A,t+1}$*, where* $r_{A,t+1}$ *is the gross real rate of return on the consumer's wealth portfolio, then the user cost of any monetary asset* i *is given by*

$$\pi_{i,t} - \pi_{i,t}^e = \beta_{i,t}(\Pi_{A,t} - \Pi_{A,t}^e), \tag{3.24}$$

where $\pi_{i,t}$ *and* $\Pi_{A,t}$ *are the user costs of asset* i *and of the asset wealth portfolio, respectively, and* $\pi_{i,t}^e = \frac{r_t^f - E_t r_{i,t+1}}{r_t^f}$ *and* $\Pi_{A,t}^e = \frac{r_t^f - E_t r_{A,t+1}}{r_t^f}$ *are the certainty-equivalent user costs of asset* i *and the asset wealth portfolio, respectively. The "beta" of asset* i *in equation (3.24) is given by*

$$\beta_{i,t} = \frac{Cov_t(r_{A,t+1}, r_{i,t+1})}{Var_t(r_{A,t+1})}. \tag{3.25}$$

Proof. From (3.23) and the definition of $\Pi_{A,t}$, we have for the wealth portfolio that

$$\Pi_{A,t} = 1 - E_t Q_{t+1} E_t r_{A,t+1} - Cov_t(Q_{t+1}, r_{A,t+1}). \tag{3.26}$$

Given the risk-free rate r_t^f, we have that $E_t Q_{t+1} = \frac{1}{r_t^f}$ from equation (3.18). Hence

$$\Pi_{A,t} = 1 - \frac{E_t r_{A,t+1}}{r_t^f} - Cov_t(Q_{t+1,} r_{A,t+1}) = \Pi_{A,t}^e - Cov_t(Q_{t+1,} r_{A,t+1}) . \tag{3.27}$$

Using the assumption that $Q_{t+1} = a_t - b_t r_{A,t+1}$, so that $Cov_t(Q_{t+1}, r_{A,t+1}) = -b_t Var_t(r_{A,t+1})$, it follows that

$$\Pi_{A,t} = \Pi_{A,t}^e + b_t Var_t(r_{A,t+1}) . \tag{3.28}$$

On the other hand, for any asset i we have from (3.19) and $Q_{t+1} = a_t - b_t r_{A,t+1}$ that

$$\pi_{i,t} = \pi_{i,t}^e - Cov_t(Q_{t+1}, r_{i,t+1}) = \pi_{i,t}^e + b_t Cov_t(r_{A,t+1}, r_{i,t+1}) . \tag{3.29}$$

Hence, from equations (3.28) and (3.29), we can conclude that

$$\frac{\pi_{i,t} - \pi_{i,t}^e}{\Pi_{A,t} - \Pi_{A,t}^e} = \frac{Cov_t(r_{A,t+1}, r_{i,t+1})}{Var_t(r_{A,t+1})} , \tag{3.30}$$

and the proposition follows. +

In the approximation, $Q_{t+1} = a_t - b_t r_{A,t+1}$, to the theoretical pricing kernel, Q_{t+1}, the reason for the minus sign is similar to the reason for the minus signs before the own rates of return within monetary asset user costs: the intent in the finance literature is to measure a "price," not a rate of return. In particular, with the minus sign in front of b_t and with b_t positive, we can interpret b_t in equations (3.28) and (3.29) as a "price" of risk. This interpretation can be seen from the fact that b_t then measures the amount of risk premium added to the left hand side per unit of covariance (in (3.29)) or variance (in (3.28)). Also recall that the pricing kernel itself, as a subjectively-discounted marginal rate of substitution, should be positive. Hence the signs of a_t and b_t must both be positive, and a_t must be sufficiently large so that the pricing kernel is positive for all observed values of $r_{A,t+1}$.

Proposition 3 is very similar to the standard CAPM formula for asset returns. In CAPM theory, the expected excess rate of return, $E_t r_{i,t+1} - r_t^f$, on an individual asset is determined by its covariance with the excess rate of return on the market portfolio, $E_t r_{M,t+1} - r_t^f$, in accordance with

$$E_t r_{i,t+1} - r_t^f = \beta_{i,t}(E_t r_{M,t+1} - r_t^f), \qquad (3.31)$$

where $\beta_{i,t} = \frac{Cov_t(r_{i,t+1} - r_t^f, r_{M,t+1} - r_t^f)}{Var_t(r_{M,t+1} - r_t^f)}$.

This result implies that asset *i*'s risk premium depends on its market portfolio risk exposure, which is measured by the *beta* of this asset. Our proposition shows that the risk adjustment to the certainty equivalent user cost of asset i is determined in that manner as well. The larger the *beta*, through risk exposure to the wealth portfolio, the larger the risk adjustment. User costs will be adjusted upwards for those monetary assets whose rates of return are positively correlated with the return on the wealth portfolio, and conversely for those monetary assets whose returns are negatively correlated with the wealth portfolio. In particular, if we find that $\beta_{i,t}$ is very small for all the monetary assets under consideration, then the risk adjustment to the user cost is also very small. In that case, the unadjusted Divisia monetary index would be a good proxy for the extended index in Barnett, Liu, and Jensen (1997).

Notice that Proposition 3 is a conditional version of CAPM with time-varying risk premia. Lettau and Ludvigson (2001) have shown that a conditional version of CAPM performs much better empirically in explaining the cross section of asset returns, than the unconditional CAPM. In fact, the unconditional CAPM is usually rejected in empirical tests. See, e.g., Breeden, Gibbons, and Litzenberger (1989) and Campbell (1996).

4. Concluding remarks

Simple sum monetary aggregates treat monetary assets with different rates of return as perfect substitutes. Barnett (1978, 1980) showed that the Divisia index, with user cost prices, is a more appropriate measure for monetary services, and derived the formula for the user cost of monetary asset services in the absence of uncertainty. Barnett, Liu, and Jensen (1997) extended the Divisia monetary quantity index to the case of uncertain returns and risk aversion. For risky monetary assets, however, the magnitude of the risk adjustment to the certainty equivalent user cost is unclear. Using a standard time-separable power utility function, Barnett, Liu, and Jensen (1997) showed that the difference between the unadjusted Divisia index and the index extended for risk is usually small. However, this result could be a consequence of the same problem that causes the equity premium puzzle in the asset pricing literature. The consumption-based asset pricing model with more general utility functions, most notably those that are intertemporally non-separable, can reproduce the large and time-varying risk premium observed in the data. We believe that similarly extended asset pricing models will provide larger and *more accurate* CCAPM adjustment to the user costs of monetary assets than those found in Barnett, Liu, and Jensen (1997). The current paper extends the basic result in Barnett, Liu, and Jensen (1997) in that manner.

How big the risk adjustment should be is an empirical issue. We show in this paper that for any individual monetary asset, the risk adjustment to its certainty equivalent user cost can be measured by its *beta*, which depends on the covariance between the rate of return on the monetary asset and on the wealth portfolio of the consumer. This result is analogous to the standard Capital Asset Pricing Model (CAPM). In practice, if the *beta* is found to be very small, then the certainty equivalent user cost would be a good approximation to the true user cost price of the monetary asset services under uncertainty. In that special case, the unadjusted Divisia index could still be used for monetary aggregation. We are currently conducting research on the empirical implications of the models proposed in this paper. Relevant modeling and inference methodology are available in Barnett and Binner (2004).

Another extension of the current paper could be to introduce heterogeneous investors, as considered for the case of perfect certainty by Barnett (2003). This extension can be of particular importance in multicountry or multiregional applications, in which regional or country-specific heterogeneity cannot be ignored. In such cases regional or national subscripts must be introduced to differentiate goods and assets by location. With the possibility of regulatory and taxation differences across the heterogeneous groups, arbitrage cannot be assumed to remove the possibility of multiple benchmark assets, even under perfect certainty, except under special assumptions on institutional convergence. See Barnett (2004) regarding those assumptions.

References

[1] Barnett, W. A. (1978), "The user cost of money," *Economic Letters* **1**, 145-149. Reprinted in W. A. Barnett and A. Serletis (eds.) (2000), *The Theory of Monetary Aggregation*, North-Holland, Amsterdam, Chapter 1, 6-10.

Barnett, W. A. (1980), "Economic monetary aggregates: An application of index number and aggregation theory", *Journal of Econometrics* **14**, 11-48. Reprinted in W. A. Barnett and A. Serletis (eds.) (2000), *The Theory of Monetary Aggregation*, North-Holland, Amsterdam, Chapter 2, 11-48.

Barnett, W. A. (1995), "Exact aggregation under risk," in W. A. Barnett, M. Salles, H. Moulin, and N. Schofield (eds), *Social Choice, Welfare and Ethics*, Cambridge University Press, Cambridge, 353-374. Reprinted in W. A. Barnett and A. Serletis (eds.) (2000), *The Theory of Monetary Aggregation*, North-Holland, Amsterdam, Chapter 10, 195-206.

Barnett, W. A. (1997), "The microeconomic theory of monetary aggregation," in W. A. Barnett (eds), *New Approaches to Monetary Economics*, Cambridge University Press, Cambridge, 115-168. Reprinted in W. A. Barnett and A. Serletis (eds.) (2000), *The Theory of Monetary Aggregation*, North-Holland, Amsterdam, Chapter 3, 49-99.

Barnett, W. A. (2003), "Aggregation-theoretic monetary aggregation over the Euro area when countries are heterogeneous," *ECB Working Paper No. 260*, European Central Bank, Frankfurt.

Barnett, W. A. and J. Binner (eds.) (2004), *Functional Structure and Approximation in Econometrics*, North-Holland, Amsterdam.

Barnett, W. A., M. Hinich, and P. Yue (2000), "The exact theoretical rational expectations monetary aggregate," *Macroeconomic Dynamics* **4**, 197-221.

Barnett, W. A. and Y. Liu (2000), “Beyond the risk neutral utility function,” in M. T. Belongia and J. E. Binner (eds.), *Divisia Monetary Aggregates: Theory and Practice*, London, Palgrave, 11-27.

Barnett, W. A., Y. Liu and M. Jensen (1997), “CAPM risk adjustment for exact aggregation over financial assets,” *Macroeconomic Dynamics* **1**, 485-512. Reprinted in W. A. Barnett and A. Serletis (eds) (2000), *The Theory of Monetary Aggregation*, North-Holland, Amsterdam, Chapter 12, 245-273.

Barnett, W. A. and A. Serletis (eds) (2000), *The Theory of Monetary Aggregation*, North-Holland, Amsterdam, Section 3.3, 195-295.

Breeden, D., M. Gibbons and R. Litzenberger (1989), “Empirical tests of the consumption CAPM,” *Journal of Finance* **44**, 231-262.

Campbell, J. Y. (1996), “Understanding risk and return,” *Journal of Political Economy* **104**, 298-345.

Campbell, J. Y. and J. H. Cochrane (1999), “By force of habit: A consumption-based explanation of aggregate stock market behavior,” *Journal of Political Economy* **107**, 205-251.

Chen, N. F., R. Roll and S. Ross (1986), “Economic forces and the stock market,” *Journal of Business* **29**, 383-403.

Cochrane, J. H. (2000), *Asset Pricing*, Princeton Press, Princeton, NJ.

Divisia, Francois (1925), “L’Indice Monétaire et la Théorie de la Monnaie,” *Revue d’Economie Politique* **39**, 980-1008.

Fama, E. F. and K. French (1992), “The cross-section of expected stock returns,” *Journal of Finance* **47**, 427-465.

Kocherlakota, N. (1996), “The equity premium: It's still a puzzle,” *Journal of Economic Literature* **34**, 43-71.

Lamont, O. (2000), “Economic tracking portfolio”, *Journal of Econometrics* **105**, 161-184.

Lettau, M. and S. Ludvigson (2001) “Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying,” *Journal of Political Economy* **109**, 1238-1287

Lintner, J. (1965), “The valuation of risky assets and the selection of risky investments in stock portfolios and capital budgets,” *Review of Economics and Statistics* **47**, 13 - 37.

Marshall, D. (1997), “Comments on CAPM risk adjustment for exact aggregation over financial assets,” *Macroeconomic Dynamics* **1**, 513-523.

Sharp, W. F. (1964), “Capital asset prices: A theory of market equilibrium under conditions of risk,” *The Journal of Finance* **19**, 425–442.