Entrepreneurial Financing and Project Choice
under Non-diversifiable Idiosyncratic Risk*

Hui Chen†      Jianjun Miao‡      Neng Wang§

July 24, 2008

Preliminary Version

Abstract

Entrepreneurs face significant non-diversifiable idiosyncratic business risks. In a dynamic incomplete-markets model of entrepreneurial finance, we show that such risks have important implications for their interdependent consumption/saving, portfolio choice, financing, and endogenous default/cash-out decisions. The entrepreneur issues non-recourse secured debt collateralized by his firm’s assets. In addition to the standard tradeoff between tax benefits and financial distress/agency costs of debt, risky debt also provides diversification benefits. Even though more risk-averse entrepreneurs default earlier for given debt service, they choose higher leverage ex ante for diversification benefits. The entrepreneur demands not only the systematic risk premium but also an idiosyncratic risk premium (due to lack of diversification). We derive an analytical formula for the idiosyncratic risk premium whose key determinants are risk aversion, idiosyncratic volatility and the sensitivity of entrepreneurial value of equity with respect to cash flow. Giving the entrepreneur an extra option to cash out in the future increases his diversification benefits and private value of firm, but crowds out the value of diversification via external risky debt. Finally, we extend the model to allow the entrepreneur to choose the project’s idiosyncratic volatility after debt is in place. We show that entrepreneurial risk aversion dominates his option-payoff-induced risk-shifting incentives: Only entrepreneurs with low risk aversion engage in risk shifting activities.

Keywords: Default, diversification benefits, entrepreneurial risk aversion, incomplete markets, private equity premium, hedging, capital structure, cash-out option, precautionary saving

JEL Classification: G11, G31, E2

*We thank ...
†MIT Sloan School of Management, 50 Memorial Drive, Cambridge, MA 02142. Email: huichen@mit.edu. Tel.: 617-324-3896.
‡Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215. Email: miaoj@bu.edu. Tel.: 617-353-6675.
1 Introduction

Entrepreneurship plays an important role in fostering innovation and economic growth (Schumpeter (1934)). Entrepreneurial investment activities are quite diverse, ranging from the creation of the state-of-the-art high-tech products to daily operations of small business (e.g. restaurants). The entrepreneurial business may fundamentally differ from each other, but importantly share one common feature: the entrepreneurs cannot diversify idiosyncratic risks from their investment projects.

For incentive alignment and informational asymmetry between the entrepreneur and the financiers, entrepreneurs inevitably face non-diversifiable idiosyncratic risks and hence hold an undiversified portfolio. Moskowitz and Vissing-Jorgensen (2002) document that about 75 percent of all private equity is owned by households for whom it constitutes at least half of their total net worth. Hall and Woodward (2008) document that the idiosyncratic risks that entrepreneurs of startups are facing are so large that, even with moderate risk aversion, they will be better off passing up projects with high expected payoffs. Moreover, households with entrepreneurial equity invest on average more than 70 percent of their private holdings in a single private company in which they have an active management interest.

The significant lack of diversification invalidate the standard finance textbook valuation analysis designed for firms owned by diversified investors. Unlike the standard argument that a firm’s idiosyncratic risks carry no risk premium for investors holding a diversified portfolio, the entrepreneur’s non-diversifiable position in his own investment project inevitably makes his business decisions (financing and project choice) and his household decisions (consumption, saving, and asset allocation) interdependent. This interdependence arises in our model because markets are no longer complete for the entrepreneur. The standard (two-step) complete-market analysis (i.e. first value maximization and then optimal consumption allocation) no longer applies. This non-separability between value maximization and consumption smoothing has potentially important implications on real economic activities and the pricing of financial claims that the entrepreneur issues to finance his investment activities.

To the best of our knowledge, this paper provides the first dynamic incomplete-markets trade-off model of capital structure for entrepreneurial firms. In this model, the entrepreneur makes his interdependent real investment, financing, and consumption-saving/portfolio choice decisions.
The infinitely-lived risk-averse entrepreneur has expected utility over intertemporal consumption. He has both liquid financial wealth and an illiquid (entrepreneurial) investment project, which he sets up as a firm with limited liability (the entrepreneurial firm). He can invest his liquid financial wealth in both the risk-free asset earning a constant rate of return, and the market portfolio yielding an independently and identically distributed (iid) rate of return with constant risk premium (as in the classic consumption/portfolio choice problem (Merton (1971))). The entrepreneurial firm’s investment project generates a stream of stochastic cash flows, which is imperfectly correlated with the market portfolio. Therefore, the entrepreneur cannot fully hedge his project risk by dynamically adjusting his exposure to the market portfolio (i.e. markets are incomplete), and hence faces non-diversifiable idiosyncratic risks.

In our model, outside risky debt and the cash-out option (to sell the entrepreneurial firm to diversified outside investors) are the two channels for the entrepreneurs to reduce his idiosyncratic risk exposure. This modelling choice is motivated by the empirical evidence that debt is the primary source of outside financing for small businesses (Heaton and Lucas (2004)).

The entrepreneur can borrow on the firm’s balance sheet using the illiquid project as collateral. The endogenous default option on outside debt (limited liability) gives the entrepreneur an insurance to walk away from his business, if the firm’s performance is sufficiently poor. Moreover, debt is potentially less information sensitive, and may be preferred in some settings with asymmetric information (Myers and Majluf (1984)). The entrepreneur can also sell the firm to diversified investors via either direct placement or initial public offering (IPO). Such sales are costly, and our model allows the entrepreneur to optimally time his cash-out decision. Moreover, we incorporate the effect of leverage adjustment for firm upon cash-out on the ex ante entrepreneur’s capital structure decision and valuation for his firm.

Default triggers costly liquidation as in standard tradeoff models of capital structure. In addition to potential tax benefits as in standard tradeoff models, the risky debt also offers diversification benefits for the entrepreneur. The entrepreneur chooses the debt policy by trading off tax and diversification benefits of debt with bankruptcy and agency costs. We show that the entrepreneur’s utility maximization problem (with interdependent consumption/saving, portfolio and default decisions) can be simplified to a problem of maximizing the entrepreneur’s private value of firm. The

1The non-diversifiable idiosyncratic risk that entrepreneurs bear also has implications on the ex ante career choice. See Kihlstrom and Laffont (1979) and Rampini (2004) building on the ideas in Knight (1921).
entrepreneur’s risk attitude and his exposure to the project’s idiosyncratic risks have significant effects on the private valuation of the entrepreneurial firm and its equity, and hence influence the firm’s initial capital structure and subsequent default decisions.

Unlike the standard tradeoff models of capital structure, the natural measure of leverage for an entrepreneurial firm is given by the ratio of “public” value of debt and “private” value of firm, given by the sum of the entrepreneur’s private value of equity and the public value of debt. We dub this ratio “private leverage” to highlight the impact of entrepreneurial risk aversion and idiosyncratic risk exposure on the firm’s leverage decision. Naturally, our model includes the standard complete-markets (contingent claim) trade-off model of capital structure (i.e. Leland (1994)) as a special case.

Our main findings are as follows. First, diversification benefits of debt are large. Even when the tax rate is zero (i.e. no tax shields for debt), the entrepreneurial firm still issues a significant amount of outside debt for diversification benefits. The more risk-averse the entrepreneur, the more he values the diversification benefit of risky debt, hence the higher his private leverage. At the first glance, this result might appear counterintuitive: more risk-averse entrepreneurs ought to be less aggressive in terms of financial policies (e.g. a lower leverage). We may reconcile this result as follows. The more risk-averse entrepreneur has a lower subjective valuation of his business project and has a stronger incentive to sell his firm. Since the (diversified) lenders demand no premium for bearing the entrepreneurial firm’s idiosyncratic risk, we naturally expect that more risk-averse entrepreneurs sell a higher fraction of their firms via outside debt.

While the effect of risk aversion on leverage is monotonically increasing, there are two opposing effects of risk aversion on debt coupon. On the one hand, the diversification benefits suggests that coupon increases with entrepreneurial risk aversion. On the other hand, the more risk-averse entrepreneur \textit{ex post} defaults earlier for a given level of coupon, lowering the firm’s ability to issue debt. We refer to the latter effect as the distance-to-default effect. The net impact of risk aversion on debt coupon is therefore ambiguous. For example, when the asset recovery rate is high (i.e. when loss given default is low), then the distance-to-default effect is less relevant, which in turn makes debt coupon increasing with risk aversion. However, when loss given default is high, debt coupon may decrease with risk aversion.

Second, the entrepreneur’s private value of his equity and firm in our incomplete-markets model behave differently from equity and firm values for (public) firms held by diversified investors. In-
spired by important insights in Black and Cox (1976), Leland (1994) initiated strong interests on structural models of credit risk and capital structure. In a complete-markets credit risk/capital structure model (e.g. Leland (1994)), equity value is convex, which follows from the Black-Scholes-Merton insight that equity is a call option on firm value. Moreover, firm value is concave in cash flow $y$ due to \textit{ex post} costly liquidation. Intuitively, the (public) firm is long an unlevered cash flow generating machine, long a tax shield, and \textit{short} a liquidation option, which makes \textit{ex ante} firm value concave in cash flow $y$.

Unlike the complete-markets model, our model predicts that the private value of equity (after debt issuance) is not necessarily globally convex in cash flow $y$. Indeed, the entrepreneur’s precautionary saving demand makes his private value of equity potentially concave in cash flow $y$ when precautionary saving motive and/or idiosyncratic volatility is sufficiently high. Similarly, the private value of firm (the sum of private value of equity and the public value of debt) is not necessarily globally concave as in complete-markets models because of the additional diversification option under incomplete markets in our model.

Third, we show that the leverage ratio can drop substantially when the entrepreneur also has the cash-out option to diversify his project’s idiosyncratic risks. Intuitively, the incremental value of debt financing is lower when the entrepreneur substitutes some debt with the expected use of the cash-out option in the future. A higher debt value increases the “strike” price of exercising the cash-out option because debt needs to be called back at par when the firm cashes out. Hence, the distance to cash-out is higher and the present value of the cash-out option lower \textit{ceteris paribus} (due to debt overhang). The entrepreneur maximizes his \textit{ex ante} private value of firm by trading off debt issuance/default option and the value of cash-out option. Intuitively, the more attractive the cash-out option is, the lower the firm’s private leverage and debt coupon are.

\textit{Related Literature}

The key feature of our model is the interdependence among the entrepreneur’s consumption/saving, portfolio choice, and default/cash-out option exercising decisions. Naturally, our model draws on insights from and contributes to the following six strands of literature: \textit{(i)} structural credit risk/dynamic capital structure, \textit{(ii)} incomplete-markets consumption smoothing/precautionary saving, \textit{(iii)} portfolio choice and dynamic hedging, \textit{(iv)} option exercising and pricing under incomplete markets, \textit{(v)} risk seeking/asset substitution models (Jensen and Meckling (1976)), and \textit{(vi)}
entrepreneurial finance.

Our model provides a generalized tradeoff model for the entrepreneurial firm’s capital structure under incomplete markets, where the risky outside debt offers an additional diversification benefit over inside equity. We show that precautionary saving demand plays an important role in determining the entrepreneurial firm’s leverage and default strategies. Naturally, our model includes the structural (complete-markets) credit risk/capital structure models such as the workhorse Leland (1994) model as a special case.

Our model relates to the incomplete-markets consumption smoothing/precautionary saving literature. For analytical tractability reasons, we adopt the expected constant-absolute-risk-averse (CARA) utility specification as Merton (1971), Caballero (1990), Kimball and Mankiw (1989), and Wang (2006). Our model contributes to this literature by extending the CARA-utility-based precautionary saving problem to allow the entrepreneur to diversify his idiosyncratic risks via exit strategies such as cash-out and default.

Our model also links to the dynamic portfolio choice/hedging literature (Merton (1971, 1973)). Unlike the standard portfolio choice literature, our model predicts an interdependence between portfolio choice/hedging demand and the entrepreneur’s optimal default and leverage policies. The model also contributes to the option exercising/valuation problem under incomplete markets. Miao and Wang (2007) analyze the impact of the entrepreneur’s non-diversifiable idiosyncratic risk on his growth option exercising decision. We model and focus on the entrepreneurial firm’s capital structure and endogenous default decisions.

We extend our model to allow the entrepreneur to have a project choice decision after debt is in place in order to understand the entrepreneur’s tradeoff between risk shifting (induced by the option payoff) and risk sharing (due to incomplete markets). We show that the entrepreneur’s risk-shifting incentives (Jensen and Meckling (1976)) are substantially reduced when we take the entrepreneur’s non-diversifiable idiosyncratic risk into account. Our model provides one potential explanation for the (weak) empirical and survey evidence on the quantitative importance of asset substitution and risk seeking incentive (Graham and Harvey (2001) and other related papers).

Finally, our paper builds on recent work on entrepreneurial finance, particularly Heaton and Lucas (2004), which explicitly model the impact of non-diversifiable idiosyncratic risk on the en-

---

2 Hall (1978) pioneered the Euler equation approach to analyze intertemporal consumption decision and showed that consumption is a martingale (under certainty equivalence (i.e. quadratic utility)). See Deaton (1992) and Attanasio (1999) for recent surveys.
One key difference is that our model is dynamic and their model is static. As a result, we capture the entrepreneur’s interdependent consumption/saving, portfolio choice, and financing decisions. Since our model is dynamic, the key driving force in our model is the entrepreneur’s consumption smoothing/precautionary saving motive, which translates into an intertemporal consumption Euler equation. In their static model, consumption is equal to terminal wealth and hence risk aversion plays the key role. Our dynamic model also naturally gives rise to an endogenous default as a perpetual American option exercising problem under incomplete markets. Our model rules out wealth effect in order to focus on the intertemporal tradeoffs, while Heaton and Lucas (2004) capture the wealth effects using constant relative risk aversion (CRRA) utility in a static setting. Results of the two papers are complementary.

Other differences between the two papers include: we allow for tax benefits of debt and hence allows us to include Leland (1994) (the tradeoff model of capital structure in contingent claim setting) as special cases, while they do not. We parameterize the cost of bankruptcy as in Leland (1994) and other tradeoff models, while they use adverse selection as the cost of external financing. We allow for the entrepreneur to invest in the risky asset to partially hedge his project risk and hence include the complete-markets setting (Leland (1994)) as a special case, while they assume away hedgable component of risk. We acknowledge that their model can be extended along these dimensions.

The remainder of the paper is organized as follows. Section 2 describes the entrepreneur’s interdependent dynamic decision problem. Section 3 uses the backward induction to solve the entrepreneur’s consumption, saving, portfolio choice and the firm’s capital structure and cash-out/default decisions. Section 4 analyzes the effect of risky debt for the entrepreneur’s financing decisions. Section 5 studies the effect of cash-out on entrepreneurial financing and firm value as an additional diversification channel. Section 6 extends the model to allow for the entrepreneur’s endogenous project choice after debt is issued. Section 7 concludes.

2 Model Setup

Consider an infinitely-lived risk-averse entrepreneur’s decision problem in a continuous-time setting. The entrepreneur derives utility from a consumption process \( \{c_t : t \geq 0\} \) according to the following
time-additive utility function:

$$E \left[ \int_0^\infty e^{-\delta t} u(c_t) \, dt \right],$$

(1)

where $\delta > 0$ is the entrepreneur’s subjective discount rate and $u(\cdot)$ is an increasing and concave function. For analytical tractability, we adopt the CARA utility. That is, let $u(c) = -e^{\gamma c}/\gamma$, where $\gamma > 0$ is coefficient of absolute risk aversion, which also measures precautionary motive.

The entrepreneur has a take-it-or-leave-it project at time zero. If he launches this project by paying a lump-sum cost $I$, the project will generate a stream of stochastic cash flows $\{y_t : t \geq 0\}$. Assume that the cash flow process $\{y_t : t \geq 0\}$ follows a geometric Brownian motion (GBM) given by:

$$dy_t = \mu y_t dt + \omega y_t dB_t + \epsilon y_t dZ_t, \quad y_0 \text{ given},$$

(2)

where $\mu$ is the expected growth rate of the cash flow, $\omega$ and $\epsilon$ are volatility parameters, and $B$ and $Z$ are independent standard Brownian motions driving the market (systematic) risk and idiosyncratic risk, respectively. Intuitively, we may interpret $\omega > 0$ and $\epsilon \geq 0$ as systematic and idiosyncratic volatility parameters of the cash flow growth. The corresponding total volatility $\sigma$ is then given by $\sigma = \sqrt{\omega^2 + \epsilon^2}$. As we will show, these different volatility parameters $\omega$, $\epsilon$, and $\sigma$ have different effects on the entrepreneur’s decision making.

The entrepreneur invests his financial wealth in both the risk-free asset with a constant risk-free rate $r$ and a market portfolio to smooth his intertemporal consumption and to diversify risks. Assume that the return of the market portfolio is independently and identically distributed, in that

$$dP_t/P_t = \mu_e dt + \sigma_e dB_t,$$

(3)

where $\mu_e$ is the expected return on the risky asset and $\sigma_e$ is the return volatility. Let $\eta = (\mu_e - r) / \sigma_e$ denote the Sharpe ratio of the market portfolio. Whenever the cash flow process (2) and the market portfolio return (3) are not perfectly correlated (i.e. $\epsilon > 0$), the entrepreneur bears non-diversifiable idiosyncratic risks from owning the project. Although the key focus of our paper is the


4Precautionary saving is motivated by the consumption Euler equation analysis to focus on uncertainty of income rather than on risk attitude towards wealth and consumption. Leland (1968) is among the earliest studies on precautionary saving models. Kimball (1990) links the degree of precautionary saving to the convexity of the marginal utility function $u'(c)$. By drawing an analogy to the theory of risk aversion, Kimball (1990) defines $-u'''(c)/u''(c)$ as the coefficient of absolute prudence. For CARA utility, we have $-u'''(c)/u''(c) = \gamma$. 

7
effects of non-diversifiable idiosyncratic risk, instead of the tax effects, taxes do play a potentially important role in entrepreneurial finance. We capture the effects of tax codes in a parsimonious but also reasonably descriptive way. In particular, the interest rate and stock returns are after-tax values. That is, the parameters $r$, $\mu_e$ and $\sigma_e$ are after-tax values.

If the entrepreneur chooses to invest in the project, he sets up a separate entity such as limited liability company (LLC). Running the firm also incurs a fixed operating cost $w$ at each point in time. This cost captures operating leverage. The entrepreneur’s business income is taxed at the rate $\tau_1$ as if the firm were sole proprietorship, but enjoys the single taxation benefit. He then raises non-recourse secured debt from the lender using the project as the collateral at time zero. Since our focus is on the effect of the non-diversifiable idiosyncratic project risk on entrepreneurial financing decisions, we abstract away from other frictions for simplicity such as the borrowing constraint that the entrepreneur may face. As in Heaton and Lucas (2004), we assume that the external source of financing for the entrepreneur is debt. Accessing to external equity markets involves fixed costs such as initial public offering (IPO) fees, which particularly discourage small firms from going public, *ceteris paribus*. Empirically, entrepreneurial firms mostly use debt as external financing, particularly for smaller ones (see Vissing-Jorgensen and Moskowitz (2001), and Heaton and Lucas (2004)).

The entrepreneur has incentives to borrow against his business project because financing via risky debt allows the entrepreneur to reduce his equity exposure and partially to diversify his idiosyncratic project risk (as in Heaton and Lucas (2004)). This diversification benefit argument does not apply to firms held by owners who have diversified portfolios as in standard valuation theory that finance textbooks treat. A potentially significant fraction of the entrepreneur’s total wealth is in his business due to reasons such as incentives (Jensen and Meckling (1976)) or informational asymmetry (Leland and Pyle (1977). Diversification benefits for the entrepreneur exist only when the debt is risky. Unlike public firms (such as C corp), the entrepreneurial firm may or may not have tax benefit of debt because the entrepreneurial firm does not face double taxation like C corp does. We let $\tau_1$ denote the effective tax rate for the entrepreneurial (private) firm, and $\tau_2$ denote

---

5 The S corp, a form of corporation, is also only subject to the single-level taxation.
6 The standard finance valuation exercise starts with an exogenously specified cash flow project and a cost of capital calculation using risk-return models such as the capital asset pricing model (CAPM). Only systematic risk enters the calculation of the cost of capital and valuation as in the CAPM.
the effective tax rate for the public firm after the entrepreneur sells to the well diversified investors.

We assume that debt is issued at par and has infinite maturity for analytical tractability reasons as in Leland (1994) and Duffie and Lando (2001). Let \( b \) denote the coupon payment of debt and \( F_0 \) denote the par value of debt. The remaining cash flow from operation after debt service and tax payments accrues to the entrepreneur. The terms of debt issuance are determined at time 0. After debt is in place, at each point in time \( t > 0 \), the entrepreneur continues his project until he decides either to default on his outstanding debt which will lead to the liquidation of his firm, or to cash out by selling his firm to a diversified buyer by paying a fixed cost. We model both default and cash-out decisions as irreversible. Default and cash-out resemble American-style put and call options on the underlying cash flows. By American style, we mean that the entrepreneur can time his default and cash-out decisions, both of which depend on the firm’s operating performance. The entrepreneur chooses the default and the cash-out timing policies to maximize his own utility. As in other financing models, there is an inevitable conflict of interest between financiers and the entrepreneur (e.g. Jensen and Meckling (1976)). The entrepreneur’s default and cash-out timing strategies are not contractible at time zero.

If the entrepreneur defaults on the firm’s debt at his endogenously chosen (stochastic) time \( T_d \), the firm goes bankrupt. Following Mello and Parsons (1992), we assume that, after bankruptcy, debtholders take control of the firm and run the firm as an all equity-value public firm. Because running the firm incurs fixed operating costs \( w \), debtholders as the new owners have the option to abandon the firm and obtain an outside value normalized to zero. Bankruptcy is costly as in tradeoff models of capital structure (e.g., Leland (1994)). Assume that debtholders collect a fraction \( \alpha \) of the after-tax unlevered market value of the firm upon default. The remaining fraction \( 1 - \alpha \) accounts for bankruptcy costs. While there are potential room for the debtholders and the entrepreneur to renegotiate \( \text{ex post} \), we abstract from this issue given the focus of our paper.

In Appendix A, we show that the pre-tax unlevered market value of the firm \( A(y) \) as a function of the current project cash flow \( y \) is given by:

\[
A(y) = \Pi(y) - \Pi(y_a) \left( \frac{y}{y_a} \right)^{\theta_1},
\]

where \( \nu \equiv \mu - \omega \eta \) is the risk-adjusted expected growth rate of cash flows, \( \theta_1 \) is a parameter given by (A.8) in Appendix A, \( y_a \) is the abandonment threshold given by (A.7) in Appendix A, and

\[
\Pi(y) = \frac{y}{r - \nu} - \frac{w}{r}
\]


is the risk-adjusted present value of profit flows. Equation (4) shows that the before-tax unlevered firm value is equal to the risk-adjusted present value of profit flows plus an option value of abandonment. Note that only systematic risk demands a risk premium as in the CAPM model.

Intuitively, the entrepreneur defaults when the firm does sufficiently poorly to walk away from his liability. Assume that absolute priority is enforced. The limited liability and the entrepreneur’s voluntary default imply that the entrepreneur receives nothing upon default and the lender collects the proceeds from liquidation. After liquidation, the entrepreneur is no longer exposed to the firm’s idiosyncratic risk. In our generalized tradeoff model, the entrepreneur trades off tax and diversification benefits against the default and agency costs of debt.

While cashing out allows the entrepreneur to capitalize on his firm value and achieve diversification, he needs to pay a fixed cost \( K(y) \) to sell his firm via either an initial public offering (IPO) or direct sale to diversified investors. This fixed cost could be a fraction of the selling value of the firm, and thus may depend on the cash flow state \( y \). Importantly, the entrepreneur will generally pay capital gains taxes upon cashing out. Let \( \tau_g \) denote the capital gains tax rate. Assume that the entrepreneur retires the firm’s debt obligation at par \( F_0 \) in order to cash out. The gross proceeds that the entrepreneur obtains from selling his firm is fairly priced by the competitive capital market. Since new owners are well diversified and do not demand idiosyncratic risk premium, we will use the complete-markets solution to obtain the sale value of the firm. The new owners will optimally relever the firm by issuing a perpetual debt with a different coupon level as in Leland (1994). The entrepreneur also benefits from this relevered firm value after cash out.

Let \( T_u \) denote the endogenously chosen cash-out time. The entrepreneur pays the transaction cost \( K(y_{T_u}) \), retires debt at par \( F_0 \), and pays the capital gains taxes to capitalize on the value of the project. Let \( V^*(y_{T_u}) \) denote the selling value of the entrepreneurial firm when cashing out at time \( T_u \). We provide an explicit expression for this value in the next section. The tax liability upon cashing out is \( \tau_g (V^*(y_{T_u}) - F_0 - K(y_{T_u}) - (I - F_0)) \). Our model can be easily extended to account for other features such as depreciation, which may further complicate the tax treatment. For simplicity, we abstract away from these issues, which are not crucial to the focus of this paper. The entrepreneur’s financial wealth changes discretely from \( x_{T_u^-} \) just prior to cashing out to \( x_{T_u^+} = x_{T_u^-} + V^*(y_{T_u}) - F_0 - K(y_{T_u}) - \tau_g (V^*(y_{T_u}) - K - I) \) immediately after cashing out.

After the entrepreneur exits from his business by either defaulting on his debt or cashing out his business, he retires and has no other non-financial income. He then solves a standard complete-
markets consumption and portfolio choice problem as in Merton (1971). Extending our model to allow for sequential rounds of entrepreneurial activities will complicate our analysis. We leave this extension for future research.

3 Model Solution

We solve the entrepreneur’s optimization problem as follows. Section 3.1 summarizes the complete-markets solution for firm value and financing decisions if the firm is owned by well diversified investors. This complete-markets solution (Leland (1994)) gives the “cash-out” value for the entrepreneur from selling his firm and also serves as a benchmark for our comparison analysis. In Section 3.2 we analyze the entrepreneur’s interdependent consumption/saving, portfolio choice, and initial investment and financing decisions. In Section 3.3, we discuss some special cases.

3.1 Complete markets firm valuation and financing policy

After the entrepreneur cashes out his equity by selling his firm to well diversified investors, the new owners relever the firm by issuing a perpetual debt with a new coupon level \( b \) to maximize ex ante firm value. The new owners trade off the tax benefits of debt against bankruptcy and agency costs. For analytical convenience, following Leland (1994), we assume that there is no re-adjustment of capital structure in the future.

In Appendix A, we provide details for the complete-markets analysis, which essentially follows from Leland (1994). Here we highlight a few pricing formulae that we use in the main body of the text. Recall that the tax rate for the public firm is \( \tau_2 \). This tax rate takes into account the double taxation for the public firm. Given coupon rate \( b \) and default threshold \( y_d \), the market value of equity \( E(y; y_d) \) and the levered firm value \( V(y; y_d) \) are respectively given by

\[
E(y; y_d) = (1 - \tau_2) \left[ \Pi(y) - \frac{b}{r} - \left( \Pi(y_d) - \frac{b}{r} \right) \left( \frac{y}{y_d} \right)^{\theta_1} \right],
\]

\[
V(y; y_d) = (1 - \tau_2) A(y) + \tau_2 \frac{b}{r} \left[ 1 - \left( \frac{y}{y_d} \right)^{\theta_1} \right] - (1 - \tau_2) (1 - \alpha) A(y_d) \left( \frac{y}{y_d} \right)^{\theta_1},
\]

7We abstract away from the dynamic capital structure decisions after the entrepreneur cashes out to keep the analysis tractable and also analogous to our treatment before the entrepreneur exits. While extending the model by allowing for dynamic financing adjustments will enrich the model, it complicates our analysis without changing the key economic tradeoff that we focus: the impact of idiosyncratic risk on entrepreneurial financing decisions. We leave extensions along the line of Goldstein, Ju, and Leland (2001) for future research.
where \( A(y) \) and \( \Pi(y) \) are given by equations (4) and (5), respectively. Equation (6) shows that equity value is equal to the after-tax present value of profit flows minus the present value of the perpetual coupon payments plus an option value to default. The term \((y/y_d)^{\theta_1}\) may be interpreted as the “risk-neutral” Arrow-Debreu price of a unit of claim contingent on the event of default. Equation (7) shows that the levered market value of the firm is equal to the after-tax unlevered firm value plus the present value of tax shields minus bankruptcy costs.

Appendix A gives the analytical formulae for the optimal default threshold \( y_d \). Let \( b^* \) denote the firm-value-maximizing coupon rate and \( V^*(y) \) denote the levered market value of the firm as a function of cash flow \( y \) when the firm chooses its value-maximizing debt coupon \( b^* \). No closed form expressions for \( b^* \) and \( V^*(y) \) are available in general except for the case with \( w = 0 \). In this case, we show in Appendix A that

\[
V^*(y) = \left[ 1 - \tau_2 + \tau_2 \left( 1 - \theta_1 - \frac{(1 - \tau_2)(1 - \alpha)\theta_1}{\tau_2} \right)^{1/\theta_1} \right] \frac{y}{r - \nu}. \tag{8}
\]

This firm value formula only applies at the moment of debt issuance in Leland (1994).

### 3.2 Entrepreneur’s decision making

We solve the entrepreneur’s decision making in three steps by backward induction. First, we briefly summarize the entrepreneur’s consumption/saving and portfolio choice problem after he retires from his business either via cashing out or defaulting on debt. This optimization problem is the same as in Merton (1971), a (dynamically) complete-markets consumption/portfolio choice problem. Second, we solve the entrepreneur’s joint consumption/saving, portfolio choice, default, and cashing-out decisions after debt is in place when he still runs the firm. Finally, we solve the entrepreneur’s initial investment and financing decision at time zero.

Let \( \{x_t : t \geq 0\} \) denote the entrepreneur’s financial wealth process and let \( \pi_t \) denote the amount of his financial wealth invested in the market portfolio at time \( t \). The entrepreneur’s initial wealth \( x_0 \) is equal to his endowment \( x \) plus debt proceeds \( F_0 \) and minus the investment cost \( I \), i.e.,

\[
x_0 = x + F_0 - I. \tag{9}
\]

**Consumption/saving and portfolio choice after exit.** After he exits from his business (via either default or cash-out), the entrepreneur no longer has business income and lives on his financial
wealth. His wealth follows from the following dynamics

$$dx_t = (rx_t + \pi_t (\mu_e - r) - c_t) dt + \pi_t \sigma_e dB_t.$$  \hspace{1cm} (10)

The entrepreneur’s optimization problem becomes the standard complete-market consumption and portfolio choice problem. By Merton (1971), the entrepreneur’s value function $J_e(x)$ is given by the following explicit form:

$$J_e(x) = -\frac{1}{\gamma r} \exp \left[ -\gamma r \left( x + \eta^2 + \frac{\delta - r}{\gamma r^2} \right) \right].$$  \hspace{1cm} (11)

**Entrepreneur’s decision making while running the firm.** We now consider the decision problem when the entrepreneur runs his firm. Before exiting from his business, the entrepreneur’s financial wealth evolves as follows:

$$dx_t = (rx_t + \pi_t (\mu_e - r) + (1 - \tau_1) (y_t - b - w) - c_t) dt + \pi_t \sigma_e dB_t, \ t < \min (T_d, T_u),$$  \hspace{1cm} (12)

where $T_d$ is the default time and $T_u$ is the cash-out time. The entrepreneur receives $(1 - \tau_1) (y_t - b)$ from his business in flow terms after servicing the debt and paying taxes. Let $J^s(x, y)$ denote his value function. In Appendix B, we show that it takes the following explicit exponential form:

$$J^s(x, y) = -\frac{1}{\gamma r} \exp \left[ -\gamma r \left( x + G(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2} \right) \right],$$  \hspace{1cm} (13)

where $G(y)$ is given in Theorem 1 below. As shown in Miao and Wang (2007), $G(y)$ is the entrepreneur’s certainty equivalent wealth coming from his ownership of the firm. We will refer to $G(y)$ as the entrepreneur’s private value of equity.

At the moment of default and cash-out, the value functions $J^s(x, y)$ and $J^d(x)$ must satisfy certain value-matching and smooth-pasting conditions described in Appendix B. These conditions determine a default boundary $y_d(x)$ and a cash-out boundary $y_u(x)$. In general these boundaries depend on the wealth level $x$. Note that equation (13) implies that $G(y)$ is additively separable from financial wealth $x$ in $J^s(x, y)$. This absence of wealth effect implies that the default and cash-out boundaries $y_d(x)$ and $y_u(x)$ are independent of wealth. We thus simply use $y_d$ and $y_u$ to denote the default and cash-out thresholds, respectively.

While CARA utility does not capture the wealth effect, we emphasize that the main results and insights of our paper (the effect of non-diversifiable idiosyncratic shocks on investment timing) do not rely on the particular choice of this utility function. As we show below, the driving force of
our results is the precautionary savings effect, which is captured by utility functions with convex marginal utility such as CARA. While power utility is more commonly used and more appealing for quantitative analysis, this utility specification will substantially complicate our analysis since it will lead to a much harder two dimensional free-boundary problem.

We summarize the solution for consumption/saving, portfolio choice, default trigger \( y_d \), and cash-out trigger \( y_u \) in the following:

**Theorem 1** The entrepreneur exits from his business when the cash flow process \( \{ y_t : t \geq 0 \} \) reaches either the default threshold \( y_d \) or the cash-out threshold \( y_u \), whichever occurring the first. When the entrepreneur runs his firm, he chooses his consumption and portfolio rules as follows:

\[
\bar{c}(x, y) = r \left( x + G(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2} \right),
\]

\[
\bar{\pi}(x, y) = \frac{\eta}{\gamma r \sigma e} - \frac{\omega}{\sigma e} y G'(y),
\]

where \((G(\cdot), y_d, y_u)\) solves the free boundary problem given by the differential equation:

\[
rG(y) = (1 - \tau_1) (y - b - w) + \upsilon y G'(y) + \frac{\sigma^2 y^2}{2} G''(y) - \frac{\gamma r \epsilon^2 y^2}{2} G'(y)^2,
\]

subject to the following (free) boundary conditions

\[
G(y_d) = 0,
\]

\[
G'(y_d) = 0,
\]

and

\[
G(y_u) = V^*(y_u) - F_0 - K(y_u) - \tau g (V^*(y_u) - K(y_u) - I),
\]

\[
G'(y_u) = (1 - \tau_g) \left( V^{*\prime}(y_u) - K'(y_u) \right).
\]

Here, \( V^*(y) \) is given in (8) and \( F_0 \) is the par value of the perpetual debt.

---

\(*\) A well-known implication of CARA-utility-based models is that consumption and wealth may sometimes turn negative (see e.g. Merton (1971), Grossman (1976) and Wang (1993)). Cox and Huang (1989) provide analytical formulae for consumption under complete markets for CARA utility with non-negativity constraints. However, in our incomplete-markets setting, imposing non-negativity constraints substantially complicates the analysis. Intuitively, requiring consumption to be positive increases the entrepreneur’s demand for precautionary saving because he will increase his saving today to avoid hitting the constraints in the future. The induced stronger precautionary saving demand in turn makes our results (such as diversification benefits of outside risky debt) stronger.
The differential equation (16) provides a certainty equivalent valuation for $G(y)$ from the entrepreneur’s perspective. The last (nonlinear) term captures the intuition behind the discount due to the non-diversifiable idiosyncratic risk. Intuitively, a higher risk aversion parameter $\gamma$, a larger discount on $G(y)$ due to the non-diversifiable idiosyncratic risk. The next section provides more detailed analysis on the impact of idiosyncratic risk on the entrepreneur’s subjective valuation.

Equation (17) comes from the value-matching condition for the firm’s default decision. It states that the private value of equity upon default is equal to zero. Equation (18) follows from the smooth-pasting condition. It can be interpreted as the optimality condition from the maximization of the private value of equity. Similarly, the value-matching condition (19) at the cash-out boundary states that the private value of equity upon the firm’s cashing out is equal to the after-tax value of the public firm value after paying back the fixed costs $K(y_u)$ and $I$, and retiring outstanding debt at par $F_0$. The smooth-pasting condition (20) implies that the changes in value just before and immediately after cash-out must be equal.

**Initial financing and investment decision.** Theorem 1 characterizes the entrepreneur’s decisions after debt is in place. We now complete the model solution by endogenizing the entrepreneur’s initial investment and financing decision. Note that the entrepreneur’s initial wealth $x_0$ immediately after investment and financing is given by (9). At time zero, the entrepreneur chooses a coupon rate $b$ to solve the following problem:

$$\max_b J^s (x + F_0 - I, y_0),$$

subject to the requirement that debt be issued at par, i.e. $F_0 = F(y_0)$, where $F(y)$ denotes the market value of debt. In Appendix C, we provide an explicit formula for $F(y)$.

At time zero, the entrepreneur launches the project if his indirect utility from the project is higher than the indirect utility without the project, i.e.,

$$\max_b J^s (x + F_0 - I, y_0) > J^c (x).$$

Since equity is held by the entrepreneur and is not tradable, we only have private valuation $G(y)$. Debt is issued to diversified investors and is priced at market value $F(y)$. Intuitively, we define the private value of the entrepreneurial firm, denoted by $S(y)$, as follows:

$$S(y) = G(y) + F(y).$$
We may interpret $S(y)$ as the fair price that an investor needs to pay in order to acquire the entrepreneurial firm by paying $G(y)$ to the entrepreneur and $F(y)$ to the lender. Since the entrepreneur receives $F(y_0)$ at time 0 in exchange for future coupon payments, the private firm value for the entrepreneur at time 0 is $S(y_0) = G(y_0) + F(y_0)$. Using the exponential form for the entrepreneur’s value function $J^s(x, y)$, it is straightforward to show that the maximization problem stated in (21) is equivalent to the following one:

$$\max_b S(y_0) = G(y_0) + F(y_0).$$ (24)

Note the conflicts of interest between the entrepreneur and the lender. When choosing the debt coupon, the entrepreneur maximizes his private value of the firm, $S(y_0)$, because he internalizes the benefits and costs of debt issuance. After debt is in place, the entrepreneur chooses default and cash-out thresholds to maximize his private value of equity $G(y)$. Unlike publicly held firms, the optimal coupon in our setting maximizes the private value $S(y_0)$, but not the market value $V(y_0) = E(y_0) + F(y_0)$, which is the conventional way to calculate firm value as in Leland (1994) or any other structural default/credit risk models.

We may interpret our model’s implication on capital structure as a generalized tradeoff model of capital structure for the entrepreneurial firm, where the entrepreneur trades off the benefits of debt (tax shields and diversification) against the costs of debt (bankruptcy/inefficient liquidation and agency conflicts between the entrepreneur and outside lenders). Similar to the classic tradeoff model, the objective function in our model for debt choice is private value of firm $S(y)$, the sum of private value of equity $G(y)$ and public value of firm $F(y)$. The natural measure of leverage (based on the optimization problem) is the ratio between private value of equity $G(y)$ and the private value of firm $S(y)$, in that

$$L(y) = \frac{G(y)}{G(y) + F(y)} = \frac{G(y)}{S(y)},$$ (25)

We dub $L(y)$ as private leverage to reflect the impact of idiosyncratic risk on the leverage choice.

Given the exponential forms for the entrepreneur’s value functions, we can also deduce from (22) that the entrepreneur invests in the project if the private value of the firm is larger than the investment cost, $\max_b S(y_0) > I$. That is, the entrepreneur follows a modified net present value rule in that he computes the net present value of the project using his private

---

9 Assume that the entrepreneur and the lender do not receive any surplus from the takeover/restructuring. Hence, the entrepreneur and the lender demands $G(y)$ and $F(y)$, respectively.
valuation, instead of market valuation.

3.3 Special cases: Immediate exits via default or cash-out

There are two special cases. The first is the one where the cost of cashing out is sufficiently small so that it is optimal for the entrepreneur to sell the firm immediately to diversified investors. That is, the cash-out option is immediately worth exercising at time 0 \((y_u < y_0)\). The other special case is the one where asset recovery rate is sufficiently high or the entrepreneur is sufficiently risk averse. Then, the entrepreneur raises as much debt as possible (i.e. 100% sale to the lender) and then defaults immediately \((y_d > y_0)\). In our analysis below, we will consider parameter values that rule out these two special cases.

We next turn to the model’s predictions and results. We proceed in three steps. In Section 4 we analyze the case where the entrepreneur only has the default option. This setting allows us to focus on the economics of non-diversifiable idiosyncratic risk on the entrepreneur’s default and financing decisions. In Section 5 the entrepreneur can exit his business via either default or cash-out, allowing the entrepreneur to diversify his idiosyncratic risks. In Section 6 we endogenize the entrepreneur’s choice of the project’s idiosyncratic risk. We show that the risk averse entrepreneur optimally trades off his risk-sharing (diversification) benefits with the option-payoff induced risk seeking incentive.

4 Risky debt, endogenous default, and diversification

The diversification and tax benefits of outside risky debt induce the entrepreneur to sells as much of the firm via debt as he can. However, the more debt he issues, the riskier the debt becomes and hence the cost of liquidation (in present value) increases. The entrepreneur trades off the benefits and costs of issuing debt. When the firm’s (stochastic) cash flow \(y\) becomes sufficiently low, the entrepreneur avoids the downside risk by defaulting. The setting without the cash-out option is a special case of the optimization problem analyzed in Proposition 1 (i.e. the cash-out cost \(K = \infty\)). This special case only has one endogenous (lower default) boundary.

We calibrate the model using standard parameters in the literature: the risk-free interest rate \(r = 0.02\), the expected growth rate of cash flows \(\mu = 0.03\), systematic volatility of growth rate \(\omega = 0.1\), idiosyncratic volatility \(\varepsilon = 0.2\), and the market price of risk \(\eta = 0.4\). These numbers are

\[\text{limit}_{T \to \infty} E \left[e^{-\delta T} J^*(w_T, y_T)\right] = 0.\]

\[\text{The upper boundary is replaced by the transversality condition: limit}_{T \to \infty} E \left[e^{-\delta T} J^*(w_T, y_T)\right] = 0.\]
all annualized. The asset recovery rate is \( \alpha = 0.6 \). We calibrate the tax rates \( \tau_1 = 0 \) and \( \tau_1 = 0.2 \) to reflect the net tax benefits of debt for private and public firms. The capital gain tax rate \( \tau_g = 0.15 \). Finally, the initial level of cash flow is \( y_0 = 1 \).

\[
K(y) = 0.1 \ast V^*(y)
\]

**Private value of equity** \( G(y) \) and **private value of firm** \( S(y) \). The top left and top right panels of Figure 1 plot private value of equity \( G(y) \) and its first derivative \( G'(y) \), respectively. Analogous to Black-Scholes-Merton’s observation that firm equity is a call option on firm value, the entrepreneur’s private equity \( G(y) \) is also a call option on the entrepreneurial firm value. Unlike the standard Black-Scholes-Merton paradigm, neither the entrepreneurial equity nor the entrepreneurial firm value is tradable. Analogous to the option Delta in financial derivatives literature, \( G'(y) \) measures the sensitivity of private value of equity \( G(y) \) with respect to cash flow \( y \). As expected, private value of equity \( G(y) \) increases with cash flow \( y \), i.e. \( G'(y) > 0 \).

Insert Figure 1 here.

Now we comment on the curvature of private value of firm \( G(y) \). In the standard Black-Scholes-Merton option pricing framework, equity value is increasing and convex in cash flow \( y \) (see also our Proposition ?? and the dash-dotted lines in the top panels of Figure 1). When the risk averse entrepreneur cannot fully diversify his project’s idiosyncratic risk, the global convexity of \( G(y) \) no longer holds. Unlike the complete-market model (Leland (1994)), the entrepreneur has precautionary saving demand to partially buffer the project’s non-diversifiable idiosyncratic cash flow shocks. This precautionary saving effect is greater, when the idiosyncratic cash flow volatility \( \epsilon y \) is larger (i.e. either \( \epsilon \) or cash flow \( y \) is larger). Moreover, the option (convexity) effect is smaller, when cash flow \( y \) is higher (i.e. the default option is further out of the money.) Therefore, the precautionary saving effect dominates the option effect for sufficiently high cash flow \( y \), making \( G(y) \) concave in cash flow \( y \). When cash flow \( y \) is low, idiosyncratic cash flow volatility \( \epsilon y \) is low. Moreover, the default option is closer to be in the money. The convexity effect of the default option dominates the concavity effect of precautionary saving demand, making \( G(y) \) convex in \( y \) for sufficiently low \( y \). This explains the convexity of \( G(y) \) for low \( y \) and concavity of \( G(y) \) for high \( y \), as shown in the upper panels.

Now we analyze private value of firm \( S(y) \), the sum of private value of equity \( G(y) \) and public
debt value $F(y)$. For an investor to acquire the entrepreneurial firm, he at least needs to pay $G(y)$ to the entrepreneur and $F(y)$ to the lender. That is, $S(y)$ is the minimal amount that the firm’s claimants require to sell the firm. Naturally, private value of firm $S(y)$ increases with cash flow $y$. Now turn to $S''(y)$, the second derivative of $S(y)$. First, recall that ex ante firm value, the sum of equity and debt, is increasing and concave in cash flow $y$ in the complete-market model (i.e. Leland (1994)). The dash-dotted line depicts the monotonicity and the global concavity of firm value in cash flow $y$. The concavity of firm value follows from the fact that liquidation option is costly for the entrepreneur ex ante. The firm is long the unlevered investment project, long tax shields and short a costly liquidation option. The net (short) position in default option makes firm value concave in cash flow $y$. When markets are incomplete, the risk-averse entrepreneur is also long a diversification option unlike the complete-market benchmark model. The entrepreneur captures his diversification benefits by defaulting if the cash flow $y$ is sufficiently low. The additional diversification benefits makes private value of firm $S(y)$ potentially convex for small values of $y$, precisely when the diversification option is deep in the money. For sufficiently high values of $y$, both the diversification and the liquidation options are out of the money. Therefore, private value of firm $S(y)$ approaches to the linear function as in the complete-market benchmark. The lower two panels demonstrate the non-concavity of private value of firm $S(y)$.

Now we turn to the effect of the entrepreneur’s risk aversion $\gamma$ on his subjective valuation and default threshold $y_d$. The more risk-averse entrepreneur attaches a lower private value $S(y)$ of the whole firm (for bearing non-diversifiable idiosyncratic risks), and moreover, has a stronger incentive to diversify by retaining less of the firm, contributing to a lower private value of equity $G(y)$. There are two opposing effects on the default threshold $y_d$. On the one hand, the more risk-averse entrepreneur has stronger incentives to issue more debt, which in turn calls for a larger coupon $b$ and hence a higher default threshold, ceteris paribus. On the other hand, the more risk-averse entrepreneur is more concerned with the downside risk and hence defaults earlier given the same level of coupon $b$, which reduces the distance to default, mitigates the entrepreneur’s ability to issue outside risky debt, and giving rise to a lower coupon, ceteris paribus. Our numerical analysis indicates that the diversification effect dominates the latter effect, making the default threshold $y_d$ increasing in risk aversion $\gamma$. Figure 1 shows these three monotonic comparative statics results. Next, we analyze the impact of non-diversifiable idiosyncratic risks on the entrepreneurial firm’s capital structure.
Capital structure for entrepreneurial firms. To highlight the role of idiosyncratic risks in a simplest possible way, we first consider the special case where debt has no tax benefits (i.e. $\tau = 0$), and later incorporate the tax benefits of debt into or analysis.

The top panel in Table 1 provides results for the entrepreneurial firm’s capital structure when debt has no tax benefits (i.e. $\tau = 0$). For entrepreneurial firms that have been incurring net operating losses for a sufficiently long period, the effective tax rate is zero. If the entrepreneur demands no idiosyncratic risk premium (in the complete-market benchmark labelled as “public” in the top panel of Table 1), the standard tradeoff theory implies that the entrepreneurial firm will be purely financed by equity. The entrepreneur values the firm at its market value 33.33. Now consider the effect of the risk-averse entrepreneur’s demand for the idiosyncratic risk premium. For $\gamma = 0.5$, the entrepreneur borrows $F_0 = 10.67$ in present value (with coupon $b = 0.59$) from the lender and values his non-tradable equity $G_0$ at 11.32, giving the private value of firm $S_0 = 21.99$. Note the substantial 34% drop of $S_0$ from 33.33 to 21.99. This significant drop of $S_0$ is not due to the default risk premium of the risky debt, since the 10-year cumulative default probability is only 2% and the implied credit spread on the perpetual debt is 52 basis points. Instead, the significant drop of $S_0$ reflects the entrepreneur’s idiosyncratic risk exposure on his subjective valuation discount of his non-tradable equity position. For entrepreneurial firms, the natural measure of leverage is private leverage $L_0$, which is given by the ratio of public debt value $F_0$ and private value of firm $S_0$. As we have discussed, $L_0$ captures the entrepreneur’s tradeoff between private value of equity and public value of debt in choosing debt coupon policy. For $\gamma = 0.5$, private leverage is about 49%, implying that the entrepreneur sells half of his firm to the lender.

Insert Table 1 here.

With a higher risk aversion ($\gamma = 1$), the entrepreneur borrows more ($F_0 = 18.95$) with a higher coupon ($b = 1.38$)\footnote{This result does not always hold. We will return to this point when we analyze the setting with tax benefits ($\tau = 20\%$).} The entrepreneur values his remaining non-tradable equity with a much smaller value $G_0 = 1.27$. The implied private leverage $L_0 = 94\%$, substantially higher than 49%, the level of $L_0$ for $\gamma = 0.5$. The more risk-averse entrepreneur takes on more (private) leverage. This result is consistent with the diversification benefits argument. The more risk-averse entrepreneur has stronger incentives to sell more of the firm. The high leverage (i.e. 94%) gives
rise to a significantly higher credit spread (226 basis points over the risk-free rate), and a much
higher 10-year cumulative default probability (50%), making debt non-investment grade. Despite
the substantially higher default risk, the private value of firm $S_0$ is 20.22, only about 8% lower than
21.99, the value for $\gamma = 0.5$. We reconcile this result by noting the natural lower bound for $S_0$.
Since the entrepreneur can sell his firm to the lender and default immediately, the lender recovers
the liquidation value in exchange of his fund: $R(y_0) = \alpha y_0 / (r - \nu) = 20$, which is the lower bound
of private value of firm $S_0 = 20$ (for entrepreneurs with $\gamma = \infty$).

Now we incorporate the effect of tax benefits into our generalized tradeoff model of capital
structure for entrepreneurial firms. The lower panel in Table 1 provides results for the case with
$\tau = 20\%$. The first row in this panel gives the results for the complete-market model. The public
firm issues perpetual risky debt valued at $F_0 = 15.85$ with coupon $b = 0.95$, which gives 55% time-0
leverage and 5% 10-year cumulative default probability. Consider the case with $\gamma = 0.5$. While the
entrepreneur chooses a slightly higher coupon ($b = 0.97$) than the complete-market level ($b = 0.95$),
the debt value is 15.25, lower than 15.85, the corresponding complete-market level. The intuition
for this seemingly counter-intuitive result is as follows. While non-diversifiable idiosyncratic risk
gives the entrepreneur incentives to issue more debt (diversification benefits), it also lowers the
entrepreneur’s valuation of the firm and induces the entrepreneur to default earlier, ceteris paribus.
The latter effect reduces the distance to default and hence lowers debt value for a given coupon,
which in turn limits the entrepreneur’s ex ante ability to raise debt. Therefore, the amount of debt
$F_0$ depends on both diversification and distance-to-default incentives.

The robust results for the entrepreneurial firm with respect to risk aversion $\gamma$ are as follows:
The more risk-averse entrepreneur has a lower private value of firm $S_0$, a lower private value of
equity $G_0$ after debt issuance, chooses a higher (private) leverage $L_0$, issues debt with a larger
credit spread and higher default probability. Intuitively, outside (public) debt is relatively more
attractive for the more risk-averse entrepreneur because his motive to hedge his idiosyncratic risk
exposure is greater, ceteris paribus.

Now we turn to the effect of taxes on debt issuance and capital structure. The higher the tax
rate, the greater the tax benefits of debt as in standard tradeoff models of capital structure. On
the other hand, a higher tax rate lowers the volatility of the after-tax cash flow, hence reduces the
entrepreneur’s precautionary saving demand$^{12}$ and lowers diversification benefits. (If $\tau = 100\%$,}

$^{12}$See Kimball and Mankiw (1989) on how taxes reduce precautionary saving demand in incomplete-markets self
the volatility of after-tax cash flow is zero and hence the entrepreneur has no precautionary saving demand, since the entrepreneur receives nothing permanently regardless of $y!$ Therefore, taxes have two opposing effects: tax shields and diversification benefits. Table 1 shows that both coupon $b$ and private leverage $L_0$ are higher under $\tau = 20\%$ than $\tau = 0$ for the case with $\gamma = 0.5$, but the opposite holds for the case with $\gamma = 1$. The results make intuitive sense. Tax shields are more important for low to moderate risk aversion (i.e. $\gamma = 0.5$), and the reduction of diversification effects due to a higher tax rate is more significant for high risk aversion (i.e. $\gamma = 1$).

To understand the impact of non-diversifiable idiosyncratic risk on leverage, we next analyze the determinants of leverage.

**Decomposition of private leverage for entrepreneurial firms.** Table 2 decomposes the effect of the entrepreneur’s non-diversifiable idiosyncratic risk exposure on leverage. Recall our previous analysis for the firm with risk aversion $\gamma = 1$. The entrepreneur chooses coupon $b = 1.05$, and ex post defaults if cash flow $y$ falls below $y_d = 0.51$. The implied private leverage $L_0$, computed as the ratio between private value of equity $G_0$ and private value of firm $S_0$ by using (7), is 82%. Given coupon $b$ and default threshold $y_d$, we may calculate the implied market value of equity $E_0 = E(y_0, b, y_d)$ and the market value of firm $V_0 = V(y_0, b, y_d)$ using valuation formulae (7) and (8), respectively. The imputed (public) leverage for the entrepreneurial firm is given by the ratio between market value of debt $F_0$ and market value of firm $V_0$. Since $E(y; b, y_d) > G(y; b, y_d)$, the imputed public leverage overstates the value of equity for the entrepreneur by ignoring the idiosyncratic risk premium ($E_0 = 11.19$ and $G_0 = 3.34$), thus leading to a leverage ratio 58%, substantially lower than the firm’s private leverage $L_0 = 82\%$. The difference between the private and public leverages (82% versus 58%) highlights the economic significance of taking idiosyncratic risks into account in order to correctly interpret the determinants of leverage for entrepreneurial firms.

**Insert Table 2 here.**

To highlight the impact of the entrepreneur’s endogenous default decision on the leverage ratio, consider a public firm (held by diversified investors) that has the same technology/environment parameters as the entrepreneurial firm. Moreover, the public firm has the same debt coupon $b$ insurance models.
on the outstanding perpetual debt as the entrepreneurial firm does \((b = 1.05\) in this example). The key is that default threshold \(y_d\) is endogenously determined. The default threshold \(y_d\) for the public firm is 0.36, significantly lower than the threshold \(y_d\) for the entrepreneurial firm \((y_d = 0.51)\). Intuitively, facing the same coupon \(b\), the entrepreneurial firm defaults earlier than the public firm because defaults allows the entrepreneur to avoid the downside non-diversifiable idiosyncratic risk. The implied shorter distance-to-default for the entrepreneurial firm translates into a higher 10-year default probability (21% for the entrepreneurial firm versus 7% for the public firm) and a higher credit spread (169.09 basis points for the entrepreneurial firm versus 114.96 basis points for the public firm). Despite the significant difference in the default thresholds, the leverage for the public firm (evaluated at the optimal default threshold \(y_d = 0.36\)) is 59%, only slightly higher than 58%, the leverage for the entrepreneurial firm. Based on this decomposition, the difference between the (private) leverage for the entrepreneurial firm \((L_0 = 82\%)\) and the (public) leverage for the public firm \((i.e. 58\%)\) primarily comes from the effect of risk aversion, not from the default risk premium. While this result is parameter specific, the analysis provides some support consistent with our intuition that the entrepreneur’s subjective valuation discount for bearing non-diversifiable idiosyncratic risk is a key determinant of private leverage for the entrepreneurial firm.

**Comparative statics of coupon \(b\) and leverage \(L_0\).** First, we summarize the results under the complete-markets setting. Both coupon \(b\) and leverage \(L_0\) increase with asset recovery rate \(\alpha\), because liquidation is less costly. Without idiosyncratic risks \((\epsilon = 0)\), markets are effectively complete and the entrepreneur bears no idiosyncratic risk and hence has no subjective valuation discount. Both the optimal coupon \(b\) and the private leverage \(L_0\) are independent of the entrepreneur’s risk aversion \(\gamma\) \((i.e. b = 1.18\) and \(L_0 = 0.74\) in the right two panels). Now consider the complete-markets comparative static result with respect to idiosyncratic volatility parameter \(\epsilon\). Increasing idiosyncratic risk lowers leverage. The firm \textit{ex ante} is short a liquidation option, whose value increases with idiosyncratic volatility. Therefore increasing volatility lowers \textit{ex ante} firm value and hence the firm’s ability to lever. While leverage is monotonically decreasing in idiosyncratic volatility parameter \(\epsilon\), coupon \(b\) is not monotonically decreasing in \(\epsilon\) even when markets are complete. The intuition is as follows. After debt is in place, the higher the idiosyncratic volatility parameter \(\epsilon\), the greater the \textit{ex post} option value of default and the lower the default threshold, implying a longer distance to default, a lower default probability, and hence more capacity to issue debt with a higher
coupon $b$. The bottom right panel shows that coupon $b$ slowly increases with $\epsilon$ for sufficiently high $\epsilon$.

**Insert Figure 2 here.**

Now turn to the effect of risk aversion $\gamma$ on coupon $b$ and private leverage $L_0$ when markets are incomplete. Non-diversifiable idiosyncratic risks have two additional effects on the risk-averse entrepreneur’s issuance of debt. The larger the idiosyncratic volatility parameter $\epsilon$, the stronger the entrepreneur’s hedging demand, increasing debt issuance for diversification benefits, but the entrepreneurial firm also has stronger incentives to default to avoid downside risk *ceteris paribus*, hence constraining the entrepreneur’s ability to issue debt (distance-to-default effect).

Figure 2 captures these two additional opposing effects of the entrepreneur’s risk aversion and precautionary saving motive. The left two panels plot the comparative static results for coupon $b$ and private leverage $L_0$ with respect to asset recovery rate $\alpha$. A lower recovery rate $\alpha$ implies a more costly liquidation, making the risky debt more costly as a hedging device and hence a weaker entrepreneur’s hedging demand for risky debt. Coupon $b$ and private leverage $L_0$ increase with recovery rate $\alpha$, *ceteris paribus*. The effect of risk aversion $\gamma$ on coupon $b$ differs depending on the level of recovery rate $\alpha$. When $\alpha$ is high, the distance-to-default effect is less important because liquidation is less costly. Therefore, diversification benefits dominate, making the more risk-averse entrepreneur choose a higher coupon $b$. For sufficiently low recovery rate $\alpha$, the opposite holds and hence the more risk-averse entrepreneur chooses a lower coupon $b$. This explains that coupon $b$ is non-monotonic in risk aversion $\gamma$. However, private leverage $L_0$ is increasing in risk aversion $\gamma$ for any recovery rate $\alpha$. This is due to the fact that the risky outside debt is (relatively) more attractive to the more risk averse entrepreneur (after controlling for the level of private value of firm $S_0$).

The right two panels of Figure 2 plot the comparative static results for coupon $b$ and private leverage $L_0$ with respect to the project’s idiosyncratic volatility parameter $\epsilon$. The entrepreneur’s precautionary saving demand induces him to increase $L_0$ for diversification benefits. The higher the idiosyncratic volatility $\epsilon$, the stronger diversification benefits. Recall that leverage for public firm decreases with $\epsilon$ under complete markets. Therefore, two opposing forces determine the private leverage $L_0$ for the entrepreneurial firm. For sufficiently high risk aversion (i.e. $\gamma = 1$), diversification benefits dominate, making $L_0$ increase with $\epsilon$. For moderate risk aversion (i.e. $\gamma = 0.5$),
private leverage $L_0$ decreases with $\epsilon$ first (i.e. the complete-market effect dominates when $\epsilon$ is low) and then increases with $\epsilon$ (i.e. diversification benefits dominate when $\epsilon$ is high enough), giving rise to a non-monotonic relation between $L_0$ and $\epsilon$. While $L_0$ is not monotonic in $\epsilon$, it is monotonically increasing in risk aversion $\gamma$ for a given level of $\epsilon$. Intuitively, the more risk-averse entrepreneur takes on more leverage for the net diversification benefits.

Note that coupon $b$ is monotonic in neither $\epsilon$ nor risk aversion $\gamma$. For sufficiently high $\epsilon$, diversification benefits dominate, which makes coupon $b$ increase with both idiosyncratic volatility $\epsilon$ and risk aversion $\gamma$. For sufficiently low $\epsilon$, increasing $\epsilon$ lowers the firm’s capacity to issue debt (due to the firm’s ex ante short position in the liquidation option), which makes coupon $b$ decrease in $\epsilon$. The more risk-averse entrepreneur has stronger incentives to default and hence a shorter distance to default, making coupon $b$ decrease in risk aversion $\gamma$ (for low levels of $\epsilon$). The decreasing relation of coupon $b$ in risk aversion $\gamma$ is opposite of the (increasing) relation of private leverage $L_0$ for low levels of $\epsilon$.

Finally, note that leverage approaches 100%, when recovery rate $\alpha$ or idiosyncratic volatility parameter $\epsilon$ is sufficiently high (i.e. diversification benefits dominate). Selling the whole firm to the lender (and then immediately default) becomes optimal.

5 Cash-out as an additional channel for diversification

We now turn to a richer and more realistic setting where the entrepreneur can exit from his business and diversify away from the idiosyncratic risk by either exercising the default or the cash-out option. As in Section 3.1, the entrepreneur avoids the downside risk by defaulting if the firm’s (stochastic) cash flow $y$ falls sufficiently low. When the firm does well enough, the entrepreneur capitalizes on the upside by selling the firm to diversified investors.

Private value of equity $G(y)$ and private value of firm $S(y)$. The upper left and upper right panels of Figure 5 plot private value of equity $G(y)$ and its first derivative $G'(y)$ for two levels of the fixed cash-out cost ($K = 50, 100$). For sufficiently low values of cash flow $y$, private value of equity $G(y)$ is increasing and convex because the default option is deep in the money, generating convexity. For sufficiently high values of $y$, $G(y)$ is also increasing and convex because the cash-out option is deep in the money. For cash flow $y$ in the intermediate range, neither default nor cash-out option is deep in the money, moreover, precautionary saving motive may be large enough to induce
concavity. The higher the fixed cash-out cost $K$, the lower level of cash flow $y$ is required for the cash-out option to be deep in the money. Therefore, the convexity of $G(y)$ is more significant as we see from the upper right panel.

Insert Figure 3 here.

The lower left and lower right panels of Figure 3 plot private value of firm $S(y)$ and its first derivative $S'(y)$. As for the setting with the default option only, $S(y)$ may be convex for small values of $y$ due to the fact that the diversification option is deep in the money (for sufficiently high risk aversion $\gamma$, i.e. $\gamma = 1$). Unlike the setting with the default option only, private value of firm $S(y)$ is also convex for sufficiently high value of $y$ due to the option to cash out. In the intermediate range, $S(y)$ may be concave due to the entrepreneur’s precautionary saving motive. Therefore, there is no global convexity/concavity for private value of firm $S(y)$. Intuitively, the higher the fixed cash-out cost $K$, the lower the cash-out boundary $y_u$ and the lower the private value of firm $S(y)$/the private value of equity $G(y)$.

Next we turn to the incremental effect of the cash-out option on the entrepreneurial firm’s capital structure.

Capital structure for entrepreneurial firms with both default and cash-out options. First, consider the complete-market benchmark. With no tax benefits ($\tau = 0$) and no diversification benefits (under complete markets), the cash-out option is worthless. Hence, the results for the first row “public” (with $\tau = 0$) is identical to the corresponding first row in Table 1 for the complete-market setting where the firm only has the default option. When the public firm has tax benefits ($\tau = 20\%$), the firm’s cash-out option is really an option to adjust the firm’s capital structure since there is no diversification benefit. To focus on the economics of the cash-out option as a diversification channel, we set the fixed cash-out cost $K = 10$, which makes the 10-year cash-out probability zero and hence this (upper boundary exit) option value is close to zero for the public firm.\footnote{Firm value is 29.14, slightly higher than 28.83 (the value without the cash-out option). Leverage is 55% compared with 54% (with the default option only). Credit spreads are both equal to 98 basis points (with or without the cash-out options).} Therefore, we may interpret the bulk part of the cash-out option value for entrepreneurial firms comes from the diversification benefits, not from the option value of readjusting leverage.

Insert Table 3 here.
Now consider an entrepreneurial firm with risk aversion $\gamma = 0.5$, but with no tax benefits ($\tau = 0$). The presence of the cash-out option substantially lowers the initial coupon to $b = 0.04$ from $b = 0.59$ for the firm which only has the default option. Private leverage $L_0$ at issuance is only 3% with a credit spread at tiny 1.7 basis points (compared to private leverage $L_0 = 49\%$ and credit spread 52 basis points when the firm only has the default option). The 10-year default probability is thus essentially zero, but the 10-year cash-out probability is 62%, which is economically significant. (Recall that the 10-year cash-out probability for a public firm is zero.) For higher risk aversion ($\gamma = 1$), the private leverage is only 41%, significantly smaller than 94% for the setting with the default option only. Note the significant substitution effect between risky debt and the cash-out option as channels to diversify the entrepreneur’s idiosyncratic risk. With the cash-out option, the entrepreneur substitute away from the risky debt towards the cash-out option to diversify his idiosyncratic risk. With $\tau = 0$, the entrepreneur issues no debt when and after exercising the cash-out option. Hence, debt issuance for the entrepreneurial firm is purely driven by diversification benefits.

Now we introduce tax benefits into the model ($\tau = 20\%$). For $\gamma = 0.5$, the cash-out option lowers the initial coupon to $b = 0.85$ from $b = 0.97$ for the firm which only has the default option. Private leverage $L_0$ at issuance is 57% with a credit spread 134 basis points (compared to $L_0 = 74\%$ and a credit spread at 137 basis points when the firm only has the default option). For higher risk aversion ($\gamma = 1$), the private leverage is 60%, and initial coupon $b = 9.87$, respectively smaller than $L_0 = 82\%$ and $b = 1.05$ for the setting with the default option only, but credit spreads are almost the same (168 bps versus 169 bps (with default option only)). While the cash-out option lowers the 10-year default probability from 21% to 7%, the significant 10-year cash-out probability (70%) and the call-back feature of the debt at the cash-out boundary make the credit spreads of the risky debt comparable with or without the cash-out option. The table provides compelling results that the cash-out option is a less effective substitute for the risky debt for diversification purposes when debt has tax benefits.

**Economic values of debt financing and the cash-out option.** We now derive the incremental value of debt financing and the cash-out option for the entrepreneur. First, we construct a benchmark where the entrepreneur has neither the option to borrow nor the cash-out option. In this benchmark model, the entrepreneur effectively operates a machine generating a stochastic
stream of exogenously specified cash-flow, which naturally fits into a classic self-insurance (income-fluctuation) problem (with an added feature of portfolio choice as in Merton (1971)). The entrepreneur behaves as a consumer endowed with a stochastic stream of exogenously specified cash flow with neither leverage nor the cash-out option. The entrepreneur’s consumption and portfolio rules are given by (14) and (15) in Proposition 1, respectively. Let $S^b_0$ denote the entrepreneur’s certainty equivalent value for this baseline model, which we is also the private value of firm, equivalently private value of equity $G_0$ in this no-debt case.

Having set up the benchmark model for comparison, we now calculate the value of debt financing for the entrepreneur. Recall the model where the entrepreneur finances his project with the perpetual defaultable debt, has limited liability, but has no cash-out option (Model 1). The certainty equivalent wealth for the entrepreneur is equal to the private value of firm $S^{(1)}_0$, which we report in Table 1. Intuitively, the incremental value of debt financing, calculated as $(S^{(1)}_0 - S^b_0)$, increases with risk aversion $\gamma$. Note that $(S^{(1)}_0 - S^b_0)$ also increases with the tax rate $\tau$. This result is less obvious. The tax rate has two opposing effects on the incremental value of debt financing $(S^{(1)}_0 - S^b_0)$. The higher the tax rate $\tau$, the greater the tax benefits, but the lower the entrepreneur’s precautionary saving demand due to the tax effect (Kimball and Mankiw (1989)). Our numerical results suggest that the positive tax shields effect dominate the negative precautionary saving effect, giving the increasing relation between $(S^{(1)}_0 - S^b_0)$ and the tax rate $\tau$. This seems intuitive, since tax shields is about the expected cash flows and reduction of precautionary saving is about the volatility of cash flows.

Similarly, we calculate the value of the cash-out option by solving the model where the entrepreneur finances his project with only inside equity/no debt, but has a cash-out option to diversify the project’s idiosyncratic risk (Model 2). The certainty equivalent wealth for the entrepreneur is equal to the private value of firm $S^{(2)}_0$. The value of the cash-out option is given by $(S^{(2)}_0 - S^b_0)$. Intuitively, diversification benefits are greater for more risk-averse entrepreneurs. Note that the cash-out option value decreases in the tax rate $\tau$. For example, for $\gamma = 0.5$, this option value decreases by 0.6 from 4.26 to 3.80 when increasing the tax rate $\tau$ from zero to 20%.

Intuitively, the lower the tax rate, the less volatile the after-tax cash flow and hence the weaker the

---


15The new boundary conditions include the transversality condition $\lim_{T \to \infty} E \left[ e^{-\delta T} J^* (w_T, y_T) \right] = 0$ and the absorbing barrier condition $G(0) = 0$ for the GBM cash flow process.
entrepreneur’s precautionary saving demand. Therefore, cash outing as a channel for diversification is less valuable, the higher the tax rate $\tau$. This reduction of precautionary saving demand is greater for more risk-averse entrepreneurs. (Note that the cash-out option value decreases by 0.83 from 5.04 to 4.21 for $\gamma = 1$, when increasing the tax rate from zero to 20%.) Intuitively, precautionary saving demand is more important for more risk-averse entrepreneurs.

Finally, we calculate the value of having both debt financing and the cash-out option, the model in the previous section (we refer to it in this section as Model 3). The certainty equivalent wealth for the entrepreneur is equal to the private value of firm $S_0^{(3)}$ is reported in Table 3. Naturally, the value of having both debt financing and the cash-out option is measured by $(S_0^{(3)} - S_0^b)$. First, recall the substitution effect between debt financing and the cash-out option for diversification. This substitution effect is captured by the reduction of the value added when debt financing and the cash-out option are combined, i.e. $(S_0^{(3)} - S_0^b) < (S_0^{(1)} - S_0^b) + (S_0^{(2)} - S_0^b)$. For example, for $\gamma = 1$ and $\tau = 20\%$, the value of debt financing is 3.00, the value of the cash-out option is 4.21, while the value of both debt financing and the cash-out option 5.72, implying that the value of having both debt and the cash-out option is lower than the value of having two separate options by $3.00 + 4.21 - 5.72 = 1.49$, a significant reduction reflecting the substitution effect. Second, the incremental value $(S_0^{(3)} - S_0^b)$ increases with risk aversion $\gamma$ because diversification benefits are greater for more risk-averse entrepreneurs. Third, the incremental value $(S_0^{(3)} - S_0^b)$ increases with the tax rate $\tau$. As we have noted, increasing the tax rate lowers the after-tax cash-flow volatility and hence reduces the precautionary saving demand (Kimball and Mankiw (1989)). As a result, both the cash-out option and the default option values (for diversification purposes) are lower. But tax shields channel dominate the precautionary saving channel, making $(S_0^{(3)} - S_0^b)$ increasing with $\tau$.

Insert Table 4 here.

**Comparative statics of coupon $b$ and leverage $L_0$: both the cash-out and default options.** First, we describe the results under the complete-markets setting. Both coupon $b$ and leverage $L_0$ increase with the fixed cash-out cost $K$. Without idiosyncratic risk premium, the cash-out option is effectively an option to readjust the firm’s leverage. The higher the fixed cost $K$ is, the longer the expected time the firm takes to readjust its leverage. In order to capture the
tax benefits of debt, the firm will choose a higher coupon $b$ and a higher leverage $L_0$. When $K$ is sufficiently high, coupon $b$ and leverage asymptotically approaches to 0.94 and 54%, respectively. The 10-year cash-out and default probabilities become 0% and 4% with $K$ approaching $\infty$.

Now consider the entrepreneurial firm with $\gamma = 1$. The firm has two channels to diversify the entrepreneur’s idiosyncratic risk: cash-out or default. The higher the fixed cash-out cost $K$ is, the more important the risky debt/default channel is and hence greater incentives to take on debt. Therefore, coupon $b$ and leverage $L_0$ are higher for entrepreneurial firms with higher cash-out costs $K$. Now consider the alternative scenario. The lower the fixed cash-out cost $K$ is, the more attractive the cash-out option is. The entrepreneur rationally keeps the cash-out option higher by issuing less debt, *ceteris paribus* because he needs to call back the debt at par if he cashes out. The more debt he issues at time 0, the higher “strike” price he needs to pay to exercise the cash-out option and hence a lower cash-out option value. This explains the single-crossing in coupon $b$ and private leverage $L_0$. Not surprisingly, the 10-year default probability for the entrepreneurial firm reflects the similar properties as the one for the private leverage as a function of the fixed cash-out cost $K$. Note that the 10-year cash-out probability for the entrepreneurial firm is always higher than the one for the otherwise identical public firm. Intuitively, a significant portion of the cash-out option value comes from diversification benefits, which do not apply to the public firm.

*Insert Figure 4 here.*

**Idiosyncratic and systematic risk premia.** To understand the impact of idiosyncratic volatility $\epsilon$ on the risk premium that the entrepreneur demands, we decompose the entrepreneur’s risk premium into two components: the systematic risk premium $\pi^s(y)$ and the idiosyncratic risk premium component $\pi^i(y)$. To motivate our analysis, we rewrite the valuation ODE (16) for the entrepreneur’s private value of equity $G(y)$ as follows:

$$
\pi^s(y) + \pi^i(y) = \frac{(1 - \tau)(y - b)}{G(y)} + \frac{1}{G(y)} \left( \mu y G'(y) + \frac{\sigma^2 y^2}{2} G''(y) \right) - r, \quad (26)
$$

where the systematic risk premium $\pi^s(y)$ and the idiosyncratic risk premium $\pi^i(y)$ are respectively given by

$$
\pi^s(y) = \eta \omega \frac{G'(y)}{G(y)} y = \eta \omega \frac{d \log G(y)}{d \log y}, \quad (27)
$$

$$
\pi^i(y) = \frac{\gamma r (\epsilon y G'(y))^2}{2 G(y)}. \quad (28)
$$
The first term and the second term on the right side of (26) measure the current yield and the expected percentage change for private value of equity $G(y)$. Analogous to the standard asset pricing models, we may interpret the sum of the two terms as the total expected return for private value of equity $G(y)$. Subtracting the risk-free rate from the expected return gives the risk premium for $G(y)$, which includes both the systematic component $\pi_s(y)$ and the idiosyncratic component $\pi_i(y)$. As in standard valuation models, systematic risk premium $\pi_s(y)$ is given by the product of (market) Sharpe ratio $\eta$, systematic volatility $\omega$, and the elasticity of $G(y)$ with respect to cash flow $y$. Unlike standard valuation models, $G(y)$ is private value of equity and the entrepreneur’s non-diversifiable idiosyncratic risk commands a risk premium which is given by (28). Naturally, the idiosyncratic risk premium $\pi_i(y)$ increases with the idiosyncratic volatility of the cash flow $\epsilon y$ and $G'(y)$, which gives the sensitivity of $G(y)$ with respect to cash flow $y$. The quadratic specification comes from the fact that the idiosyncratic risk premium is linked to the precautionary saving demand, which is proportional to the conditional (idiosyncratic) variance.

Insert Figure 5 here.

First consider the behavior of risk premia at the default boundary. The entrepreneur’s equity is a levered position in the firm. When the firm approaches default, the systematic component of the risk premium $\pi_s(y)$ behaves similarly to the standard valuation model. That is, the significant leverage effect around the default boundary implies that the risk premium diverges to infinity when $y$ approaches $y_d$. The idiosyncratic risk premium $\pi_i(y)$ behaves quite differently. To highlight the intuition for the idiosyncratic risk premium $\pi_i(y)$ at the default boundary, consider the following approximation at the default boundary:

$$\pi_i(y_d) = \lim_{y \to y_d} \frac{\gamma r (\epsilon y G'(y))^2}{2 G(y)} \approx \lim_{y \to y_d} \frac{\gamma r (\epsilon y)^2 2 G''(y_d) G'(y_d) (y - y_d)}{G'(y_d) (y - y_d)} = \gamma r (\epsilon y_d) G''(y_d).$$

(29)

Using $G''(y_d)$ implied by Proposition (1), we may write $\pi_i(y_d) = -2(1 - \tau)(y_d - b)\epsilon^2/\sigma^2$, which is always positive (the option value of default (i.e. $y_d < b$) and finite. That is, the quadratic dependence of the idiosyncratic risk premium $\pi_i(y)$ on $G'(y)$ makes its value finite at the default boundary $y_d$. Intuitively, diversification benefits are realized at default. Moreover, when $y$ is close to the default boundary, the fixed cash-out cost $K$ has little impact on both systematic and idiosyncratic risk premia. Finally, the idiosyncratic risk premium $\pi_i(y)$ increases with cash flow $y$ (for sufficiently high $y$) reflecting the effect of the cash-out option.
6 Endogenous Project Choice, Risk Shifting

To be completed.

7 Conclusion

We develop an intertemporal model of entrepreneurial finance and investment. Unlike publicly held firms, the entrepreneurs face significant non-diversifiable idiosyncratic risk. We show that more risk-averse entrepreneurs have lower debt capacity since they default earlier, yet still choose a higher leverage. Cash-out option is an alternative channel for diversification, which reduces the attractiveness of leverage. We also characterize the idiosyncratic risk premium for entrepreneurs, which behaves quite differently compared to the systematic risk premium. Finally, we show that entrepreneur risk aversion dominates risk-seeking incentives: only those entrepreneurs with very low risk aversion will engage in asset substitution.

Our model can also be easily adapted to address the impact of un-diversified executives’ decisions on the firm’s capital structure and investment decision. We do not model the fundamental frictions causing markets to be incomplete. While we view endogenous incomplete markets as a complementary perspective, this issue can have fundamental impacts on questions such as promotion of entrepreneurship and contract design. We leave these important questions for future research.
Appendices

A  Market Valuation and Capital Structure of a Public Firm

Well-diversified owners of a public firm face complete markets. Given the Sharp ratio $\eta$ of the market portfolio and the riskfree rate $r$, there exists a unique stochastic discount factor (SDF) $(\xi_t : t \geq 0)$ satisfying (see Duffie (2001)):

$$d\xi_t = -r\xi_t dt - \eta\xi_t dB_t, \quad \xi_0 = 1. \quad (A.1)$$

Using this SDF, we can derive the market value of the unlevered firm, $A(y)$, the market value of equity, $E(y)$, and the market value of debt $D(y)$. The market value of the firm is equal to the sum of equity value and debt value:

$$V(y) = E(y) + D(y). \quad (A.2)$$

By the Girsanov theorem, under the risk-neutral probability measure $Q$, the standard Brownian motion $B^Q_s$ satisfies $dB^Q_t = dB_t + \eta dt$ (see Duffie (2001)). We then rewrite the dynamics of the cash flows (2) as follows:

$$dy_t = \nu y_t dt + \omega y_t dB^Q_t + \epsilon y_t dZ_t, \quad (A.3)$$

where $\nu$ is the risk-adjusted drift defined by $\nu \equiv \mu - \omega \eta$.

Using (A.3), we can derive valuation equations for firm securities by the standard asset pricing theory. We start with before-tax unlevered firm value $A(y)$. It satisfies the following differential equation:

$$rA(y) = y - w + \nu y A'(y) + \frac{1}{2} \sigma^2 y^2 A''(y). \quad (A.4)$$

This is a second-order ordinary differential equation (ODE). We need two boundary conditions to obtain a solution. One boundary condition describes the behavior of $A(y)$ when $y \to \infty$. This condition must rule out speculative bubbles. To ensure $A(y)$ is finite, we assume $r > \nu$ throughout the paper. The other boundary condition is related to abandonment. As in the standard option exercise models, the firm is abandoned whenever the cash flow process hits a threshold value $y_a$ for the first time. At the threshold $y_a$, the following value-matching condition is satisfied

$$A(y_a) = 0, \quad (A.5)$$
because we normalize the outside value to zero. For the abandonment threshold $y_a$ to be optimal, the following smooth-pasting condition must also be satisfied

$$A' (y_a) = 0.$$  \hspace{1cm} (A.6)

Solving equation \((A.4)\) and using the no-bubble condition and boundary conditions \((A.5)-(A.6)\), we obtain equation \((4)\) and

$$y_a = \frac{r - \nu}{r} \frac{\theta_1}{\theta_1 - 1} w,$$  \hspace{1cm} (A.7)

where

$$\theta_1 = -\sigma^{-2} (\nu - \sigma^2/2) - \sqrt{\sigma^{-4} (\nu - \sigma^2/2)^2 + 2r\sigma^{-2}} < 0.$$  \hspace{1cm} (A.8)

We next turn to the valuation of levered firms. First, consider the market value of equity. Let $T_d$ denote the random default time and $y_d$ be the corresponding default threshold. After default, equity is worthless, in that $E(y) = 0$ for $y \leq y_d$. This gives us the value matching condition $E(y_d) = 0$. Before default, equity value $E(y)$ satisfies the following differential equation:

$$rE(y) = (1 - \tau_2) (y - b - w) + \nu y E'(y) + \frac{1}{2} \sigma^2 y^2 E''(y), \quad y \geq y_d.$$  \hspace{1cm} (A.9)

When $y \to \infty$, $E(y)$ also satisfies a no-bubble condition. Solving this ODE and using the boundary conditions, we obtain equation \((1)\). Using the smooth-pasting condition,

$$\frac{\partial E(y; y_d)}{\partial y} \big|_{y=y_d} = 0,$$

we obtain the optimal default threshold:

$$y_d = \frac{r - \nu}{r} \frac{\theta_1}{\theta_1 - 1} (b + w).$$  \hspace{1cm} (A.10)

Similarly, the market value of debt before default satisfies the following differential equation:

$$rD(y) = b + \nu y D'(y) + \frac{1}{2} \sigma^2 y^2 D''(y), \quad y \geq y_d.$$  \hspace{1cm} (A.11)

The value-matching condition is given by:

$$D(y_d) = \alpha (1 - \tau_2) A(y_d).$$  \hspace{1cm} (A.12)

We also imposes a no bubble condition when $y \to \infty$. Solving yields:

$$D(y; y_d) = \frac{b}{r} - \left[ \frac{b}{r} - \alpha (1 - \tau_2) A(y_d) \right] \left( \frac{y}{y_d} \right)^{\theta_1},$$  \hspace{1cm} (A.13)
Using equation (A.2), we can derive equation (7). Substituting (A.10) into (7) and using the first-order condition:

\[
\frac{\partial V(y_0)}{\partial b} = 0,
\]  

(A.14)

we can derive the optimal coupon rate \( b^* \) as a function of \( y_0 \). In general there is no closed form solution. When \( w = 0 \), we can derive an explicit expression:

\[
b^* = y_0 \frac{r}{r - \nu} \left( \frac{\theta_1 - 1}{\theta_1} \left( 1 - \theta_1 - \frac{1 - \alpha}{\tau_2} \theta_1 \right) \right)^{1/\theta_1}.
\]  

(A.15)

We can also check the second order condition is satisfied. Substituting (A.10) and (A.15) into (7), we obtain equation (8). Q.E.D.

### B Proof of Theorem 1

Using a standard argument, \( J^s(x, y) \) satisfies the following HJB equation:

\[
\begin{align*}
\delta J^s(x, y) &= \max_{c, \pi} \left( uc + (rx + \pi (\mu_e - r) - c + (1 - \tau_1) (y - b)) J^s_x(x, y) + \mu y J^s_y(x, y) \\
&\quad + \frac{(\sigma_e \pi)^2}{2} J^s_{xx}(x, y) + \frac{\sigma_e^2 y^2}{2} J^s_{yy}(x, y) + \pi \sigma_e \omega y J^s_{xy}(x, y) \right),
\end{align*}
\]  

(B.1)

We follow Merton (1969) to conjecture that the value function \( J^s(x, y) \) is exponential in wealth and is given by equation (13), where \( G(y) \) is a function to be determined. Using the first-order conditions,

\[
U'(c) = J^s_x(x, y), \quad \pi = \frac{-J^s_x(x, y) \mu_e - r}{J^s_{xx}(x, y)} + \frac{-J^s_{xy}(x, y) \omega y}{J^s_{xx}(x, y)} \sigma_e,
\]  

(B.2)

we can derive the optimal consumption rule (14) and the portfolio rule (15). Substituting these expressions back into the HJB equation (B.1) gives the differential equation (16) for \( G(y) \).

We now consider boundary conditions. First, consider the lower default boundary. At the instant of default, the entrepreneur walks away from his firm’s liability and the lender liquidates the firm’s asset. The entrepreneur’s financial wealth \( x \) does not change immediately after default, in that \( x_{T_d} = x_{T_d-} \). In addition, the entrepreneur’s value function should remain unchanged at the moment of default. That is, the following value-matching condition holds along the default boundary as in standard option exercising problems:

\[
J^s(x, y) = J^e(x).
\]  

(B.3)
The above equation implicitly defines the lower default boundary for cash flow $y$ as a function of wealth $x$: $y = y_d(x)$. Note that in general, the default boundary depends on the entrepreneur’s wealth level. Because the default boundary is optimally chosen, the following smooth-pasting conditions at $y = y_d(x)$ must be satisfied:

\[
\frac{\partial J^s(x, y)}{\partial x} = \frac{\partial J^e(x)}{\partial x}, \quad (B.4)
\]

\[
\frac{\partial J^s(x, y)}{\partial y} = \frac{\partial J^e(x)}{\partial y}. \quad (B.5)
\]

The first smooth-pasting condition (B.4) states that the marginal change in cash flow $y$ has the same marginal effect on the entrepreneur’s value functions just before and immediately after defaulting on the firm’s liability. Similarly, the second smooth-pasting (B.5) states that the marginal effect of wealth must be the same on the entrepreneur’s value functions just before and immediately after defaulting on the firm’s liability. Unlike the complete-markets endogenous default models (Leland (1994)), the entrepreneur’s financial wealth enters as an additional state variable, which gives rise to the second smooth-pasting condition.

We next turn to the upper cash-out boundary. At the instant of cashing out, the entrepreneur sells his firm to well diversified investors and collects firm value $V^*(y)$ given in (7). Since the entrepreneur needs to pay the fixed cost $K(y)$, retire debt at par $F_0$, and pay capital gains taxes, his wealth $x_{T_u}$ immediately after cashing out satisfies $x_{T_u} = x_{T_u} + V^*(y_{T_u}) - F_0 - K(y_{T_u}) - \tau_g(V^*(y_{T_u}) - K - I)$. Similar to the lower default boundary, the entrepreneur’s value function must satisfy the following value-matching condition:

\[
J^s(x, y) = J^e(x + V^*(y) - F_0 - K(y) - \tau_g(V^*(y) - K(y) - I)). \quad (B.6)
\]

This equation implicitly determines the upper cash-out boundary $y_u(x)$. Using the same arguments as those for the lower default boundary, the entrepreneur’s optimality implies the following smooth-pasting conditions at $y = y_u(x)$:

\[
\frac{\partial J^s(x, y)}{\partial x} = \frac{\partial J^e(x + V^*(y) - F_0 - K(y) - \tau_g(V^*(y) - K(y) - I))}{\partial x}, \quad (B.7)
\]

\[
\frac{\partial J^s(x, y)}{\partial y} = \frac{\partial J^e(x + V^*(y) - F_0 - K(y) - \tau_g(V^*(y) - K(y) - I))}{\partial y}. \quad (B.8)
\]

Given the conjectured form (13), the default and cash-out boundaries $y_d(x)$ and $y_u(x)$ are independent of wealth. We thus simply use $y_d$ and $y_u$ to denote the default and cash-out thresholds.

\[^{16}\text{See, for example, Krylov (1980), Dumas (1991) and Dixit and Pindyck (1994).}\]
respectively. Using the value matching and smooth pasting conditions (B.3)-(B.5) at $y_d$, we can derive equations (17) and (18). Using the value matching and smooth pasting conditions (B.6)-(B.8) at $y_u$, we can derive equations (19) and (B.5). Q.E.D.

C Market Value of the Entrepreneurial Firm’s Debt

When the entrepreneur does not default or cash out, the market value of his debt $F(y)$ satisfies the following ODE:

$$rF(y) = b + 
u y F'(y) + \frac{1}{2} \sigma^2 y^2 F''(y), \quad y_d \leq y \leq y_u.$$  \hfill (C.1)

At the default trigger $y_d$, debt recovers a fraction of after-tax unlevered firm value and thus the following value-matching condition holds,

$$F(y_d) = \alpha (1 - \tau_2) A(y_d).$$  \hfill (C.2)

At the selling trigger $y_u$, debt is retired and recovers its face value. Thus, the following value-matching condition holds

$$F(y_u) = F_0.$$  \hfill (C.3)

Solving equation (C.1) subject to the boundary conditions (C.2) and (C.3) yields:

$$F(y) = \frac{b}{r} + \left( F_0 - \frac{b}{r} \right) \overline{q}(y) + \left[ \alpha (1 - \tau_2) A(y_d) - \frac{b}{r} \right] q(y),$$  \hfill (C.4)

where

$$\overline{q}(y) = \frac{y^\theta_1 y^\theta_2 - y^\theta_2 y_d}{y^\theta_1 y_d - y^\theta_2 y_d},$$  \hfill (C.5)

$$q(y) = \frac{y^\theta_2 y^\theta_1 - y^\theta_1 y^\theta_2}{y^\theta_1 y_d - y^\theta_2 y_d}. $$  \hfill (C.6)

Here, $\theta_1$ is given by (A.8) and

$$\theta_2 = -\sigma^{-2} (\nu - \sigma^2/2) + \sqrt{\sigma^{-4} (\nu - \sigma^2/2)^2 + 2r\sigma^{-2}} > 1.$$  \hfill (C.7)

Equation (C.4) admits an intuitive interpretation. It states that debt value is equal to the present value of coupon payment plus the changes in value when default occurs and when cash-out occurs. Note that $\overline{q}(y_0)$ can be interpreted as the present value of a $1 if cash-out occurs before default, and $q(y_0)$ can be interpreted as the present value of a $1 if the entrepreneur goes bankrupt before cash-out. Q.E.D.
Table 1: Capital Structure of Entrepreneurial Firms: Default Option Only

This table reports the results for the setting where the entrepreneur only has default option to exit from his project. The parameters are: \( r = 0.05, \eta = 0.4, \mu = 0.06, \omega = 0.1, \varepsilon = 0.2, \) and \( \alpha = 0.6. \) These parameters are annualized when applicable. The initial cash flow is \( y_0 = 1. \) We report results for two tax rates \((\tau = 0, 20\%)\) and two levels of risk aversion \((\gamma = 0.5, 1).\) The case “Public” corresponds to the complete-market model.

<table>
<thead>
<tr>
<th>( \tau = 0 )</th>
<th>( \tau = 0.2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma = 0.5 )</td>
<td>( \gamma = 0.5 )</td>
</tr>
<tr>
<td>( \gamma = 0.5 )</td>
<td>( \gamma = 1.0 )</td>
</tr>
<tr>
<td>( \text{Public} )</td>
<td>( \text{Public} )</td>
</tr>
<tr>
<td>( b )</td>
<td>( F_0 )</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.59</td>
<td>10.67</td>
</tr>
<tr>
<td>1.38</td>
<td>18.95</td>
</tr>
<tr>
<td>0.95</td>
<td>15.85</td>
</tr>
<tr>
<td>0.97</td>
<td>15.25</td>
</tr>
<tr>
<td>1.05</td>
<td>15.63</td>
</tr>
</tbody>
</table>
Table 2: Decomposition of Private Leverage for Entrepreneurial Firms

This table compares a private firm owned by a risk averse entrepreneur with a public firm that has the same amount of debt outstanding (coupon is fixed at $b = 1.05$). There is no option to cash out. The model parameters are: $r = 0.05$, $\eta = 0.4$, $\mu = 0.06$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$ and $\tau = 0.2$. These parameters are annualized when applicable. All the results are for initial cash flow $y_0 = 1$.

<table>
<thead>
<tr>
<th>$\gamma = 1.0$</th>
<th>0.51</th>
<th>0.21</th>
<th>15.63</th>
<th>3.34</th>
<th>11.19</th>
<th>0.82</th>
<th>0.58</th>
<th>169.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public, $b = 1.05$</td>
<td>0.36</td>
<td>0.07</td>
<td>17.01</td>
<td>-</td>
<td>11.79</td>
<td>-</td>
<td>0.59</td>
<td>114.96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>default threshold</th>
<th>10-yr default probability</th>
<th>public debt</th>
<th>equity private</th>
<th>equity public</th>
<th>leverage private</th>
<th>leverage public</th>
<th>credit spread (bp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_d$</td>
<td>$p_d(10)$</td>
<td>$F_0$</td>
<td>$G_0$</td>
<td>$E_0$</td>
<td>$L_0$</td>
<td>$\frac{F_0}{F_0+E_0}$</td>
<td>$CS$</td>
</tr>
</tbody>
</table>
Table 3: Capital Structure of Entrepreneurial Firms: Both Default and Cash-out Options

This table reports the results for the setting where the entrepreneur has both default and cash-out options to exit from his project. The parameters are: \( r = 0.05, \eta = 0.4, \mu = 0.06, \omega = 0.1, \varepsilon = 0.2, \alpha = 0.6, \) and \( K = 10. \) These parameters are annualized when applicable. The initial cash flow is \( y_0 = 1. \) We report results for two tax rates (\( \tau = 0, 20\% \)) and two levels of risk aversion (\( \gamma = 0.5, 1. \)). The case “Public” corresponds to the complete-market model, where the “cash-out” option effectively allows the firm to adjust leverage once (by paying a fixed cost \( K \)).

<table>
<thead>
<tr>
<th>coupon</th>
<th>public debt</th>
<th>private equity</th>
<th>private firm leverage</th>
<th>credit spread (bp)</th>
<th>10-yr default probability</th>
<th>10-yr cash-out probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td>( F_0 )</td>
<td>( G_0 )</td>
<td>( S_0 )</td>
<td>( L_0 )</td>
<td>( CS )</td>
<td>( p_d(10) )</td>
</tr>
<tr>
<td>Public</td>
<td>0.00</td>
<td>0.00</td>
<td>33.33</td>
<td>33.33</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>( \gamma = 0.5 )</td>
<td>0.04</td>
<td>0.80</td>
<td>25.38</td>
<td>26.18</td>
<td>0.03</td>
<td>1.70</td>
</tr>
<tr>
<td>( \gamma = 1.0 )</td>
<td>0.59</td>
<td>10.11</td>
<td>14.26</td>
<td>24.37</td>
<td>0.41</td>
<td>88.00</td>
</tr>
<tr>
<td>Public</td>
<td>0.94</td>
<td>15.73</td>
<td>13.41</td>
<td>29.14</td>
<td>0.54</td>
<td>97.98</td>
</tr>
<tr>
<td>( \gamma = 0.5 )</td>
<td>0.85</td>
<td>13.40</td>
<td>10.09</td>
<td>23.49</td>
<td>0.57</td>
<td>133.56</td>
</tr>
<tr>
<td>( \gamma = 1.0 )</td>
<td>0.87</td>
<td>13.07</td>
<td>8.62</td>
<td>21.69</td>
<td>0.60</td>
<td>168.35</td>
</tr>
</tbody>
</table>
Table 4: Net Benefits of Debt and Cash-out Options for Entrepreneurial Firms

This table reports the net benefits for the entrepreneurial firm to issue debt and/or have a cash-out option. The baseline model is the one without debt and has no cash-out option. Let $S^b_0$ denote the time-0 private value of firm for this baseline (self insurance) incomplete-markets model. Model 1 allows for debt but no cash-out option; Model 2 allows for the cash-out option but no debt; Model 3 allows for both debt and cash-out option. Let $S^{(n)}_0$ denote the private values of firm, also the entrepreneur’s certainty equivalent wealth for Model $n = 1, 2, 3$. The benefits for the three models are defined as $(S^{(n)}_0 - S^b_0)$. Naturally, the policies (such as coupon and cash-out policies) are optimized within each model and hence differ across these models. The parameters are: $r = 0.05$, $\eta = 0.4$, $\mu = 0.06$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, and $K = 10$. The initial cash flow is $y_0 = 1$.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\tau = 0$</th>
<th>$\tau = 0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>debt</td>
<td>cash-out option</td>
</tr>
<tr>
<td>$\gamma = 0.5$</td>
<td>0.27</td>
<td>4.26</td>
</tr>
<tr>
<td>$\gamma = 1.0$</td>
<td>1.13</td>
<td>5.04</td>
</tr>
</tbody>
</table>
Figure 1: **Private value of equity* $G(y)$ and private value of firm* $S(y)$ as functions of cash flow $y$: the case of default option only.** The top two panels plot $G(y)$ and its first derivative $G'(y)$. The bottom two panels plot $S(y)$ and its first derivative $S'(y)$. In each case, we plot the results for two levels of risk aversion ($\gamma = 0.5, 1$) alongside the benchmark complete-market solution. The rest of the model parameters are: $r = 0.05$, $\eta = 0.4$, $\mu = 0.06$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, and $\tau = 0.2$. 


Figure 2: Comparative statics – optimal coupon and private leverage with respect to asset recovery rates $\alpha$ and idiosyncratic volatilities $\epsilon$: the case of default option only. The top two panels plot the optimal coupon $b$ at $y_0 = 1$. The bottom two panels plot the corresponding optimal private leverage $L_0$. In each case, we plot the results for two levels of risk aversion ($\gamma = 0.5, 1$) alongside the benchmark complete-market solution. The rest of the model parameters are: $r = 0.05$, $\eta = 0.4$, $\mu = 0.06$, $\omega = 0.1$, $\epsilon = 0.2$ (when changing $\alpha$), $\alpha = 0.6$ (when changing $\epsilon$), and $\tau = 0.2$. 

43
Figure 3: Private value of equity $G(y)$ and private value of firm $S(y)$ as functions of cash flow $y$: the case with both default and cash-out options. The top two panels plot $G(y)$ and its first derivative $G'(y)$. The bottom two panels plot $S(y)$ and its first derivative $S'(y)$. We plot the results for two levels of cash-out costs ($K = 50, 100$). The remaining parameters are: $r = 0.05, \eta = 0.4, \mu = 0.06, \omega = 0.1, \varepsilon = 0.2, \alpha = 0.6, \tau = 0.2,$ and $\gamma = 1$. 


Figure 4: **Coupon $b$, private leverage $L_0$, and cash-out/default probabilities as functions of $K$, the fixed cash-out cost.** The top two panels plot optimal coupon $b$ and private leverage $L_0$ as functions of the fixed cash-out cost $K$. The bottom two panels plot 10-year cash-out and default probabilities as functions of $K$. We compare the results of an entrepreneurial firm (with $\gamma = 1$) to a public firm, for which the cash-out option is equivalent to an option to adjust leverage. The remaining parameters are: $r = 0.05$, $\eta = 0.4$, $\mu = 0.06$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, and $\tau = 0.2$. 
Figure 5: **Systematic and idiosyncratic risk premia as functions of cash flow $y$: the case with both default and cash-out options.** This figure plots the systematic and idiosyncratic risk premia that the entrepreneur demands for holding the inside equity of a private firm. We plot the results for two levels of cash-out costs ($K = 50, 100$). The remaining parameters are: $r = 0.05$, $\eta = 0.4$, $\mu = 0.06$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, $\tau = 0.2$, and $\gamma = 1$. 
References


